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THEORETICAL MECHANICS. STATICS

INDEPENDENT AND INDIVIDUAL WORK OF STUDENTS

Міністерство освіти і науки України
Вінницький національний технічний університет

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THEORETICAL MECHANICS.
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Електронний навчальний посібник
комбінованого (локального та мережного) використання

Вінниця
ВНТУ
2024

УДК 531.2(075.8)=111
К99

Рекомендовано до видання Вченою Радою Вінницького національного технічного університету Міністерства освіти і науки України (протокол № 12 від 30 травня 2024 р.).

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Кириця, І. Ю.
К99 Теоретична механіка. Статика. Самостійна та індивідуальна робота студентів : електронний навчальний посібник комбінованого (локального та мережного) використання [Електронний ресурс] / І. Ю. Кириця, О. В. Грушко. – Вінниця : ВНТУ, 2024. – 79 с.

В навчальному посібнику наведено основні теоретичні відомості з дисципліни «Теоретична механіка», а саме: розділу «Статика». Розглянуто приклади використання теоретичного матеріалу для розв’язання задач. До кожного підрозділу наведено завдання для самостійного опрацювання студентами (комплекти завдань). Кожне з завдань ілюстроване детальним прикладом розрахунку з поясненнями при розв’язуванні задачі.

Посібник призначений як для україномовних студентів, так і для іноземних здобувачів спеціальностей 131 «Прикладна механіка», 132 «Матеріалознавство», 133 «Галузеве машинобудування», 274 «Автомобільний транспорт», 275 «Транспортні технології», 192 «Будівництво та цивільна інженерія», 144 «Теплоенергетика», 141 «Електроенергетика, електротехніка, електромеханіка» закладів вищої освіти, що вивчають теоретичну механіку.

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INTRODUCTION

Theoretical mechanics studies the most general laws of movement and interaction of bodies, considering its main task to know the quantitative and qualitative regularities observed in nature. It belongs to the fundamental natural sciences, since natural science studies various forms of movement of matter.

Theoretical mechanics is of great importance in the training of engineering personnel. It is the foundation for studying such disciplines as resistance of materials, theory of vibrations, hydraulics, theory of elasticity, aero- and hydromechanics, electrodynamics, biomechanics, theory of automatic control of moving objects, theory of mechanisms and machines, devices, manipulator robots. Knowledge of the laws of theoretical mechanics makes it possible to scientifically predict the course of processes in new tasks that arise during the development of science, engineering and technology.

Theoretical mechanics is a science that provides universal methods for compiling and analyzing the equations of motion and equilibrium of complex material systems, which is the basis of their modeling.

Theoretical mechanics relies on knowledge of analytic geometry, vector algebra, mathematical analysis, physics, and computer science. The first explanations of the general concepts of mechanics are contained in the works of the ancient Greek philosopher Aristotle (384–322 BC), who considered the solution of practical problems with the help of a lever. For the first time, the scientific justification of mechanics appears in the work of the Syracuse geometer and mechanic Archimedes (287–212 BC), who made an attempt to axiomatize mechanics (statics), gave a number of scientific generalizations related to the doctrine of equilibrium, the center of gravity and hydrostatics (Archimedes' law).

Theoretical mechanics is based on Newton's laws, which is why it is called *Newtonian* or *classical*. The laws of theoretical mechanics were formulated thanks to the productive work of many generations of scientists.

According to the nature of the problems being studied, theoretical mechanics consists of three sections:

- **statics**, in which methods of equivalent transformations of force systems are studied, as well as conditions of equilibrium of material bodies;
- **kinematics**, in which the mechanical movement of material bodies is studied from a geometric point of view, that is, regardless of masses and forces acting on them;
- **dynamics**, in which the movement of material bodies under the action of forces is studied.

In addition to these three sections, elements of **analytical mechanics** are also studied in theoretical mechanics, which is a set of the most generalized analytical methods for solving mechanics problems, which allow not only to solve dynamics problems in the same way, but also to spread them to such fields as classical theory fields and quantum mechanics.

1 BASIC CONCEPTS OF THEORETICAL MECHANICS

Mechanical movement is the simplest form of movement of matter, which is reduced to the simple movement of physical bodies over time from one position in space to another. While studying the movement of material bodies, turning away from everything partial, theoretical mechanics considers only those properties that are decisive in this problem. This leads to consideration of various models of material bodies, which represent one or another level of abstraction. The main abstractions of theoretical mechanics include the concepts of a material point and an absolutely solid body.

A material point is a body whose dimensions can be neglected when solving certain problems, or a geometric point endowed with a certain mass. For example, with a close study of the movements of the planets, they can be considered as material points.

A system of material points is a collection of material points whose positions and movements are interconnected.

A body whose distance between any points does not change during equilibrium or motion is called **absolutely solid.**

Theoretical mechanics widely uses not only the method of abstractions, but also generalization, mathematical methods and methods of formal logic. The application of these methods and generalizations of the results of direct observations, production practice and experience made it possible to establish certain general laws that play the role of axioms. All further conclusions of theoretical mechanics can be obtained from these axioms with the help of logical reasoning and mathematical statements. At the same time, the reliability of the provisions of theoretical mechanics is verified by experiment and practice.

2 STATICS. BASIC DEFINITIONS OF THE SECTION OF STATICS

Statics is a branch of mechanics that studies the methods of force transformation and elucidates the conditions of equilibrium of bodies.

Real objects are replaced b y models – absolutely solid bodies, the distance between two points of which does not change.

The real interaction of bodies (objects) is replaced by a model, the mechanical interaction of bodies is replaced by force.

The main concepts of statics are a force, a system of forces, an absolutely solid body.

Force is a vector quantity that is a quantitative measure of mechanical interaction between material bodies. A force $\overline{F}$ as a vector is considered given if its modulus, direction and point of application are determined. The force can be specified graphically or analytically.

Graphically, the force is depicted as a vector with the definition of its modulus (the absolute value of the length of the vector) and direction. The direction is indicated by an arrow, and the length of the straight segment in the scale corresponds to the length of the vector. The module of a vector is denoted like the vector itself, but with letters of a regular font and without a dash above. The line along which the force is directed is called the line of action of the force. The basic unit of force is 1 newton (N). This is the force that gives a mass of 1 kg an acceleration of 1 m/s^2 (1H = 1 kg · 1 m/s^2 = 1 kg·m/s^2).

Analytically, the force is set by projections on the coordinate axis. Projection of the force on the axis is called a scalar value equal to the product of the modulus of the force by the cosine of the angle between the positive direction of the axis and the direction of the force, for example, on the «x» axis (Fig. 2.1). The projection is given a «+» sign if it forms an acute angle with the positive direction of the axis, and a «–» sign if it is obtuse.

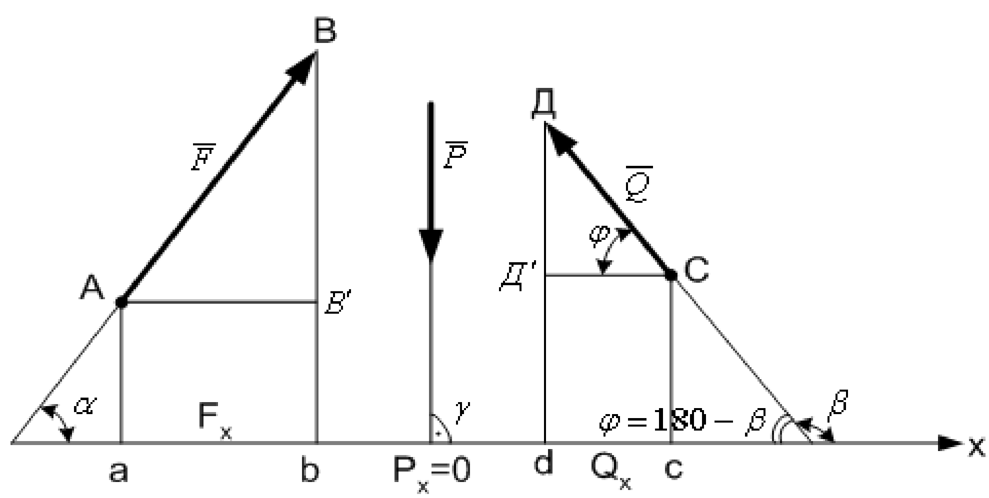


Figure 2.1

$$F_X = ab = F \cdot \cos\alpha = AB'; \; Q_X = cd = CD\cos\beta = -Q \cdot \cos\varphi$$

The projection of the force $\overline{P}$ onto the axis perpendicular to it is zero.

$$P_X = P \cdot \cos\gamma = P \cdot \cos 90^\circ = 0.$$

The projection of the force $\overline{F}$ on the plane $OXУ$ is called the vector F_{xy}, which is contained between the projections of the beginning and end of the force on this plane (Fig. 2.2).

Vector module $F_{xy} = F \cdot \cos\varphi$

$$\overline{F} = F_X \cdot \overline{i} + F_y \cdot \overline{j} + F_Z \cdot \overline{k},$$

where $\overline{i}, \overline{j}, \overline{k}$ – unit vectors directed along the axes;

F_x, F_y, F_z – projections of the force vector $\overline{F}$ on the corresponding axes.

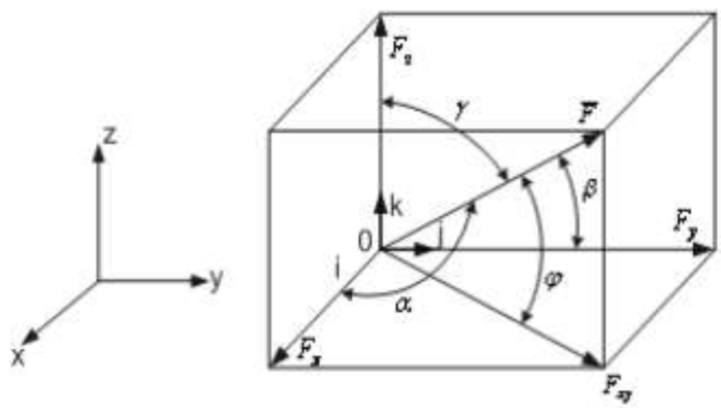


Figure 2.2

Projections of the force: $F_x = F \cdot \cos\alpha$; $F_y = F \cdot \cos\beta$; $F_Z = F \cdot \cos\gamma$.

Force module: $F = \sqrt{F_x^2 + F_y^2 + F_Z^2}$, its direction is the angle between the positive direction of the axis and the direction of the force

$$\cos\alpha = \frac{F_X}{F}; \ \cos\beta = \frac{F_y}{F}; \ \cos\gamma = \frac{F_Z}{F}$$ – direction cosines.

A system of forces is a set of forces acting on a solid body or material point. There are three systems of forces acting on a solid body in a plane and in space: a convergent system of forces, the lines of action of which intersect at one point; a parallel system of forces whose lines of action are parallel to each other; an arbitrary system of forces, the lines of action of which are not parallel to each other and do not all intersect at the same point.

An equivalent system of forces is one that can be used to replace the system of forces acting on a solid body without changing the nature of the motion or equilibrium. It is indicated by «≈».

One force equivalent to a given system of forces $(\overline{F}_1........\overline{F}_n)$ is called **equivalent** $\overline{R}$.

Balanced (or equivalent to zero) is a system of forces that keeps in balance the material point on which it acts $\left(\overline{F}_1,...,\overline{F}_n\right) \approx 0$. The balanced force is equal in magnitude and opposite in direction to the equivalent force $\overline{R}$.

The force that is applied to the body at a point is called **concentrated** (Fig. 2.3). The point of force application is the material part of the body to which this force is directly applied.

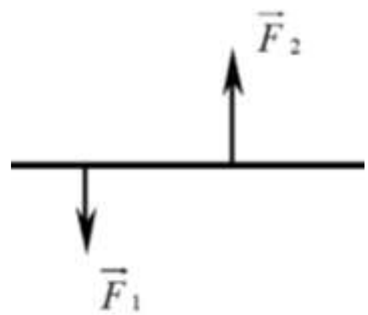


Figure 2.3

Forces acting on all points of length, surface or volume are called **distributed** (Fig. 2.4).

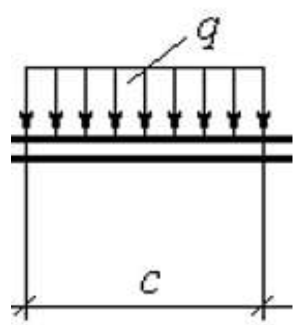


Figure 2.4

The magnitude of force per unit length, area, or volume is called intensity. Usually, the distributed force is denoted by the letter q, which has dimensions N/m, N/m^2, N/m^3, respectively. Examples of distributed forces are: the pressure of a cylindrical roller on the road surface; the pressure of the tram wheel on the rail; the pressure of snow on the roof; liquid pressure on the walls of pipelines, vessels, dams; forces of body weight, etc. The character of the action of the distributed forces is indicated by a graph (epura) (Fig. 2.5). In fig. 2.5 shows the graphs of uniform, triangular and arbitrary intensities of acting forces, respectively.

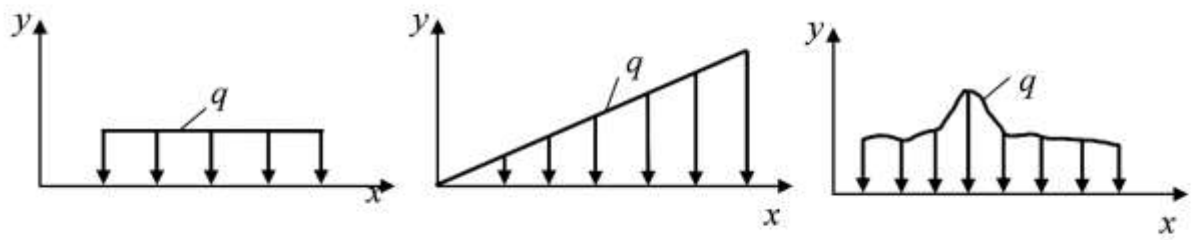


Figure 2.5

Forces acting on a body or mechanical system from material points or other bodies that are not part of this system are called **external forces.**

Interaction forces between points of one mechanical system are called **internal forces.**

Questions for self-testing

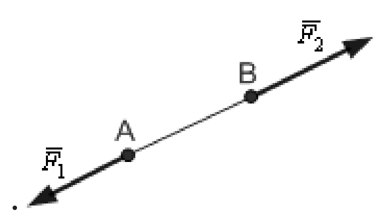
1. What does theoretical mechanics study?
2. The history of the development of theoretical mechanics.
3. What is called a material point and an absolutely solid body?
4. What is force?
5. How to determine the projection of the force on the axis, the force module and direction cosines?
6. What is an equivalent force, an equivalent system of forces?
7. What are the different systems of forces?

3 AXIOMS OF STATICS

Statics is based on a number of axioms, which are the result of generalizations of numerous experiments and observations on the balance and movement of bodies, repeatedly confirmed by practice. The axioms of statics are initial propositions of an experimental nature that are accepted without proof.

Axiom 1. About two forces

Two forces acting on a completely solid body are balanced if and only if they act along the same line in opposite directions and are equal in magnitude (Fig. 3.1)



$$\overline{F}_1 = -\overline{F}_2$$

$$|\overline{F}_1| = |\overline{F}_2|$$

Figure 3.1

This axiom defines the simplest balanced system of two forces, since experiments show that a free body, on which only one force acts, cannot be in equilibrium.

Axiom 2. Addition (subtraction) of a balanced system of forces

The action of a given system of forces on an absolutely solid body is not disturbed if a balanced system of forces is added or subtracted from it (Fig. 3.2).

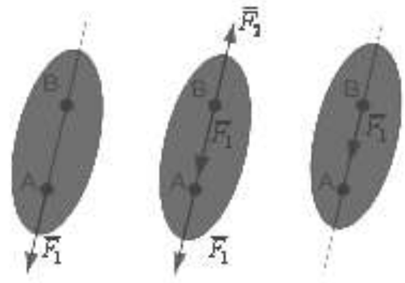
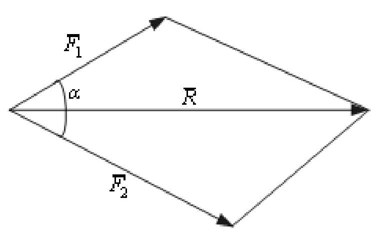


Figure 3.2

Axiom 3. About the parallelogram of forces

The equivalent of two forces applied to the body at one point is equal to the vector sum of these forces and applied at the same point (Fig. 3.3).



$$\overline{R} = \overline{F}_1 + \overline{F}_2$$

Figure 3.3

The modulus of the uniform force $\overline{R}$ is determined by the theorem of cosines, where α – the angle between the vectors $\overline{F}_1$ and $\overline{F}_2$.

At the same direction of forces $(\cos\alpha = 1)$ $R = F_1 + F_2$, and on the contrary $(\cos\alpha = -1)$ $R = F_1 - F_2$.

The direction of the equivalent action of two forces is determined by the diagonal of the parallelogram built on these forces.

Based on axiom 3, any number of forces applied at one point can be added geometrically. Equivalent forces are defined as the vector sum of these forces (Fig. 3.4).

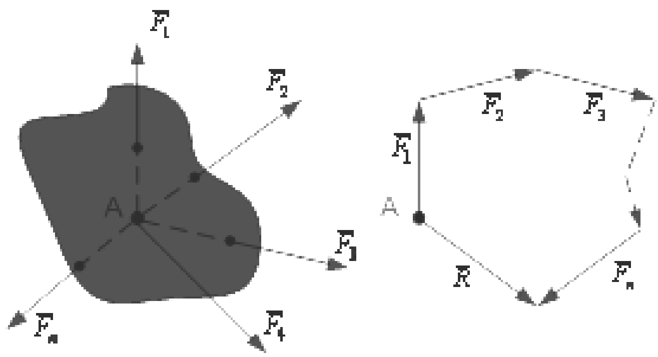


Figure 3.4

For this, from the end of the vector equal to the first force $\overline{F}_1$, we set aside the vector equal to the force $\overline{F}_2$, etc. Connecting the beginning of the first vector $\overline{F}_1$ with the end of the last one $\overline{F}_n$, we find the equivalent force

$$\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + ... + \overline{F}_n = \sum_{i=1}^{n} \overline{F}_i \ .$$

This polygon is called a **polygon of forces**.

Axiom 4. The law of action and counteraction (Newton's 3rd law)

At some action of one body on another, there is a counteraction, numerically equal, but opposite in direction (Fig. 3.5).

Action and counteraction forces $(\overline{F}_{12}, \overline{F}_{21})$ are equal in magnitude, act along the same line *aa* in the opposite direction, but are applied to different bodies. Therefore, the forces of action and counteraction are not balanced.

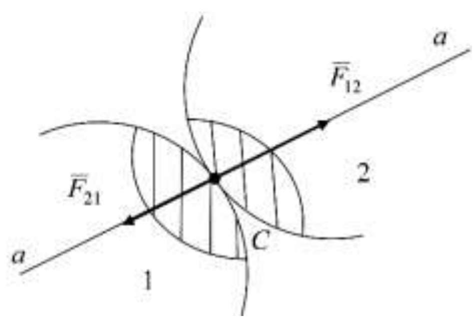


Figure 3.5

Axiom 5. The principle of solidification

The equilibrium of a deformed (changing) body, which is under the action of a given system of forces, is not disturbed if it is considered to be solidified (absolutely solid).

The meaning of the axiom is that when studying the movement of deformed bodies, the rules of theoretical mechanics obtained for solid bodies can be used.

Axiom 6. The principle of release from linkages

Without changing the mechanical state (motion or equilibrium) of a system of material points or a solid body, a linkage imposed on a system or a solid body can be discarded by replacing the action of the linkage with its reaction applied to this body or system at the point of interaction of the body and linkage.

On the basis of axiom 6, non-free material points, a system of material points, or a solid body can be considered free if they are freed from the linkages, replacing the action of the latter with their reactions.

4 LINKAGES. REACTIONS OF LINKAGES

A system of material points is called **free** if there are no restrictions on the movement of these points. In the opposite case, the system of material points is called **unfree.**

Bodies or a set of bodies that limit the movement of a given body or a given material system are called **linkages.**

According to axiom 6, a non-free material body can be considered as free if the linkages are replaced by reactions. Release from the linkages makes it possible to reduce the equilibrium of a non-free solid body to the corresponding question about the equilibrium of a free solid body, which is under the influence of external forces and reactions of the linkages at the same time.

The force with which the linkage acts on the body is called **the reaction of the linkage** and is directed in the direction opposite to that in which the linkage prevents the body from moving.

Forces that are not reactions of the linkage are called **active forces**.
The peculiarity of the active force is that its module and direction do not directly depend on other forces acting on the body. Reactions of linkages differ from active forces acting on the body in that their direction and magnitude always depend on these forces and are unknown in advance.

By their nature, linkages can be divided into **two classes.**

The first class includes linkages, the direction of reactions of which does not depend on the magnitude and direction of the active forces applied to the body in a state of equilibrium.

For example: flexible linkage (ropes, threads, chains) (Fig. 4.1); perfectly smooth surfaces (Fig. 4.2); ideal rods (Fig. 4.3).

Let's consider in more detail how the reactions of some main types of linkages are directed.

Flexible linkage (threads, ropes, cables, chains)

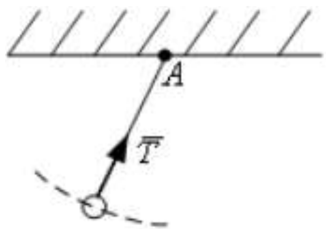


Figure 4.1

The reactions of the linkages of the cable, thread, and chain are directed along the cable, thread, and chain, and these bodies can only be stretched, that is, they counteract only the action of stretching.

The thread reaction is directed along the thread to its attachment point A and is denoted by $\overline{T}$ or $\overline{S}$.

In the problems of theoretical mechanics, it is assumed that the thread is weightless and flexible.

Perfectly smooth surfaces

The reaction of a perfectly smooth surface is directed along the normal to the surface and is denoted by $\overline{N}$ or $\overline{R_n}$.

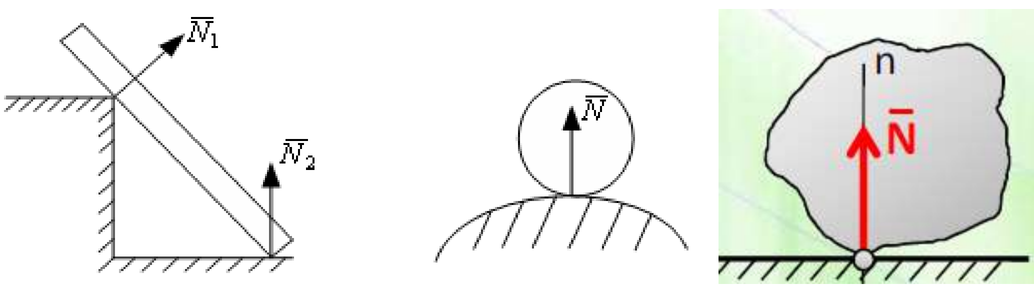


Figure 4.2

From the point of view of statics, such surfaces are called smooth, in which the reactions of the linkages at the point of contact with other bodies are directed along the general normal to the tangent surfaces.

Remark

In the case of ***a rough surface*** (Fig. 4.3), the reaction $\overline{R_A}$ is divided into two components

$\overline{R_n}$ – normal and $\overline{R_\tau}$ – tangential, directed along the tangent τ to the surface.

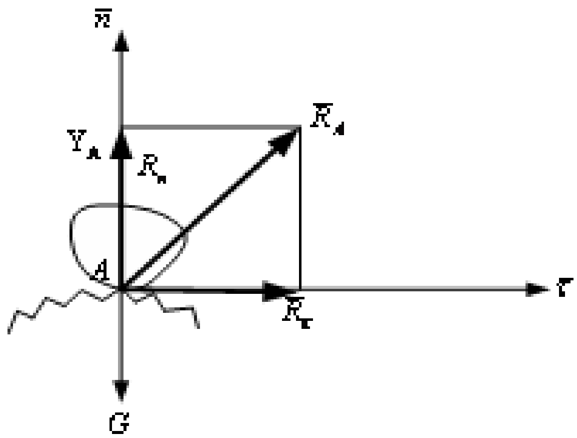


Figure 4.3

The tangential component of the reaction $\overline{R_\tau}$ is the force of friction. The force of friction is equal to $R_\tau \le f \cdot R_n$, where f is the coefficient of sliding friction.

An ideal (weightless rod) connecting two hinges A and B (Fig. 4.4).

The reaction is directed along the line connecting the hinges

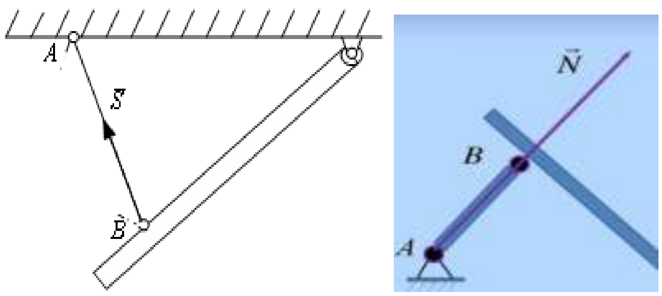
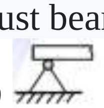


Figure 4.4

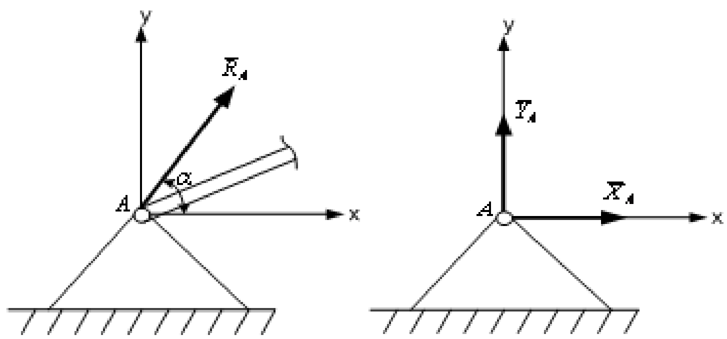
A weightless rod, to which no forces are applied (active and reaction linkages), is called **ideal**. The reactions of the linkage of an ideal rod are directed along the line connecting the beginning and end of the rod (Fig. 4.4), and the ideal rod can be compressed or stretched.

The second class includes linkages, the direction of reactions of which is completely determined by the direction and magnitude of active forces.

Such linkages are fixed (Fig. 4.5) and movable (Fig. 4.6) hinges (supports), spherical joint (Fig. 4.7), cantilever (Fig. 4.8), and thrust bearing (Fig. 4.9).

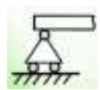
Fixed cylindrical hinge (bearing, fixed support) 

The direction of reactions of such linkages cannot be determined in advance. The unknown linkage reaction vector $\overline{R}_A$ in the plane is determined by two components $\overline{X}_A$ and $\overline{Y}_A$ along the axes OX and OY.



$$\overline{R}_A = \overline{Y}_A + \overline{X}_A$$

Figure 4.5

Movable hinge  

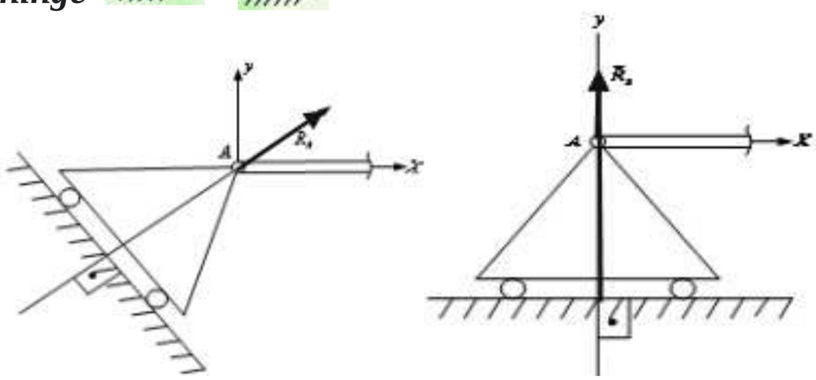
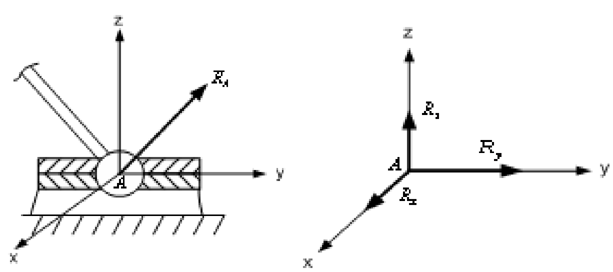


Figure 4.6

The reaction is directed perpendicular to support surface (plane).

Spherical joint. The reaction $\overline{R}_A$ of a spherical joint consists of its three projections on three coordinate axes $\overline{R}_X, \overline{R}_y, \overline{R}_Z$.



$$\overline{R}_A = \overline{R}_X + \overline{R}_y + \overline{R}_Z$$

Figure 4.7

Cantilever. The cantilever reaction consists of an equivalent force $\overline{R}_A$ ($\overline{R}_A = \overline{Y}_A + \overline{X}_A$) and a couple of forces with a moment $\overline{M}_A$ (three unknown quantities).

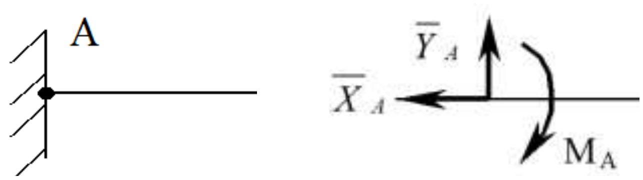
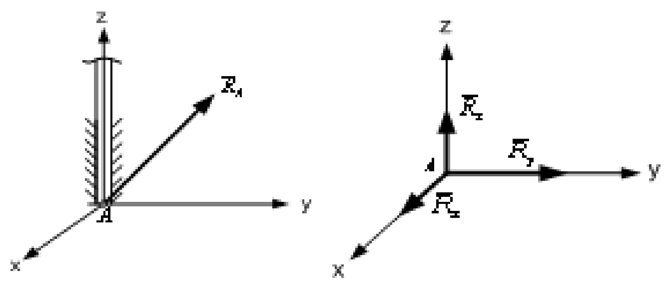


Figure 4.8

Thrust bearing (рис. 4.9)

Like the spherical joint, the thrust bearing has three spatial components: $\overline{R}_x, \overline{R}_y, \overline{R}_z$.



$$\overline{R}_A = \overline{R}_X + \overline{R}_y + \overline{R}_Z$$

Figure 4.9

Remark

A movable hinge is equivalent to one ideal rod; fixed cylindrical hinge – two ideal rods; cantilever – three ideal rods.

Questions for self-testing

1. Axiom about two forces.
2. Axiom of addition (subtraction) of a balanced system of forces.
3. Axiom about the parallelogram of forces.
4. The law of action and counteraction.
5. The principle of solidification.
6. The principle of freedom from linkages.
7. What is an linkage?
8. What is called a linkage reaction?
9. Types of the linkages and their reactions.

5 SIMPLEST THEOREMS OF STATICS

Theorem about force as a sliding vector. The action of a force on a solid body will not change if the force is transferred along the line of its action to any point (Fig. 5.1) (for example, from point A to point B).

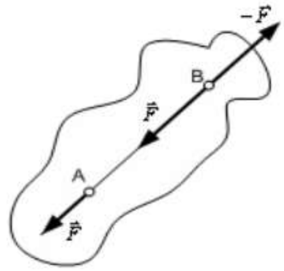


Figure 5.1

Theorem about three forces. If a completely solid body is in equilibrium under the action of three non-parallel forces $(\overline{F}_1, \overline{F}_2, \overline{F}_3)$ and the lines of action of the two forces $(\overline{F}_1, \overline{F}_2)$ intersect, then all forces lie in the same plane and their lines of action intersect at the same point p. O (Fig. 5.2).

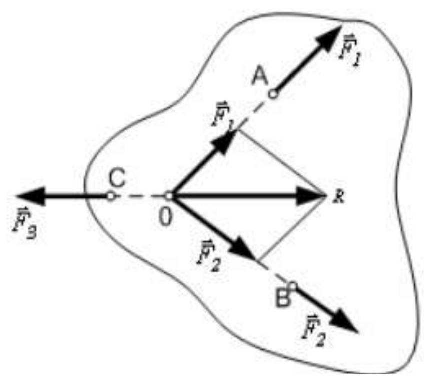


Figure 5.2

Theorem about the projection of an equivalent on an axis. The projection of the vector (geometric) sum onto the axis is equal to the algebraic sum of the projections of the components of the vector onto the same axis (Fig. 5.3).

$$\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4$$

$$R_x = F_{1x} + F_{2x} + F_{3x} + F_{4x} = oa + ab - bc + cd$$

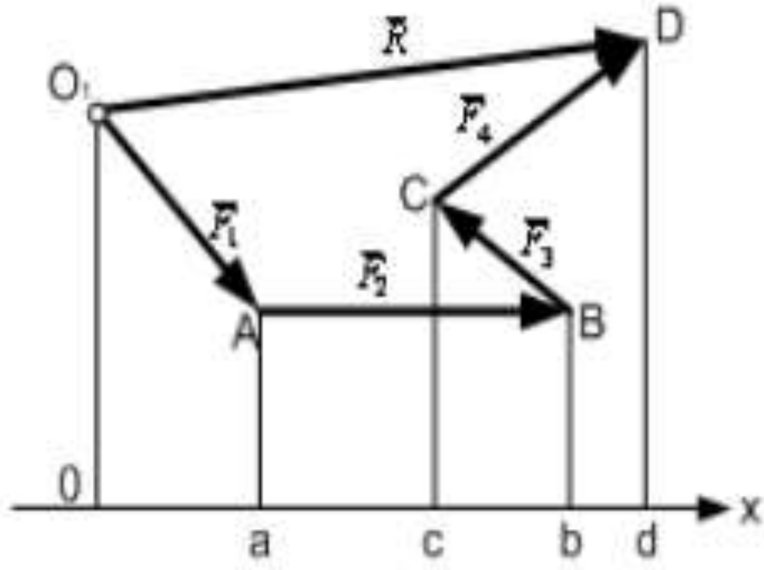


Figure 5.3

6 FORCE SYSTEMS

Force systems are:
1. A system of forces lying on the same straight line;
2. Plane parallel system of forces;
3. Plane convergent system of forces;
4. Plane arbitrary system of forces;
5. Spatial parallel system of forces;
6. Spatial convergent system of forces. Spatial arbitrary system of forces;
7. Plane system of force pairs;
8. Spatial system of force pairs.

6.1 System of convergent forces. Equilibrium conditions of the system of convergent forces

The simplest is the system of convergent forces. It can be spatial or plane (Fig. 6.1). In the latter case, all the lines of action of the forces of the system belong to one plane.

A system of converging forces is a system of forces whose lines of action intersect at one point (the point O of the convergence of forces).

One force that is equivalent to a given system of forces $(\overline{F}_1........\overline{F}_n)$ is called **equivalent** $\overline{R}$

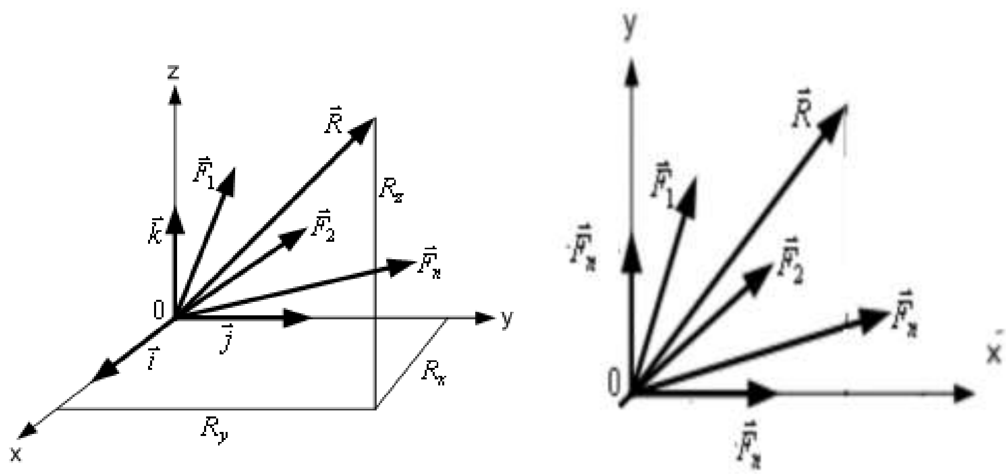
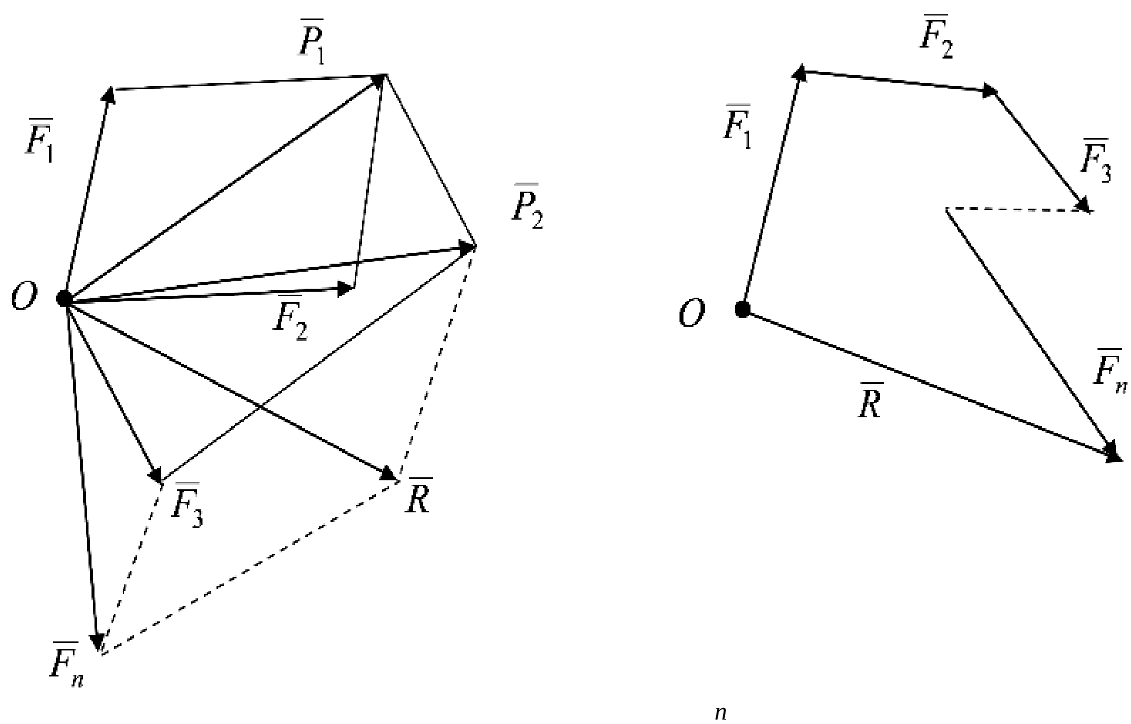


Figure 6.1

Using the axiom about the parallelogram of forces, the equivalent $\overline{R}$ system of convergent forces $\left(\overline{F}_1,...,\overline{F}_n\right)$ is defined graphically as the closing side of the polygon of forces (Fig. 6.2) $\left(\overline{R} = \sum_{i=1}^{n} \overline{F}_i\right)$.



$$\overline{R} = \overline{F}_1 + ... + \overline{F}_n = \sum_{i=1}^{n} \overline{F}_i$$

Figure 6.2

The value (module) of the equivalent force $\overline{R}$ is determined

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \ .$$

Equilibrium conditions of the system of convergent forces. Theorem. For the equilibrium of the spatial system of convergent forces, it is *necessary* and *sufficient* that the equivalent force is equal to zero

$$\overline{R} = 0 \, .$$

For the equilibrium of the spatial system of convergent forces, it is necessary and sufficient that the algebraic sums of force projections on three mutually perpendicular axes are equal to zero:

$$\left.\begin{aligned} R_x &= \sum_{i=1}^{n} F_{ix} = F_{1x} + F_{2x} + \ldots + F_{nx} = 0; \\ R_y &= \sum_{i=1}^{n} F_{iy} = F_{1y} + F_{2y} + \ldots + F_{ny} = 0; \\ R_z &= \sum_{i=1}^{n} F_{iz} = F_{1z} + F_{2z} + \ldots + F_{nz} = 0. \end{aligned}\right\}$$

If the system of convergent forces is plane, then only two of the three equilibrium conditions remain, for example

$$\sum_{i=1}^{n} F_{ix} = 0 \, ; \sum_{i=1}^{n} F_{iy} = 0 \, .$$

Statically determined and statically undetermined systems. The problem of statics can be solved only when the number of unknowns does not exceed the number of equilibrium equations. That is, in the case of a spatial system of convergent forces, the number of unknowns should not exceed three, and in a plane system – two. Such problems are called statically determinate, if these conditions are not fulfilled, they are called statically indeterminate, that is, such problems cannot be solved using only static equilibrium conditions. Solving statically indeterminate problems requires equations that can only be obtained using methods of applied mechanics.

Questions for self-testing

1. Theorem about three forces.
2. Theorem about force as a sliding vector.
3. What is called a convergent system of forces?
4. Formulate the conditions for the equilibrium of convergent forces.
5. What systems are called statically determined?

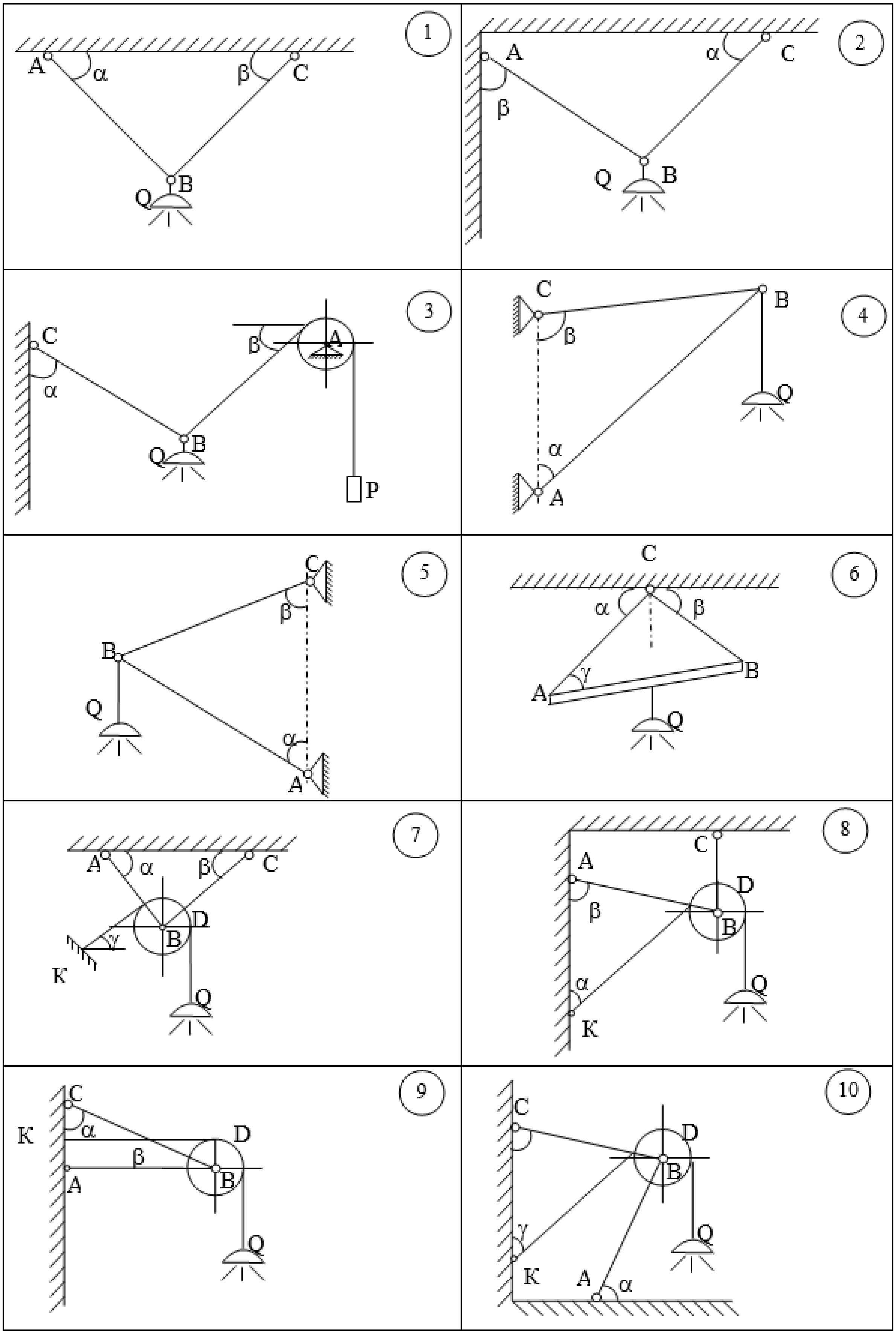
6.2 Calculation-graphic and control tasks

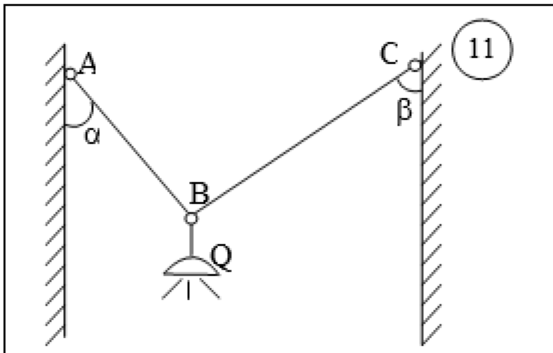
S1. Convergent system of forces

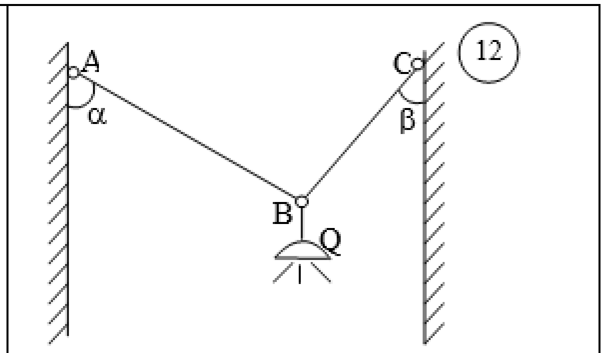
The schemes (P. 25–27) show options for hanging a lantern with weight Q. Find the force in the cable BC and rod AB. Data for calculation are given in table 6.1.

Table 6.1

Task	***Q, N***	***α, °***	***β, °***	***γ, °***
1	50	30	45	30
2	40	30	60	45
3	60	60	30	60
4	30	30	120	75
5	45	30	60	15
6	60	30	60	30
7	70	60	30	45
8	30	60	75	60
9	80	30	60	75
0	150	60	30	45


1
A α β C
B
Q
2
A α C
β
Q B
3
C
β A
α
B
Q
P
4
C
B
β
Q
α
A
5
C
β
B
Q
α
A
6
C
α β
γ B
A
Q
7
A α β C
D
γ
B
К
Q
8
C
A
D
β
B
α
Q
К
9
C
К α D
β
A B
Q
10
C
D
B
γ
Q
К A α


11
A
C
α
β
B
Q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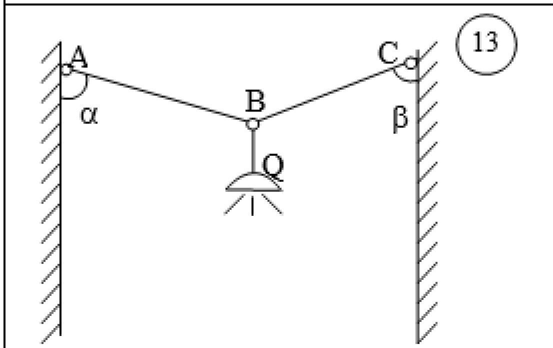
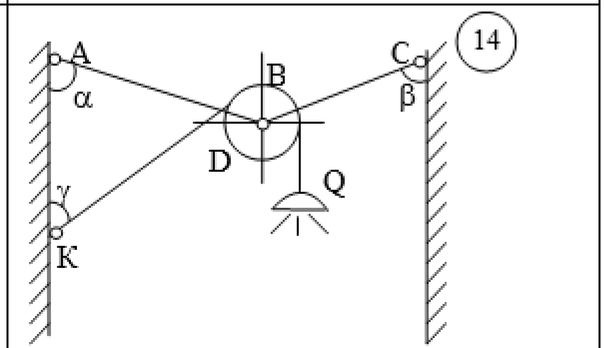
A
C
α
β
B
Q


13
A
C
α
β
B
Q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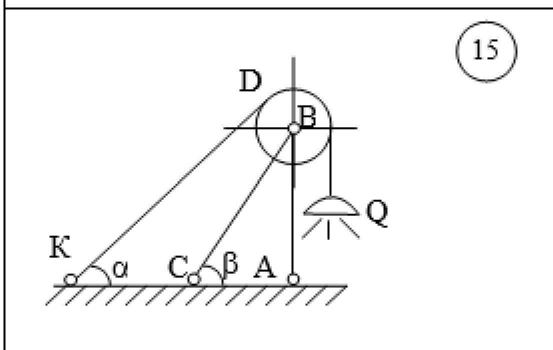
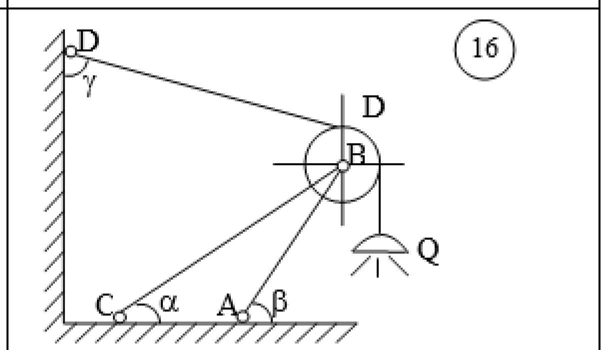
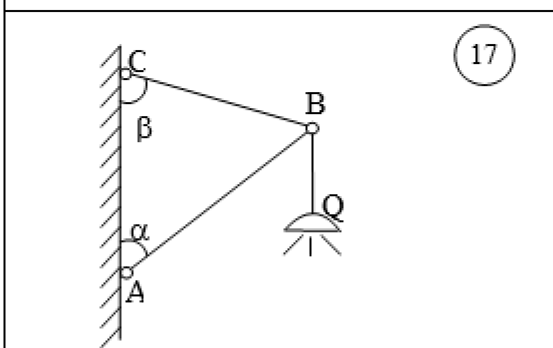
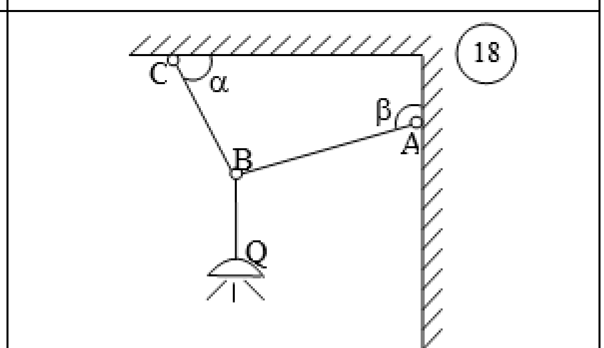
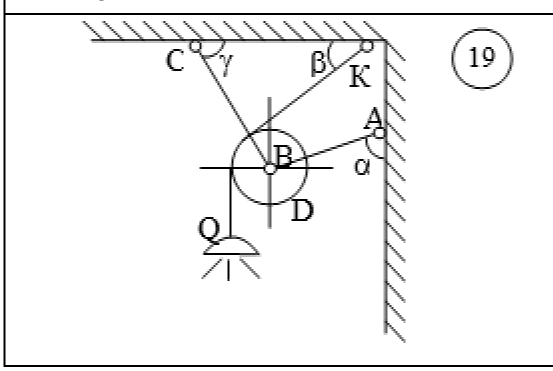
γ
K


15
D
B
Q
K
α
C
β
A


16
D
γ
D
B
Q
C
α
A
β


17
C
β
B
Q
α
A


18
C
α
β
A
B
Q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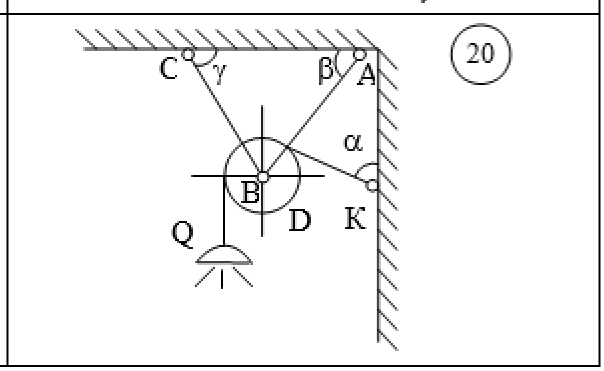
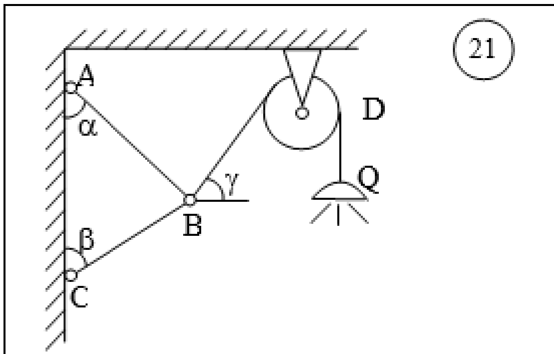
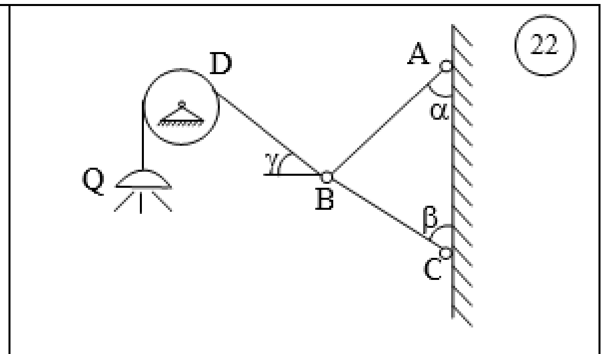
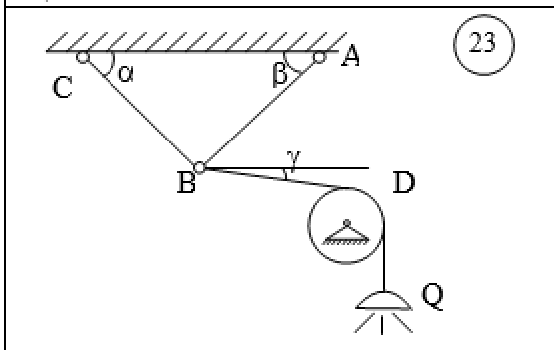
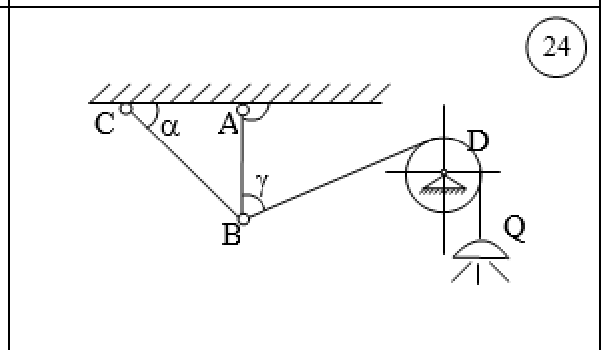
C
γ
β
K
A
B
α
Q
D

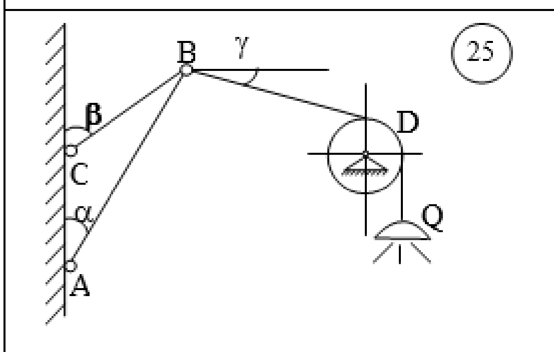

20
C
γ
β
A
α
B
D
K
Q

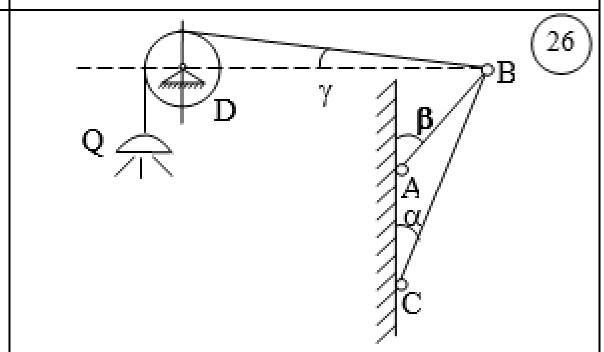

21
A
α
D
γ
Q
B
β
C

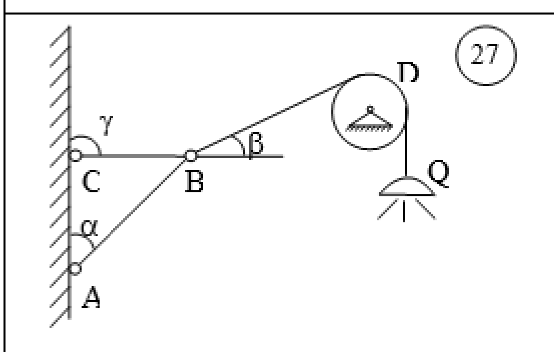

22
D
A
α
γ
Q
B
β
C

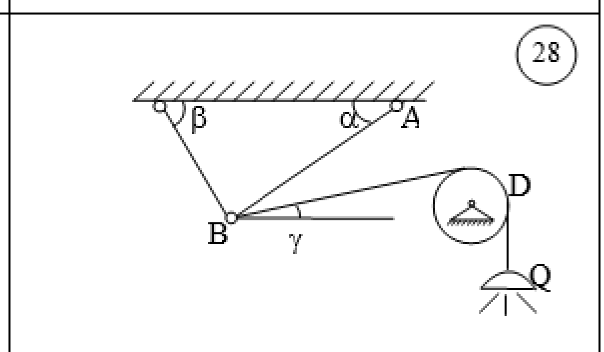

23
α
β
A
C
γ
B
D
Q

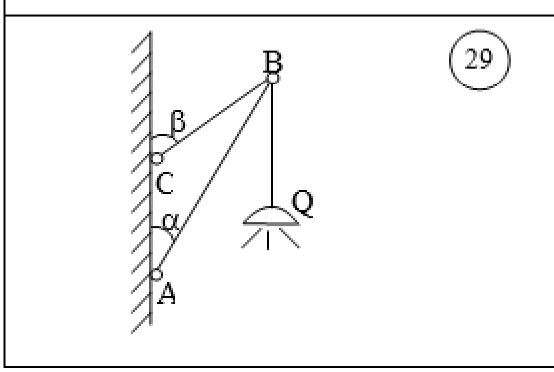

24
C
α
A
D
γ
Q
B

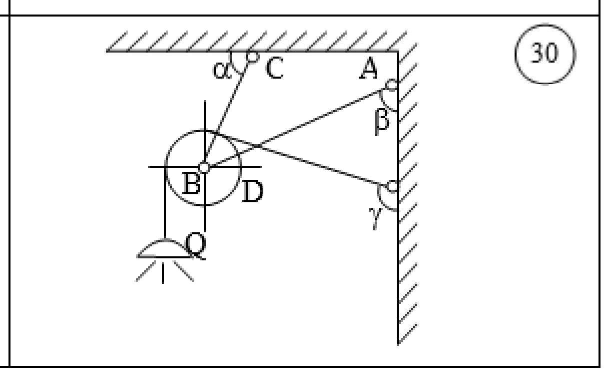

B
γ
25
β
D
C
α
Q
A


26
B
γ
D
Q
β
A
α
C


27
D
γ
β
Q
C
B
α
A


28
β
α
A
D
B
γ
Q


29
B
β
C
α
Q
A


30
α
C
A
β
B
D
γ
Q

6.3 Example of task execution

Given: a diagram of hanging a lantern (Fig. 6.3); Q = 165H; α = 60˚; β = 45˚; γ = 150˚.

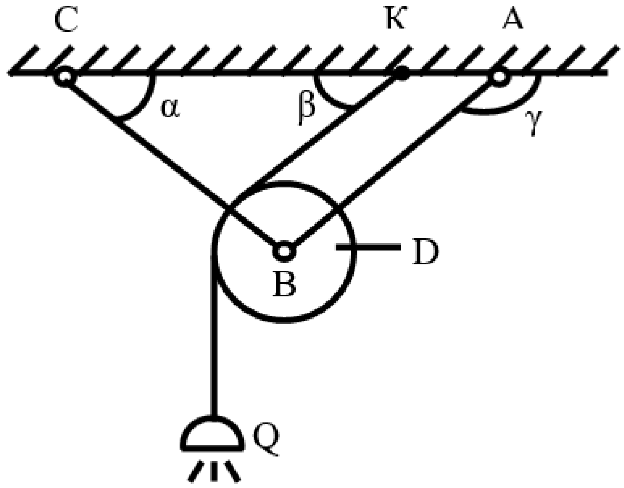


Figure 6.3

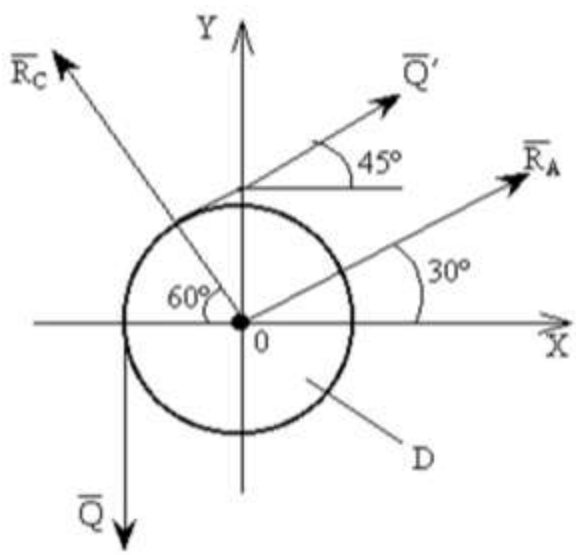


Figure 6.4

The solution. Consider the equilibrium of block D (Fig. 6.4). The force in the cable on which the lantern hangs is equal to the weight of the lantern Q (neglect the friction of the cable on block D). We discard the rod AB and cable BC and replace their action with the forces $\overline{R_A}, \overline{R_C}, \overline{Q}, \overline{Q}^{/}$ with which they act on block D.

At the same time, we take into account that the cable works only for stretching, and the connecting rod OA can be both compressed and stretched.

Block D is in equilibrium under the action of the system of forces $\{\overline{R_A}, \overline{R_C}, \overline{Q}, \overline{Q}^{/}\}$, $(\{\overline{R_A}, \overline{R_C}, \overline{Q}, \overline{Q}^{/}\} \sim 0)$, and besides $\overline{Q} = \overline{Q}^{/}$. Let's write the equilibrium equation for the forces applied to block D.

Equilibrium equation

$$\Sigma F_x = 0; \qquad \overline{Q} \cdot \cos 45^o + R_A \cdot \cos 30^o - R_C \cdot \cos 60^o = 0;$$

$$\Sigma F_y = 0; \qquad -\overline{Q} + \overline{Q}^{/} \cdot \sin 45^o + R_C \cdot \sin 60^o + R_A \cdot \sin 30^o = 0.$$

From here we find

$$R_A = Q\frac{1 - \sin 45 - \cos 45 \cdot tg60}{\sin 30 + \cos 30 \cdot tg60} = -76,87 \quad N,$$

$$R_C = \frac{Q \cdot \cos 45 + R_A \cdot \cos 30}{\cos 60} = 100,2 \quad N.$$

Answer: RA = –76,87 N;   RC = 100,2 N.

7 MOMENT OF FORCE RELATIVE TO THE CENTER

The moment of force $\overline{F}$ (Fig. 7.1) relative to the center is called the vector product of the radius-vector $\overline{r}$, drawn from point O to the point of application of force $\overline{F}$, by the force vector $\overline{F}$

$$M_0\left(\overline{F}\right) = \overline{r} \times \overline{F}.$$

The module of the vector product

$$\left|M_0\left(\overline{F}\right)\right| = r \cdot F \cdot \sin\alpha = F \cdot h,$$

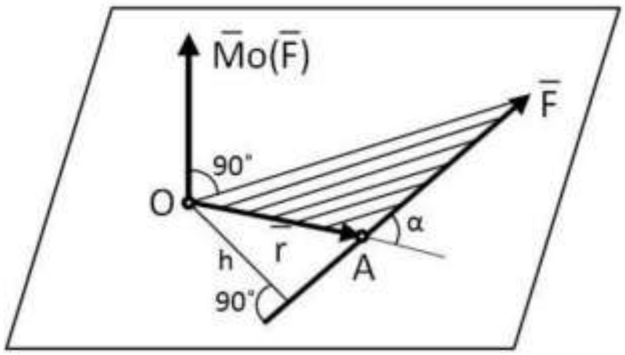


Figure 7.1

where ***h*** is the shoulder of the force, that is, the perpendicular drawn from the center to the line of action of the force.

The rule of signs for moments

The moment of the force relative to the point is considered positive if the force tries to rotate relative to the point counterclockwise, and negative if it tries to rotate clockwise (Fig. 7.2).

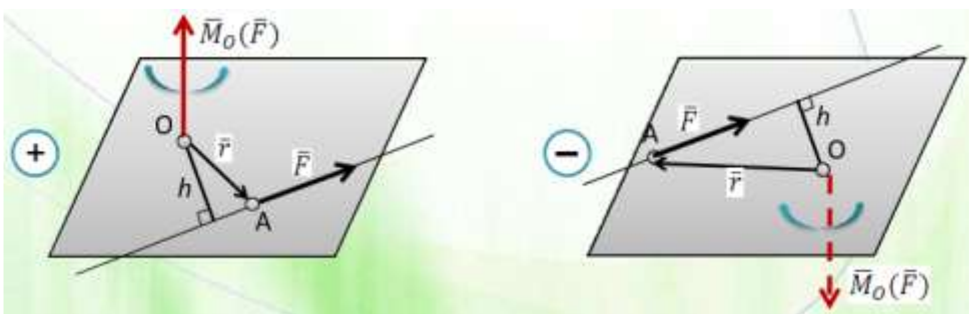


Figure 7.2

Properties of the moment of force relative to a point

1. If you move the force along the line of its action, the moment of the force relative to the point will not change (Fig. 7.3).

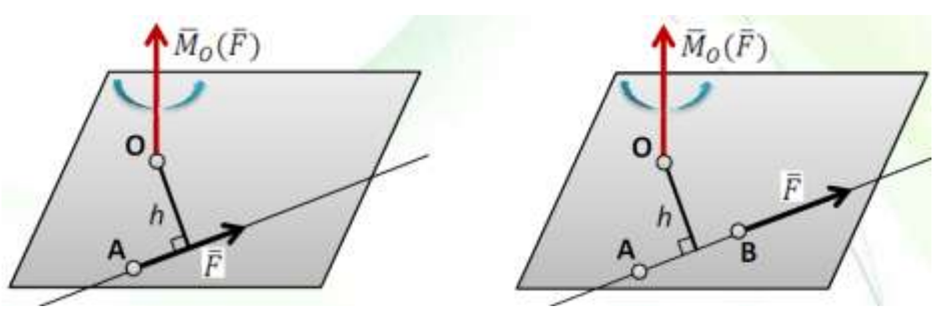


Figure 7.3

2. The moment of force relative to a point is zero if the line of action of the force passes through this point (Fig. 7.4).

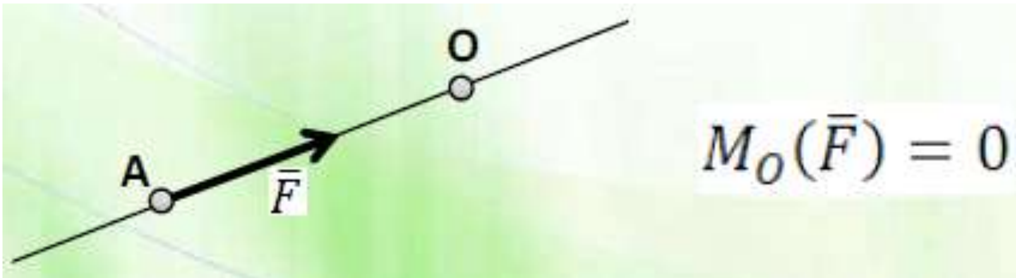


Figure 7.4

8 COUPLE OF FORCES. MOMENT OF COUPLE OF FORCES

A couple of forces is a system of two equal-module, parallel and oppositely directed forces (Fig. 8.1).

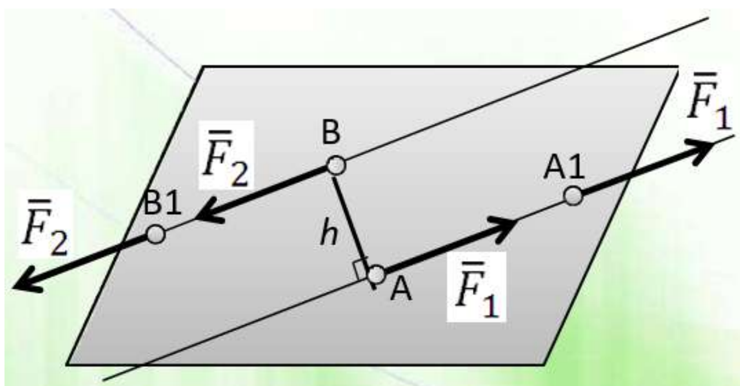


Figure 8.1

$(\overline{F}_1, \overline{F}_2)$ – a couple of forces;

$\overline{F}_1 = -\overline{F}_2$ – opposite directions;

$F_1 = F_2$ – are the same in modulus;

AB = h – the shoulder of the couple of forces.

The shoulder of the couple of forces is the shortest distance between the lines of action of the forces.

The couple of forces there is no equivalent force, but the couple forces try to rotate the body to which they are applied.

The rule of signs (Fig. 8.2).

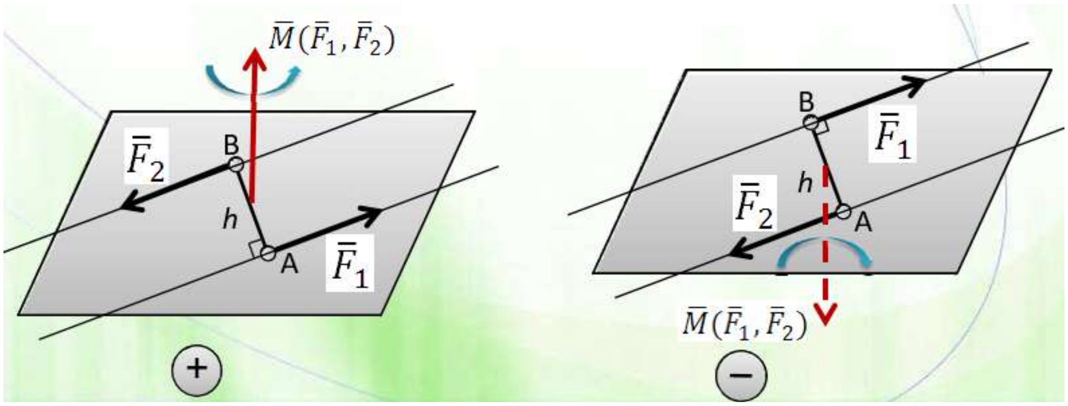


Figure 8.2

The moment of a couple of forces is the product of the modulus of one of the couples of forces on the shoulder.

$$\left|M\left(\overline{F_1}, \overline{F_2}\right)\right| = \pm F_1 \cdot h = \pm F_2 \cdot h$$

9 PLANE PARALLEL SYSTEM OF FORCES

A plane parallel system of forces is a system of forces in which all the forces of the system lie in the same plane and their lines of action are parallel (Fig. 9.1).

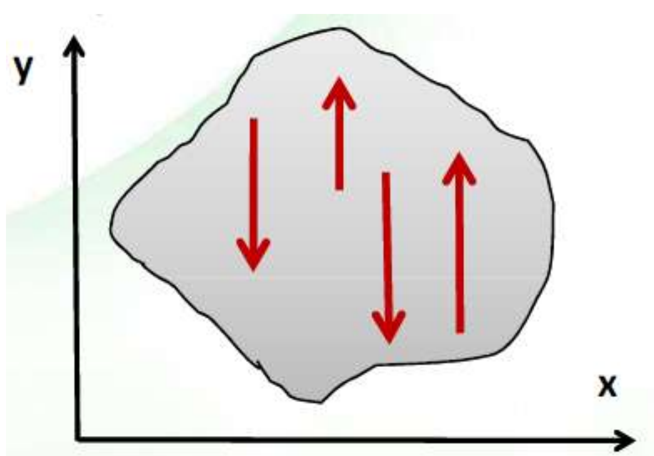


Figure 9.1

For such a system of forces, two equilibrium equations can be drawn up.

The main form of the equilibrium equations

$$\sum_{i=1}^{n} F_{iy} = 0; \quad \sum_{i=1}^{n} M_A\left(\overline{F_i}\right) = 0.$$

An additional form of the equilibrium equations

$$\sum_{i=1}^{n} M_A\left(\overline{F_i}\right) = 0; \quad \sum_{i=1}^{n} M_B\left(\overline{F_i}\right) = 0.$$

9.1 Calculation-graphic and control tasks

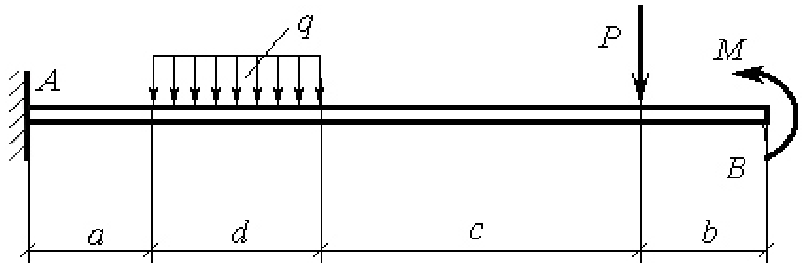
S2. A plane parallel system of forces

The beam (P. 35–40) is loaded with a force P, a distributed load of intensity q and a couple of forces with a moment M. Find the reactions of the linkages. Data for calculations are given in table 9.1.

Table 9.1

Task	*M, kN·m*	*P, kN*	*q, kN/m*	*a, m*	*b, m*	*c, m*	*d, m*
1	5	4	2	2	3	4	2
2	6	3	1	3	2	5	1
3	1	5	2	1	2	4	3
4	4	1	2	2	3	5	1
5	6	8	1	3	2	4	2
6	3	5	2	1	3	5	2
7	4	6	1	2	3	4	1
8	5	7	2	3	2	5	2
9	6	8	1	1	2	4	2
0	5	3	2	2	3	5	3

1



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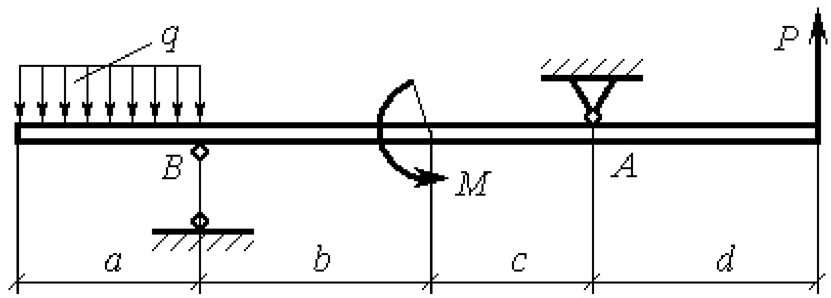

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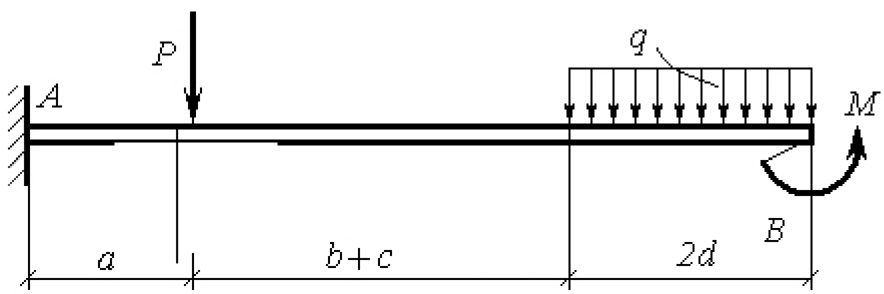

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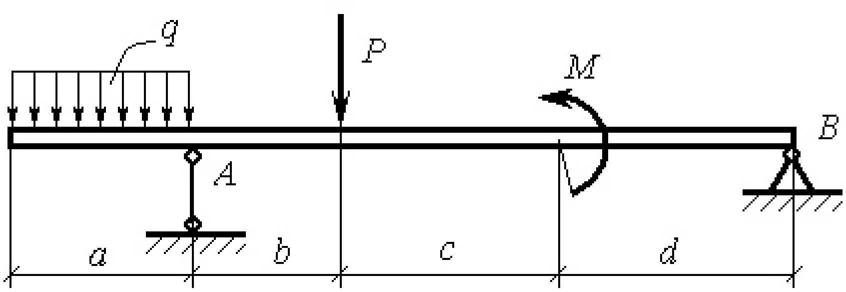

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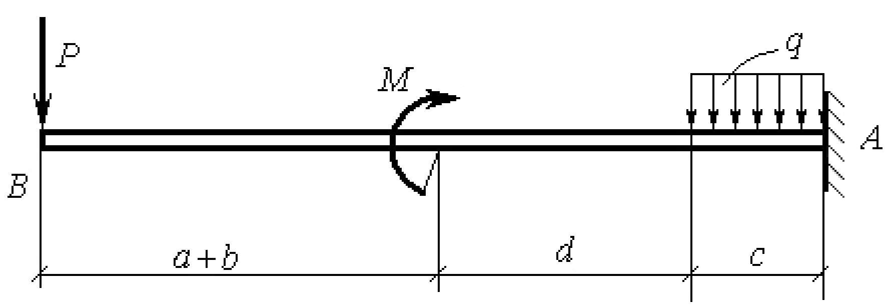

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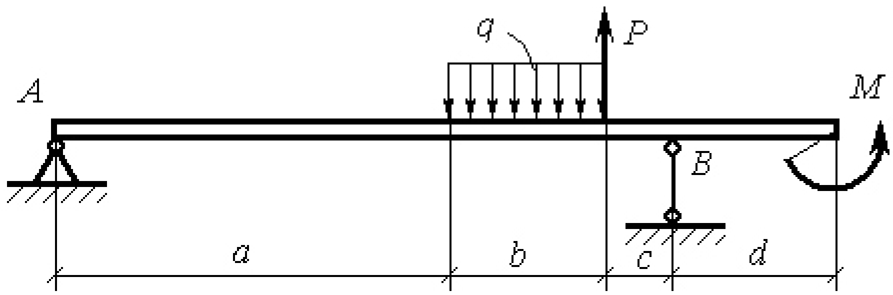

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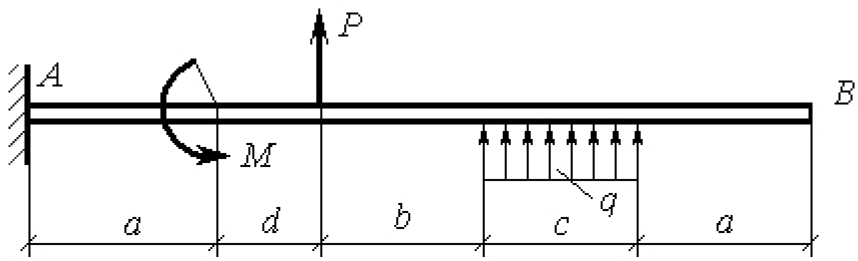

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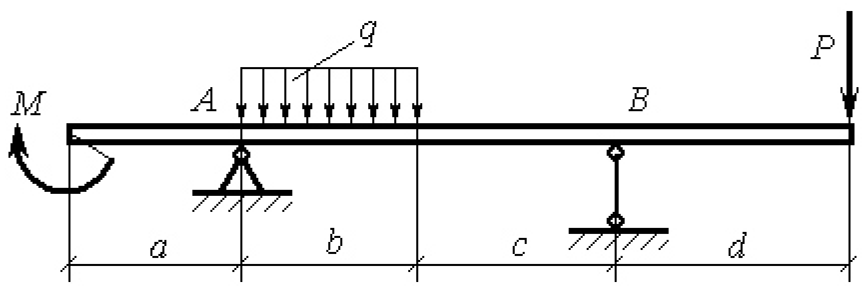

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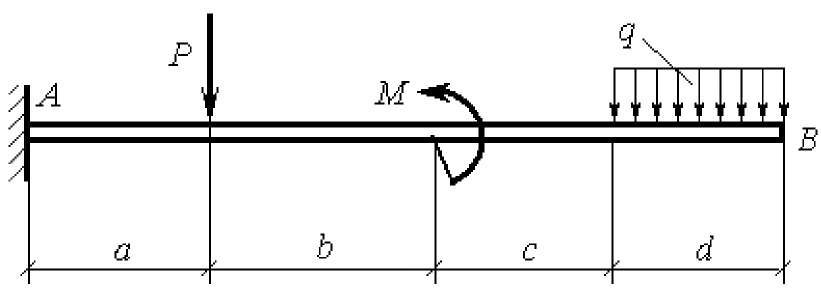

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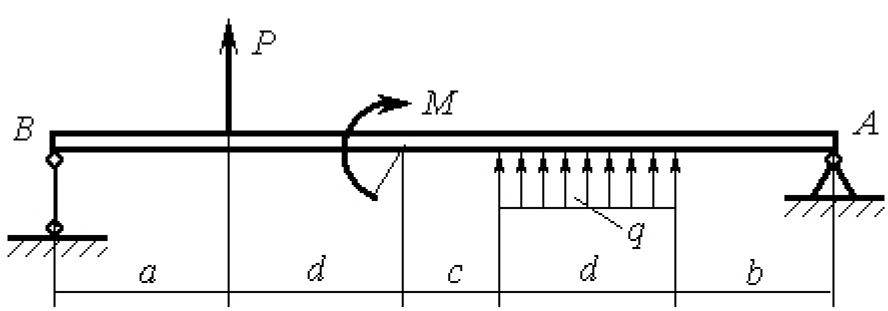

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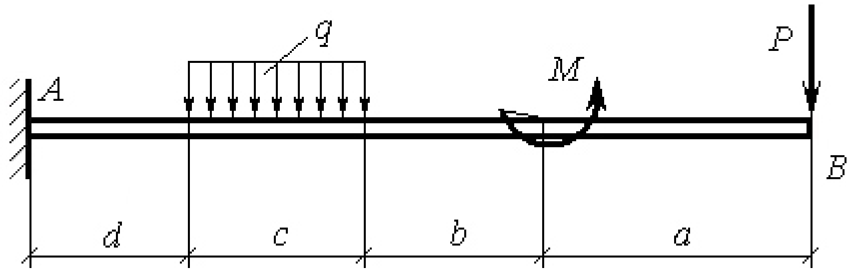

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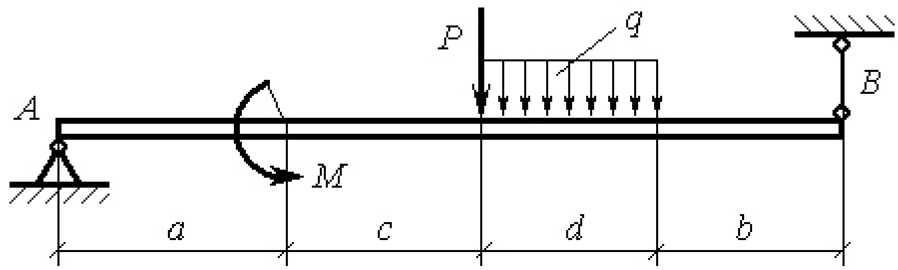

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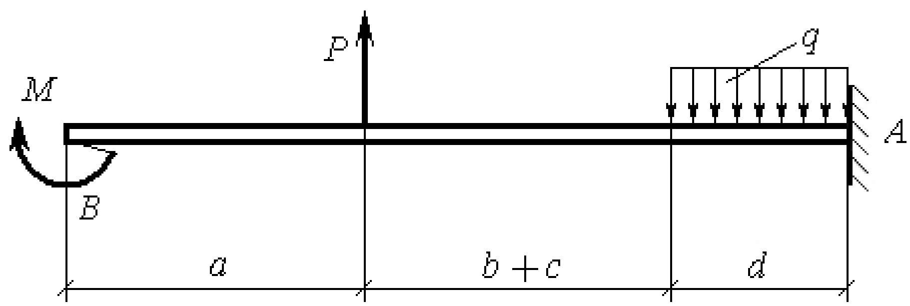

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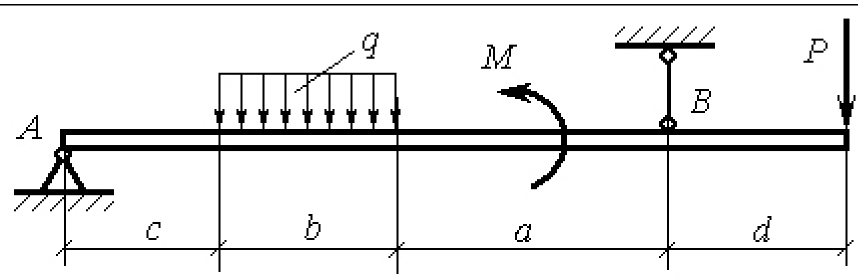

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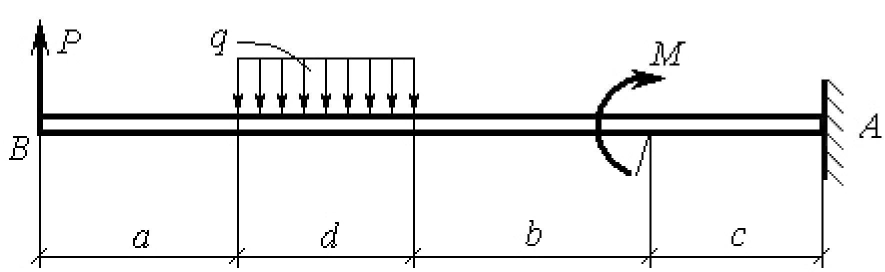

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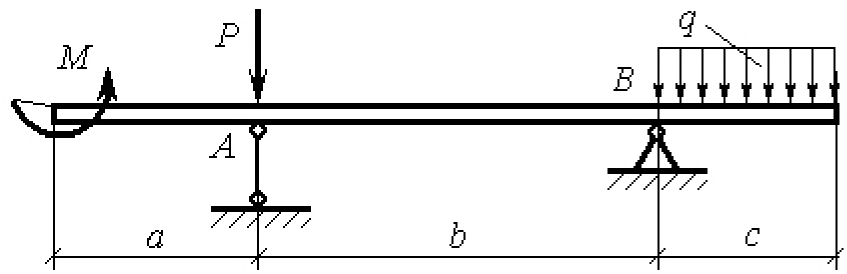

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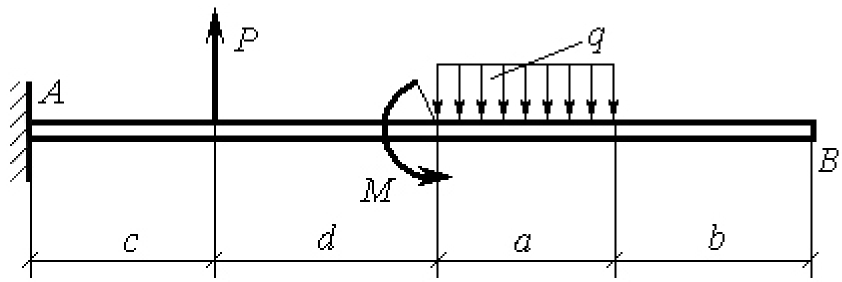

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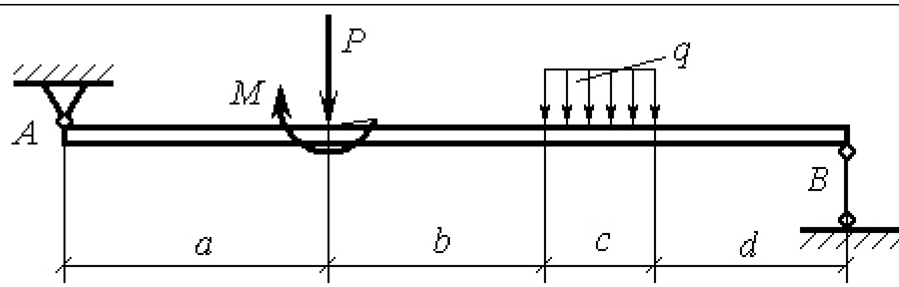

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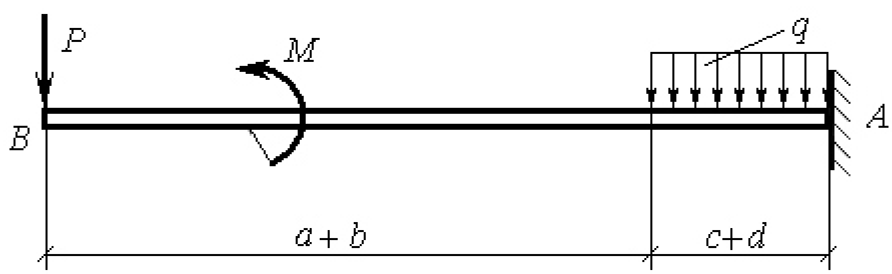

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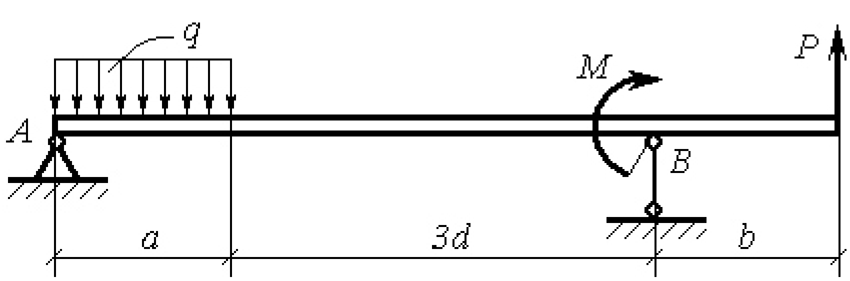

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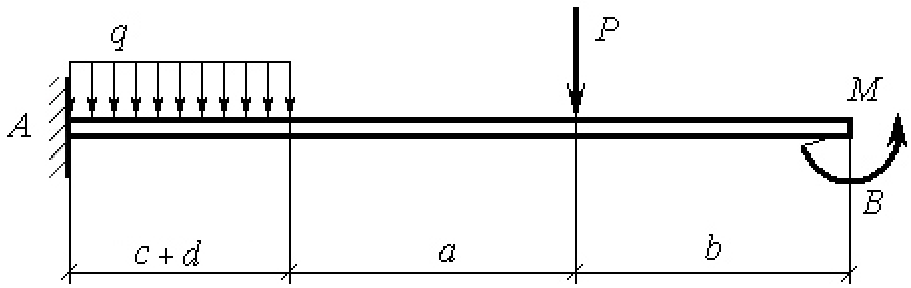

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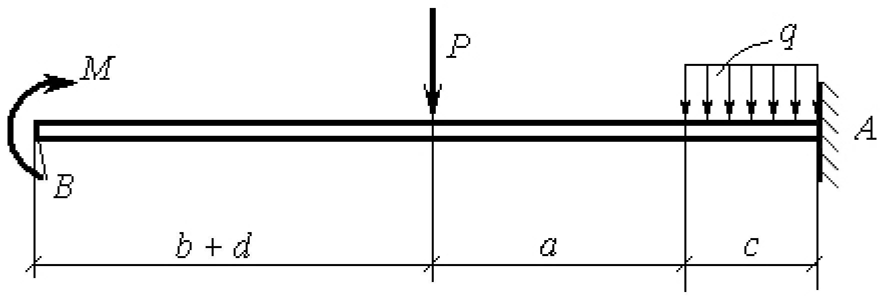

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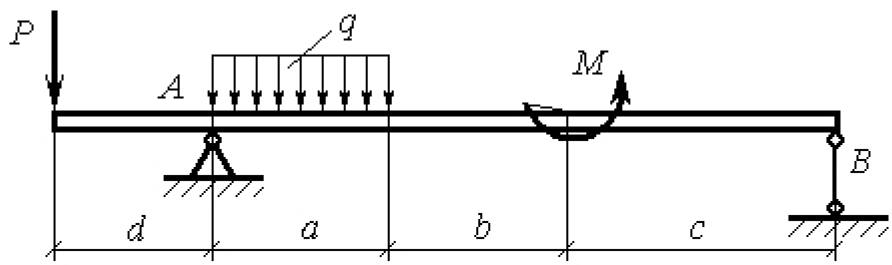

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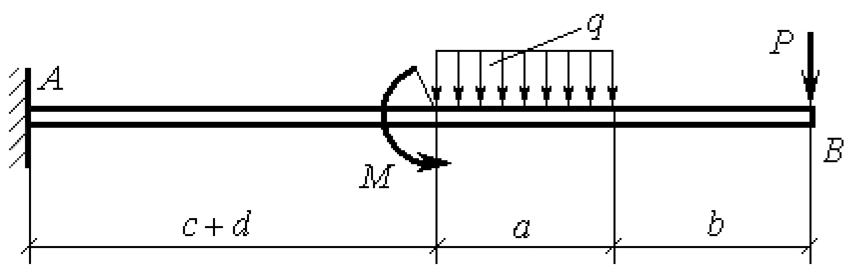

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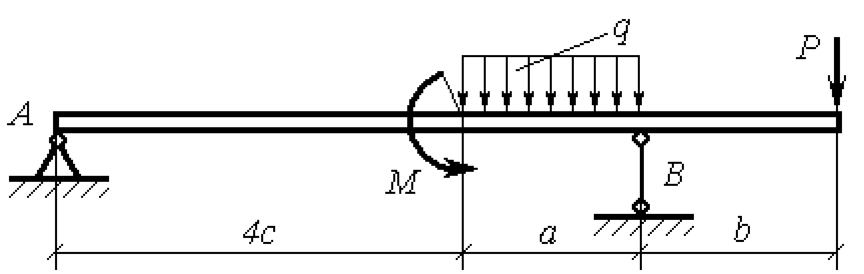

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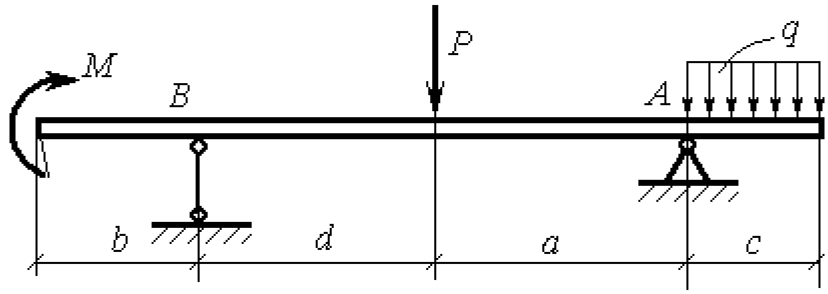

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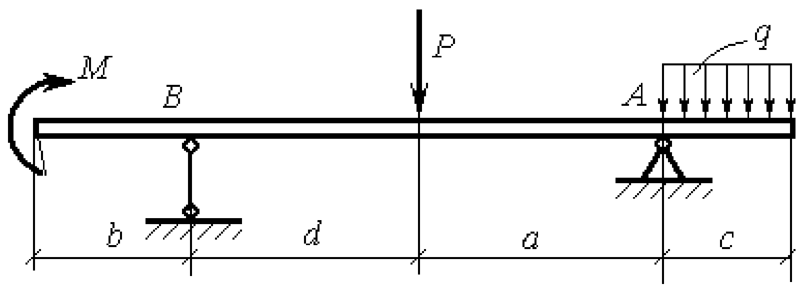

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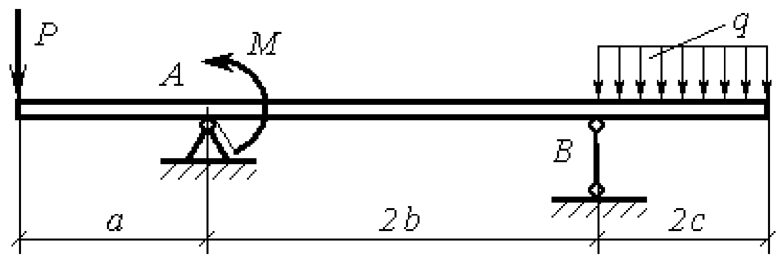

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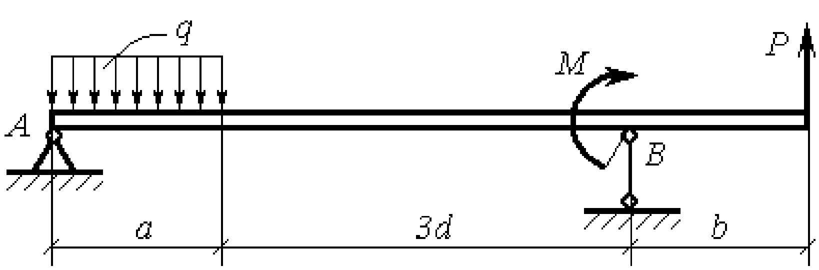

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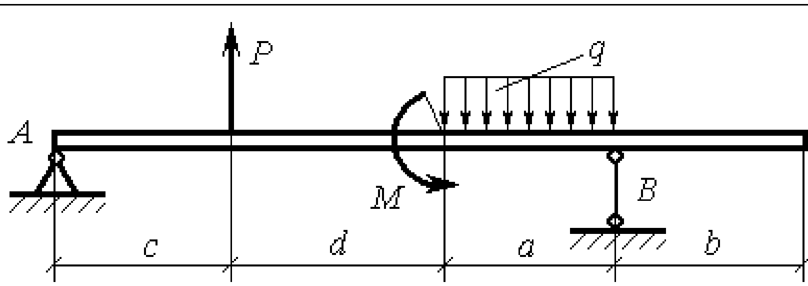


9.2 Example of task execution

Example 1. For a beam (Fig. 9.2), find the support reactions if F = 3 kN, q = 1 kN /m.

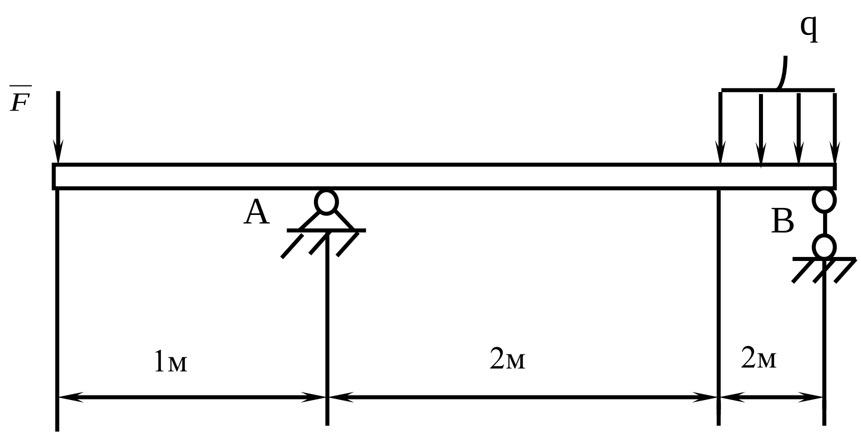


Figure 9.2

Consider the beam AB, which is in equilibrium (Fig. 9.3).

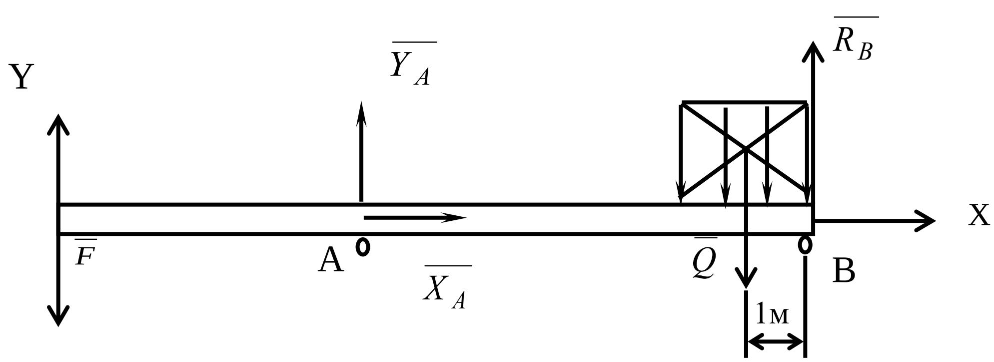


Figure 9.3

A concentrated force $\overline{F}$ and a distributed load q are acted on the beam. A distributed load q is equivalent to a concentrated force $\overline{Q}$.

$$Q = q \cdot 2 = 1 \cdot 2 = 2 \ kN.$$

We replace the action of cylindrical hinge A and movable B with their reactions $\overline{Y_A}, \overline{X_A}, \overline{R_B}$ (Fig. 9.3). For a balanced system of forces $\{\overline{F}, \overline{X_A}, \overline{Y_A}, \overline{Q}, \overline{R_B}\} \sim 0$, we write down the equilibrium conditions

$$\sum F_X = 0; \; X_A = 0;$$

$$\sum M_A(\overline{F}) = 0; \; F \cdot 1 - Q \cdot 3 + R_B \cdot 4 = 0;$$

$$\sum M_B(\overline{F}) = 0; \; F \cdot 5 - Y_A \cdot 4 + Q \cdot 1 = 0.$$

Then: $R_B = 0{,}75\ kN$, $Y_A = 4{,}25\ kN$, $X_A = 0$.

The reliability of the obtained results can be checked by writing down another equilibrium equation:

$$\sum F_Y = 0; \; -F + Y_A - Q + R_B = 0,$$
$$-3 + 4{,}25 - 2 + 0{,}75 = 0.$$

So, the problem is solved correctly.

Answer: RB = 0,75 kN, YA = 4,25 kN, XA =0.

Example 2. For a cantilever beam (Fig. 9.4), find the reaction of rigid restraint if $F = 2kN$, $M = 5kN{\cdot}m$.

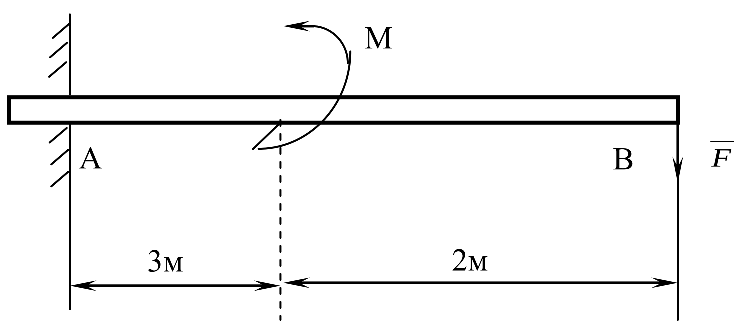


Figure 9.4

Beam AB is acted upon by: active force F, couple of forces with moment M; its movement is hindered by a linkage – a rigid linkage at point A. We discard the linkage at point A, and based on the axiom of freedom from the linkages, we replace its action with forces $\overline{X_A}, \overline{Y_A}$ and moments M_A (Fig. 9.5).

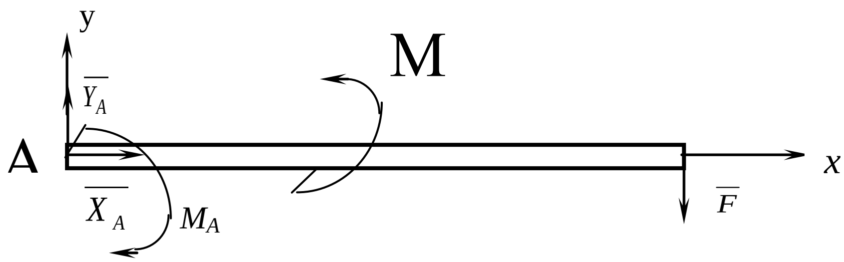


Figure 9.5

Write down the equilibrium conditions for the system of forces $\{\overline{X_A}, \overline{Y_A}, M_A, M, \overline{F}\} \sim 0$.

$$\sum F_X = 0;\ X_A = 0;$$
$$\sum F_Y = 0;\ Y_A - F = 0;$$
$$\sum M_A(\overline{F}) = 0;\ -M_A + M - F \cdot 5 = 0.$$

Where we find

$$Y_A = F = 2kN$$
$$M_A = M - 5 \cdot F = 5 - 5 \cdot 2 = -5kN \cdot m.$$

The negative sign of M_A indicates that the actual direction of the force moment in a rigid clamp is opposite to that shown in Fig. 9.5.

Let's perform the check. The problem is solved correctly if the condition is realized

$$\sum M_B(\overline{F}) = 0.$$

Let's make this equation using Fig. 9.4 and 9.5.

$$\sum M_B(\overline{F}) = 0;\ M - M_A - Y_A \cdot (3+2) = 0;$$
$$5 - (-5) - 2 \cdot (3+2) = 0.$$

So, the problem is solved correctly.

Answer: $X_A = 0;\ Y_A = 2kN;\ M_A = -5kN \cdot m.$

10 PLANE ARBITRARY FORCE SYSTEM

A plane arbitrary system of forces is a system of forces in which all the forces of the system lie in the same plane and their lines of action can be parallel for some forces, and intersect at one point for others.

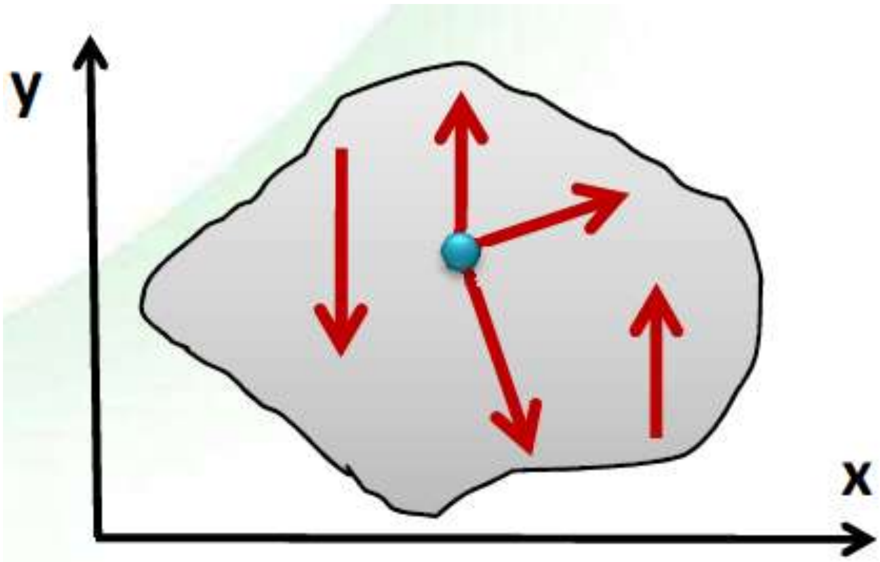


Figure 10.1

That is, we have a combination of parallel and convergent systems of forces. For such a system of forces, three equations of equilibrium can be drawn up.

The first or main form

$$\sum_{i=1}^{n} F_{ix} = 0; \quad \sum_{i=1}^{n} F_{iy} = 0; \quad \sum_{i=1}^{n} M_A\left(\overline{F_i}\right) = 0.$$

The second form

$$\sum_{i=1}^{n} F_{ix} = 0; \quad \sum_{i=1}^{n} M_A\left(\overline{F_i}\right) = 0; \quad \sum_{i=1}^{n} M_B\left(\overline{F_i}\right) = 0.$$

The third form

$$\sum_{i=1}^{n} M_A\left(\overline{F_i}\right)=0;\quad \sum_{i=1}^{n} M_B\left(\overline{F_i}\right)=0;\quad \sum_{i=1}^{n} M_C\left(\overline{F_i}\right)=0.$$

10.1 Calculation-graphic and control tasks

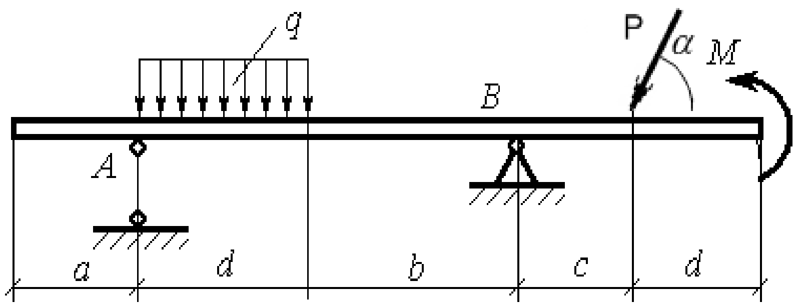
S3. The plane arbitrary system of forces

A beam (P. 46–51) is loaded with a force *F* applied at an angle *α*, a distributed load of intensity *q* and a pair of forces with moment *M*. Find the reactions of the linkages. The data for the calculations are given in Table 10.1.

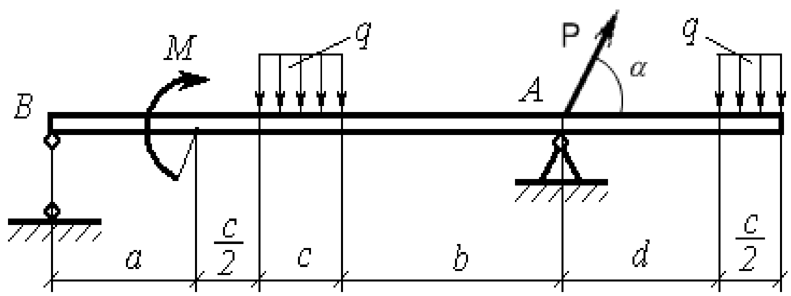
Table 10.1

Task	*M*, kN·m	*F*, kN	*q*, kN/m	*a*, m	*b*, m	*c*, m	*d*, m	α , ^{0}
1	5	4	2	2	4	3	2	30
2	6	3	1	3	5	2	1	45
3	1	5	2	1	4	2	3	60
4	4	1	2	2	5	3	1	30
5	6	8	1	3	4	2	2	45
6	3	5	2	1	5	3	2	60
7	4	6	1	2	4	3	1	30
8	5	7	2	3	5	2	2	45
9	6	8	1	1	4	2	2	60
0	5	3	2	2	5	3	3	30

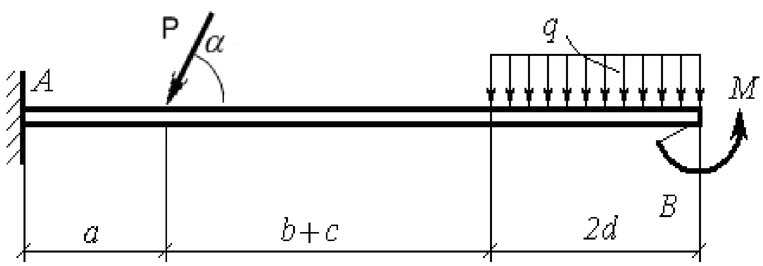
1


q
P
α
M
B
A
a
d
b
c
d

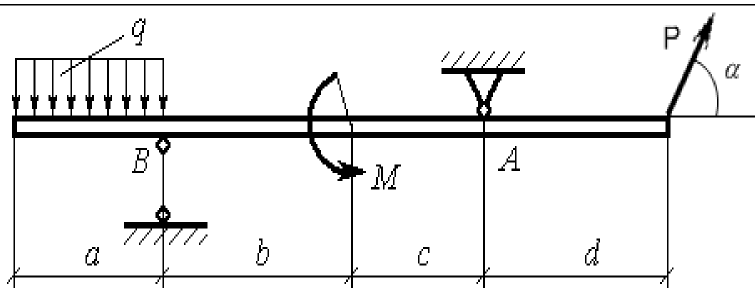
2


M
q
P
q
B
A
α
a
c/2
c
b
d
c/2

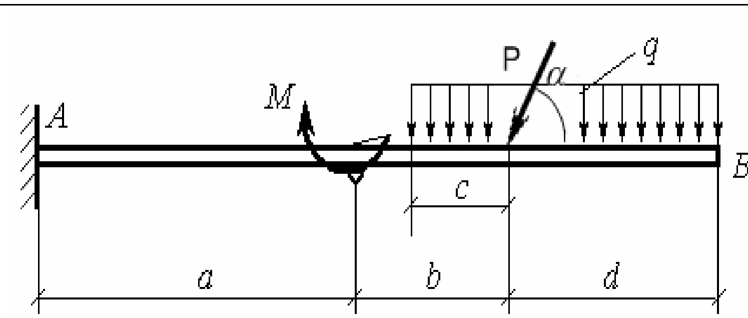
3


P
α
q
A
M
B
a
b+c
2d

4


q
P
α
B
M
A
a
b
c
d

5


P
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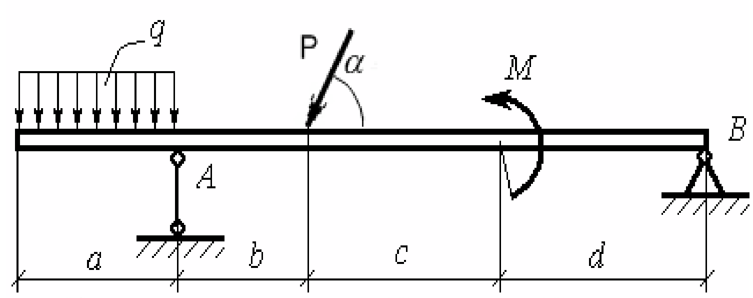
6


q
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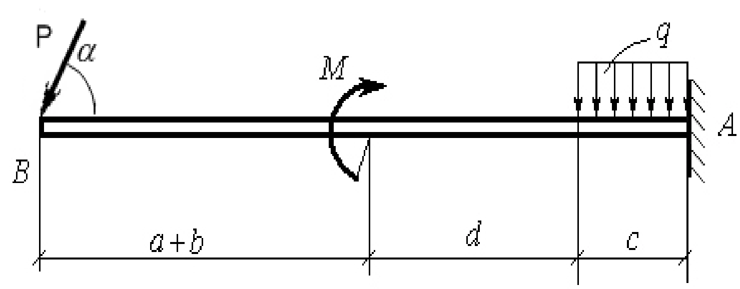
d

7


P
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A
B
a+b
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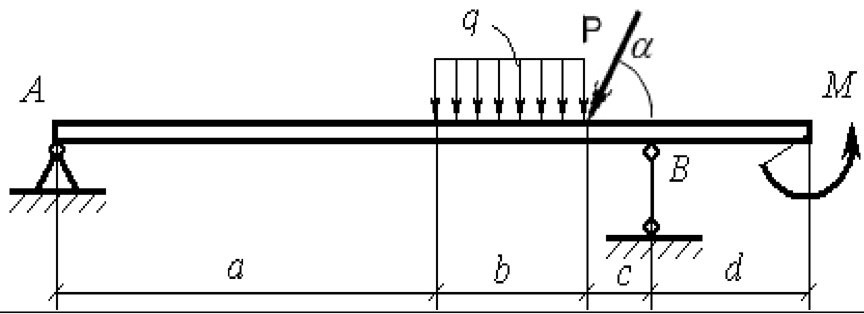
c

8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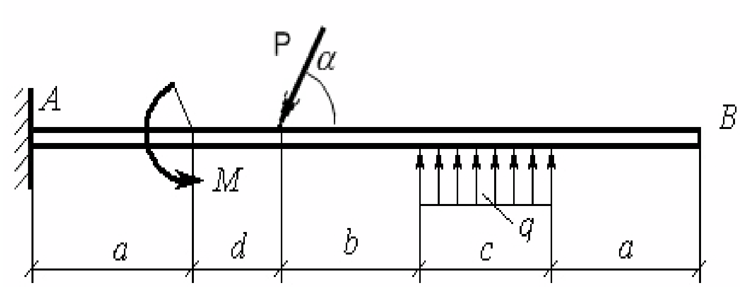
9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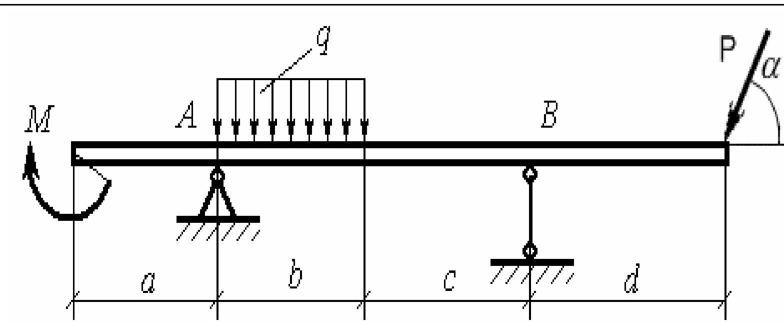
a

10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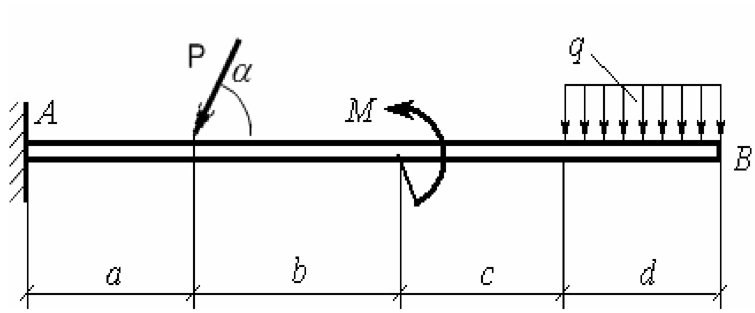
11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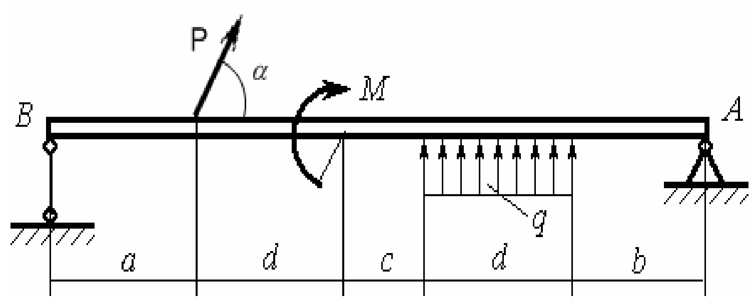
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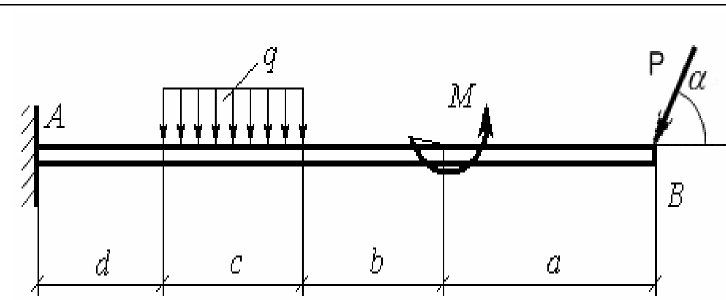
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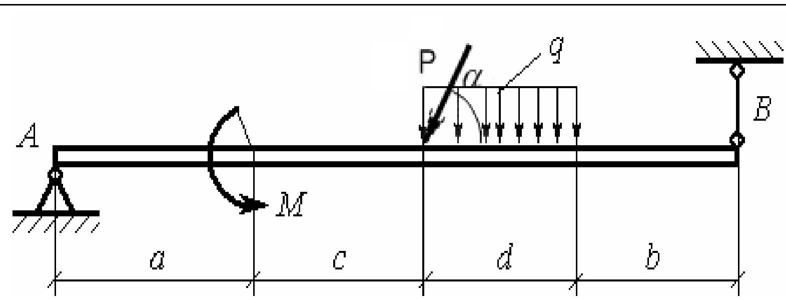
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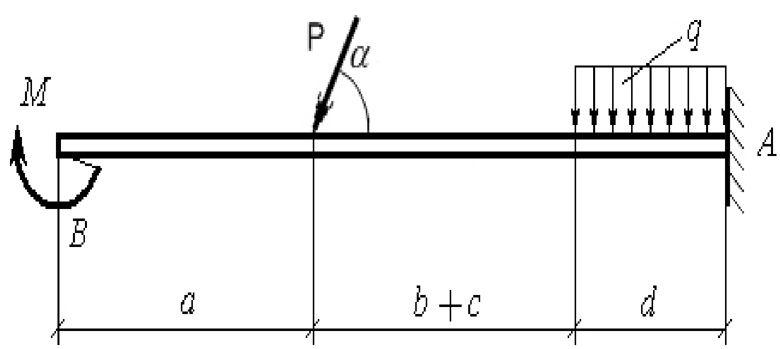
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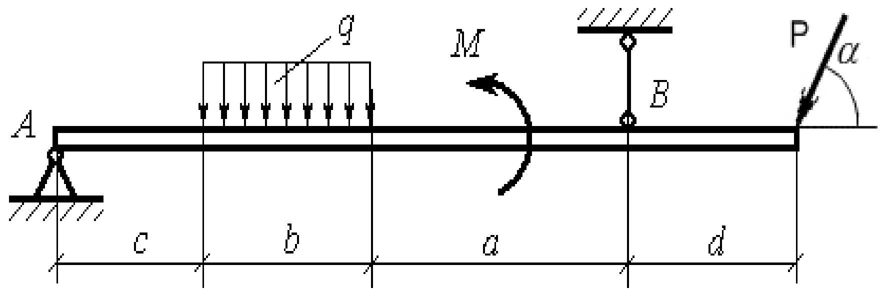

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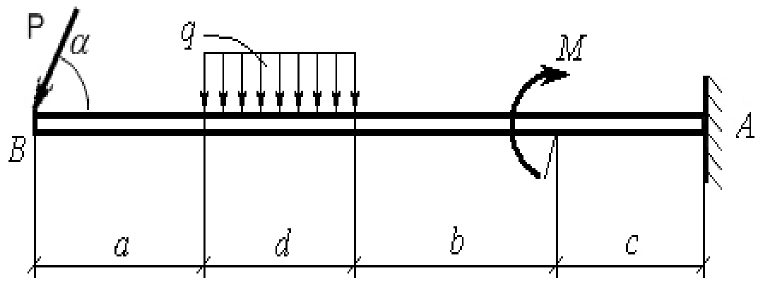

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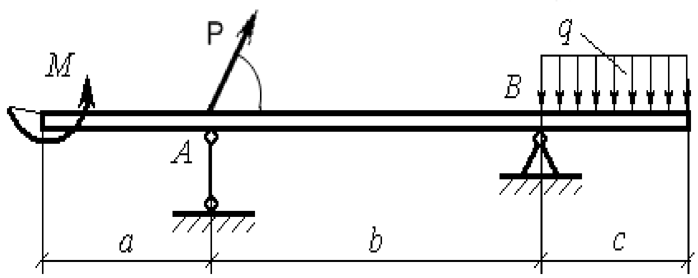

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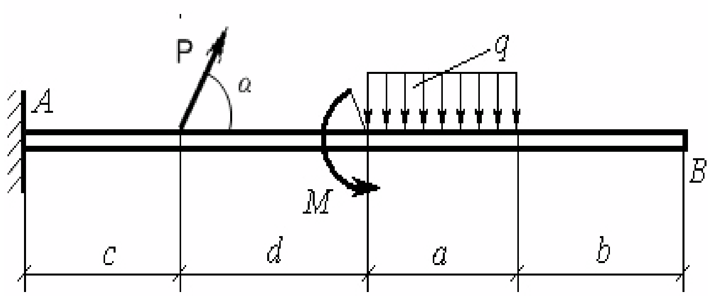

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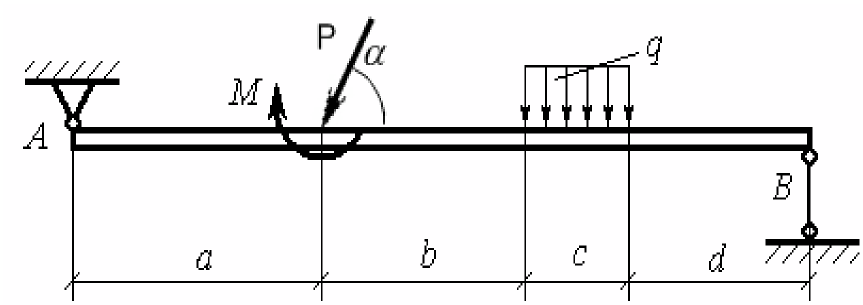

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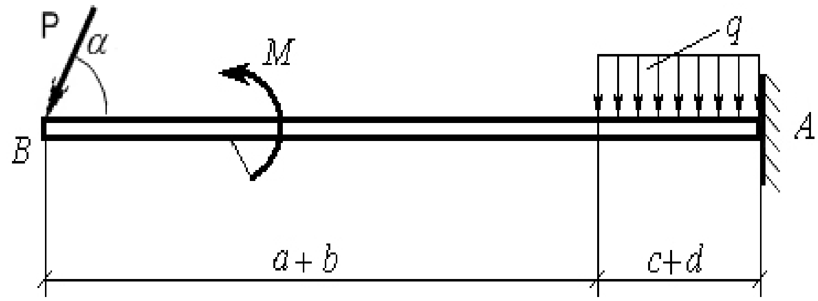
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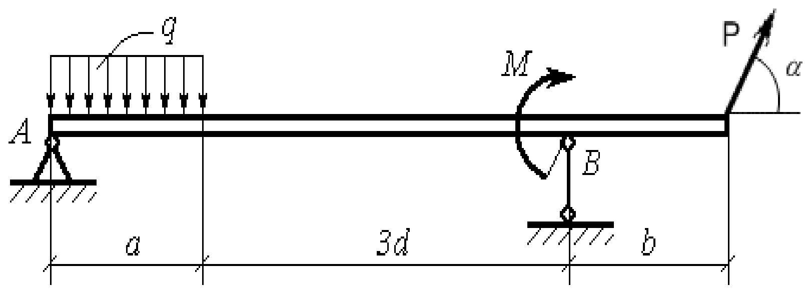
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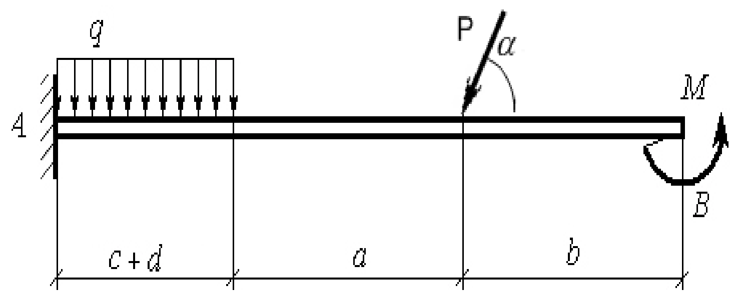
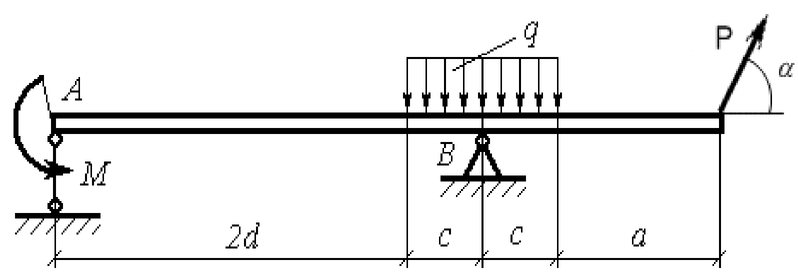

q
P
α
M
A
B
c + d
a
b

24


q
P
α
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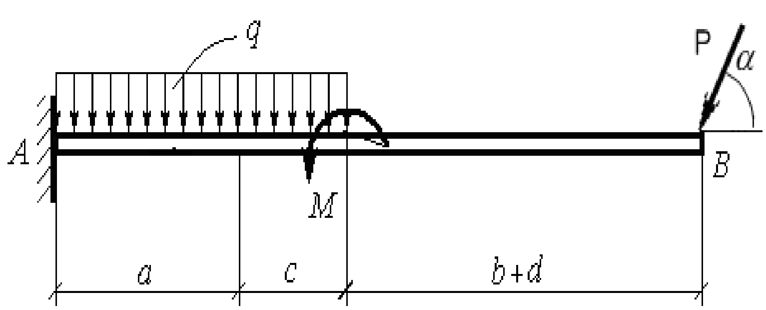
M
B
2d
c
c
a

25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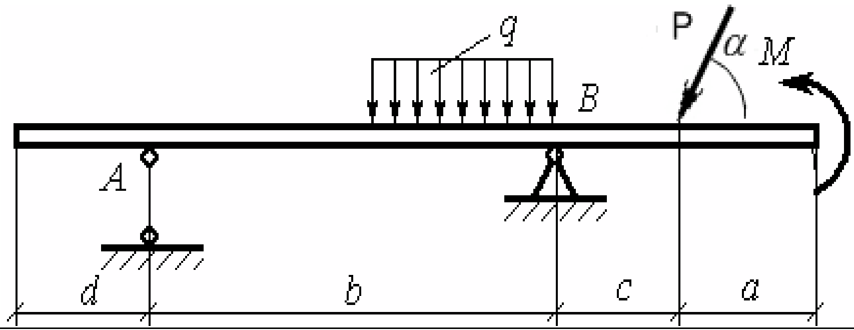
26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P
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b
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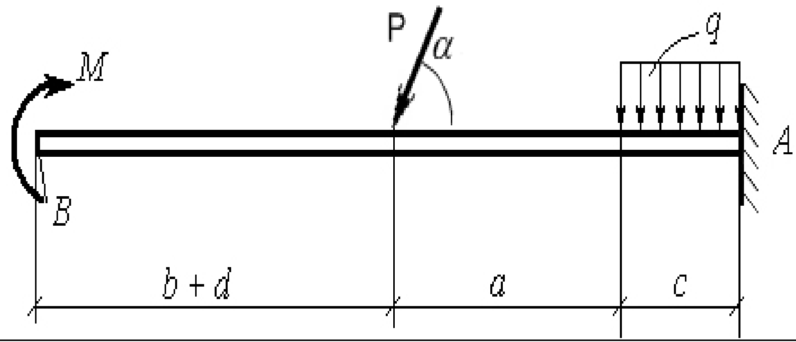
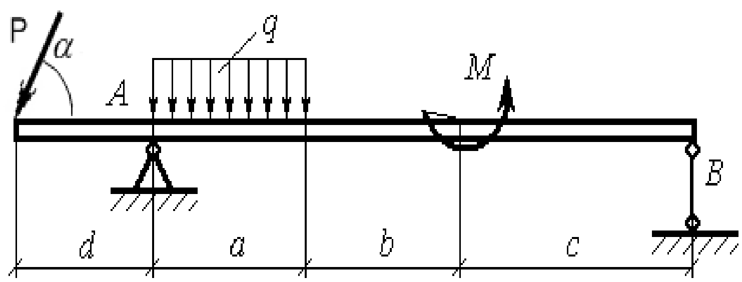
a

27


P
α
q
M
A
B
b + d
a
c

28


P
α
q
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M
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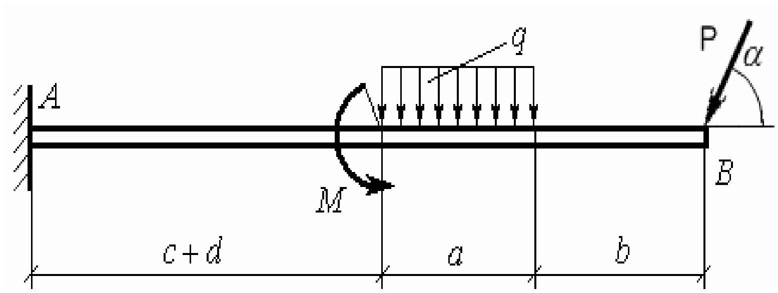
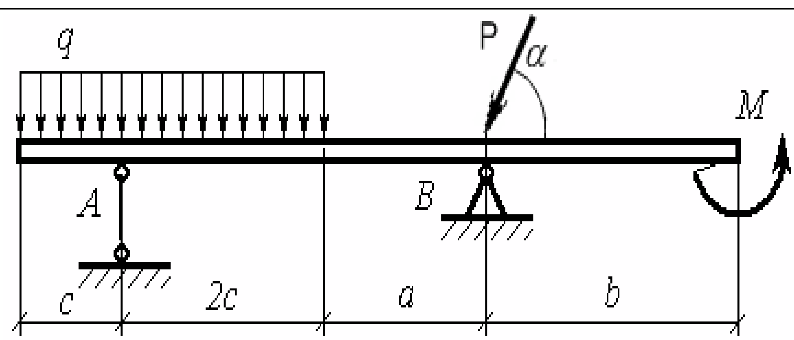
c

29


q
P
α
A
B
M
c + d
a
b

30


q
P
α
M
A
B
c
2c
a
b

10.2 Example of task execution

The plane frame shown in Fig. 10.2 is in equilibrium under the action of forces F_1 = 10 N, F_2 = 20 N and a pair of forces with moment M = 60 kN·m. The angle $\varphi = 30^o$ and the angle $\gamma = 45^o$. The dimensions are: a = 5 m, b = 8 m, c = 3 m. The supports of the frame are: at point A – fixed plane hinge, at point B – a movable support.

Find the reactions of the supports A and B.

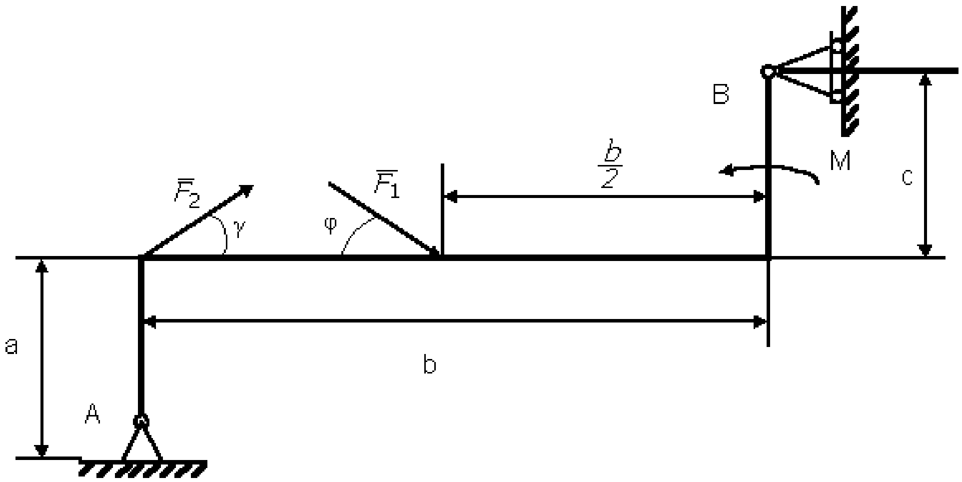


Figure 10.2

The solution. We position the given frame in the coordinate system with the origin at point A, as shown in Fig. 10.3. Indicate the reactions that appear in the supports. At point A, the reaction is decomposed into components $\overline{X_A}$ and $\overline{Y_A}$, and at point B, the reaction $\overline{R_B}$ is directed perpendicular to the support plane.

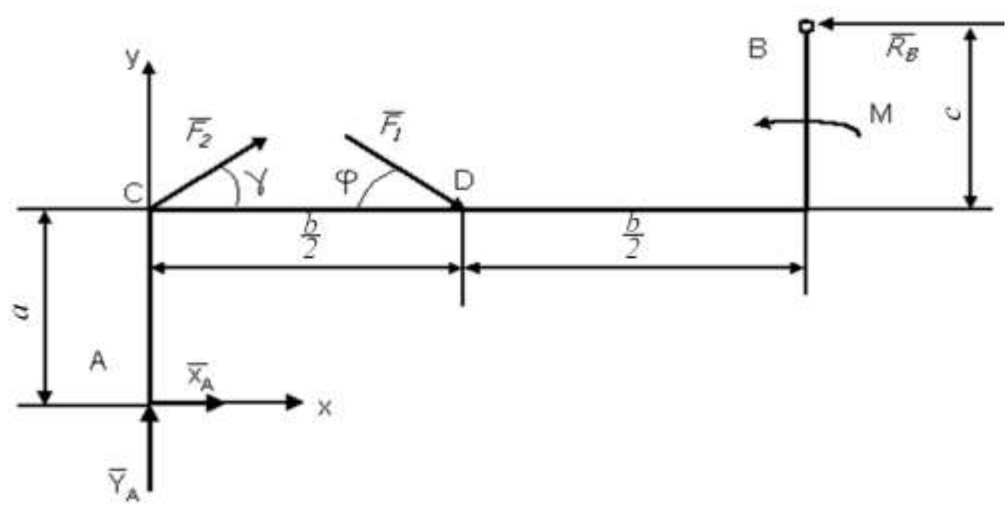


Figure 10.3

Write an equilibrium equation for a plane arbitrary system of forces:

$$\Sigma F_{xi} = 0, \qquad \Sigma F_{yi} = 0, \qquad \Sigma M_{Ai} = 0,$$

$$\Sigma F_{xi} = 0;\ X_A + F_2 \cos\gamma + F_1 \cos\varphi - R_B = 0, \tag{10.1}$$

$$\Sigma F_{yi} = 0;\ Y_A + F_2 \sin\gamma - F_1 \sin\varphi = 0, \tag{10.2}$$

$$\Sigma M_{Ai} = 0;\ - F_2\, a \cos\gamma - F_1\, a \cos\varphi - F_1\, (b / 2) \sin\varphi + M + R_B\, (a + c) = 0. \tag{10.3}$$

From equation (10.2) we find Y_A

$$Y_A = -F_2 \sin\gamma + F_1 \sin\phi = -20 \sin 45^0 + 10 \sin 30^0 = -9{,}14N. \tag{10.4}$$

From equation (10.3) we find R_B

$$R_B = \frac{F_2 a \cos\gamma + F_1 a \cos\varphi + F_1 \left(\frac{b}{2}\right) \sin\varphi - M}{(a+c)} =$$

$$= \frac{20 \cdot 5 \cos 45^0 + 10 \cdot 5 \cos 30^0 + 10 \left(\frac{8}{2}\right) \sin 30^0 - 60}{5+3} = 9{,}25N \tag{10.5}$$

From equation (10.1) we find X_A

$$X_A = -F_2 \cos\gamma - F_1 \cos\varphi + R_B = -20 \cos 45^0 - 10 \cos 30^0 + 9{,}25 = -13{,}55N \tag{10.6}$$

The modulus of reaction at point A is found by the formula

$$R_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{13{,}55^2 + 9{,}14^2} = 16{,}34N. \tag{10.7}$$

Let's make a check. The problem is solved correctly if the condition is met

$$\Sigma M_{D\,i} = 0.$$

Let's write this equation using Figure 10.3.

$$\sum_{i=1}^{n} M_D\left(\overline{F}_i\right) = -Y_A \frac{b}{2} + X_A a - F_2 \frac{b}{2} \sin\gamma + M + R_B c =$$

$$= -(-9{,}14) \cdot \frac{8}{2} + (-13{,}55) \cdot 5 - 20 \cdot \frac{8}{2} \sin 45^0 + 60 + 9{,}25 \cdot 3 = 0 \tag{10.8}$$

So, the problem is solved correctly.

Questions for self-testing

1. How to determine the moment of force relative to the center?
2. When the moment of force relative to the center is zero?
3. Why the moment of force relative to the center does not change when moving the force along the line of action?
4. What is called a pair of forces?
5. What is called the moment of a pair of forces?
6. What is called a plane parallel system of forces?
7. What is called a plane arbitrary system of forces?

11 MOMENT OF FORCE RELATIVE TO THE AXIS

The **moment of a force relative to an axis** is a scalar quantity that is numerically equal to the moment of projection of that force onto a plane perpendicular to the axis, relative to the point of intersection of the axis with the plane.

To determine the moment of a force $\overline{F}$ relative to any axis (for example, the *Oz* axis, Fig. 11.1), it is necessary to project the force onto a plane perpendicular to this axis and determine the moment of the resulting projection F_{xy} relative to the point of intersection of the axis with the plane (point O).

$$M_z\left(\overline{F}\right) = \pm F_{xy} h.$$

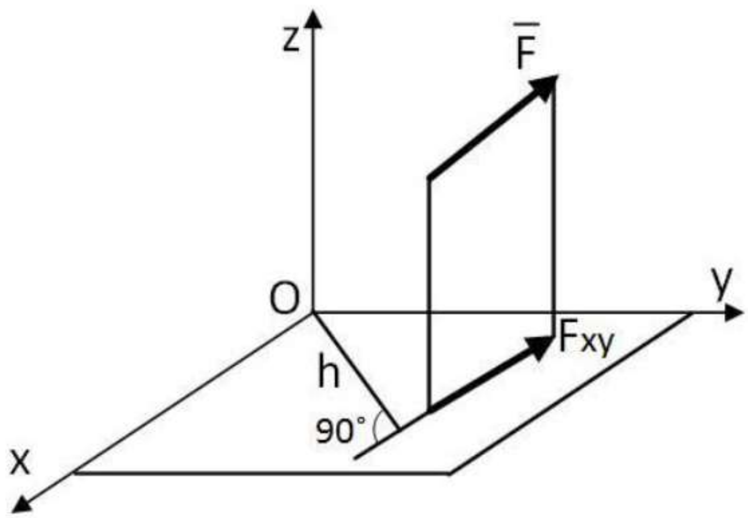


Figure 11.1

The rule of signs for the moment of force relative to the axis

The moment of force relative to the axis is considered **positive** if, when observed from the positive direction of the axis, it is clear that the force is trying to turn the body counterclockwise, otherwise it is **negative**.

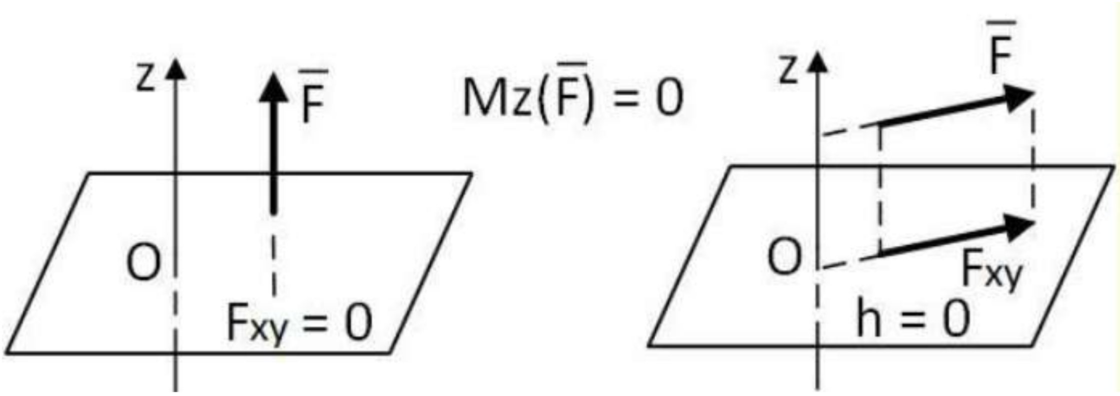
Properties of the moment of force relative to the axis

The moment of force relative to the axis is zero if:

1) the projection of the force on the plane perpendicular to the axis is zero, i.e., the force $\overline{F}$ is parallel to the axis (Fig. 11.2, a);

2) the shoulder h is zero, i.e., the line of action of the force (projection) crosses the axis (Fig. 11.2, b).

Thus, the moment of a force relative to an axis is zero if the force and the axis lie in the same plane.



a) b)

Figure 11.2

11.1 Equilibrium conditions for an arbitrary spatial system of forces

An arbitrary spatial system of forces is a system of forces anyhow arranged in space.

For **an arbitrary spatial system of forces** to be in equilibrium, it is necessary and sufficient that the principal vector and principal momentum of this system relative to any point *O* are equal to zero, i. e.

$$\overline{R}_0 = \sum_{i=1}^{n} \overline{F}_i = 0 \,;\; \overline{M}_0 = \sum_{i=1}^{n} \overline{M}_0\left(\overline{F}_i\right) = 0 \,.$$

These conditions are called equilibrium conditions for an arbitrary system of forces in **vector** (geometric) **form**.

Conditions of equilibrium of an arbitrary spatial system of forces in **analytical form**

$$\overline{R}_{0x} = \sum_{i=1}^{n} \overline{F}_{ix} = \overline{F}_{1x} + \overline{F}_{2x} + \ldots + \overline{F}_{nx} = 0;$$

$$\overline{R}_{0y} = \sum_{i=1}^{n} \overline{F}_{iy} = \overline{F}_{1y} + \overline{F}_{2y} + \ldots + \overline{F}_{ny} = 0;$$

$$\overline{R}_{0z} = \sum_{i=1}^{n} \overline{F}_{iz} = \overline{F}_{1z} + \overline{F}_{2z} + \ldots + \overline{F}_{nz} = 0;$$

$$M_{0x} = M_{0x}(\overline{F}_1) + M_{0x}(\overline{F}_2) + \ldots + M_{0x}(\overline{F}_n) = \sum_{i=1}^{n} M_{0x}(\overline{F}_i) = 0;$$

$$M_{0y} = M_{0y}(\overline{F}_1) + M_{0y}(\overline{F}_2) + \ldots + M_{0y}(\overline{F}_n) = \sum_{i=1}^{n} M_{0y}(\overline{F}_i) = 0;$$

$$M_{0z} = M_{0z}(\overline{F}_1) + M_{0z}(\overline{F}_2) + \ldots + M_{0z}(\overline{F}_n) = \sum_{i=1}^{n} M_{0z}(\overline{F}_i) = 0.$$

Thus, for an arbitrary spatial system of forces to be in equilibrium, it is necessary and sufficient that the sums of the projections of all forces on the coordinate axes and the sums of the moments of these forces relative to the coordinate axes are equal to zero.

11.2 Example of solving the problem

A rectangular homogeneous plate of weight P is fixed at point A by a spherical hinge, at point B by a cylindrical hinge, and held horizontally by a cable CC' (Fig. 11.3). Determine the reactions of the linkages if P = 100 N, F = 40 N, α=30°, β=60°, $\overline{F} \parallel zAy$.

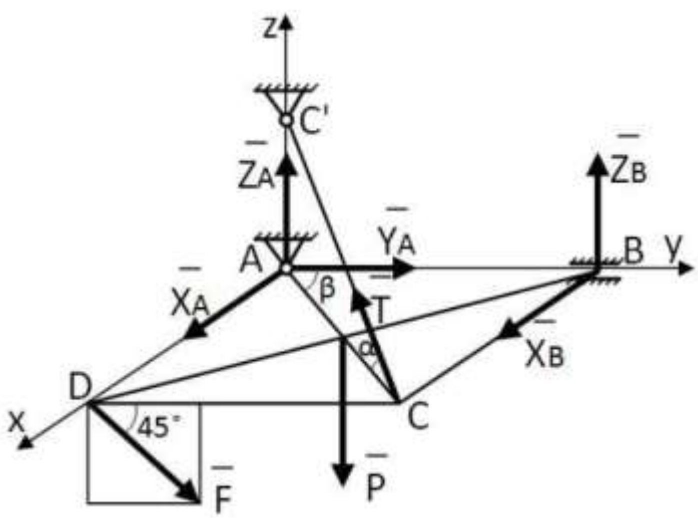


Figure 11.3

The solution: Using the principle of release from linkages, we replace the action of the linkages imposed on the plate with their reactions. At point *A* we have three components of the reaction of a spherical hinge: $\overline{X_A}$, $\overline{Y_A}$, $\overline{Z_A}$. At point *B* , we have two components of the reaction of a cylindrical hinge: $\overline{X_B}, \overline{Z_B}$. The reaction of the cable $\overline{T}$ is directed along the line CC'.

For the resulting arbitrary spatial system of forces, we write six equations of equilibrium:

$\Sigma Fix = 0$; $X_A + X_B - T\cos\alpha\sin\beta = 0$;

$\Sigma Fiy = 0$; $Y_A + F\cos 45° - T\cos\alpha\cos\beta = 0$;

$\Sigma Fiz = 0$; $Z_A + Z_B + T\sin\alpha - P - F\cos 45° = 0$;

$\Sigma Mx(\overline{F_i}) = 0$; $-P(AB/2) + Z_B AB + T\sin\alpha AB = 0$;

$\Sigma My(\overline{F_i}) = 0$; $F\sin 45° AD + P(AD/2) - T\sin\alpha AD = 0$;

$\Sigma Mz(\overline{F_i}) = 0$; $F\cos 45° AD - X_B AB = 0$.

Now we solve the equations for the unknown reactions of the linkages.

$X_B = (F\cos 45° AD)/AB = (F\cos 45° AD)/AD tg\beta = 16{,}33$ N;

$T = (F\sin 45° AD + P(AD/2))/AD\sin\alpha = 156{,}56$ N;

$Z_B = (P(AB/2) - T\sin\alpha AB)/AB = -28{,}28$ N;

$X_A = -X_B + T\cos\alpha\sin\beta = 101{,}09$ N;

$Y_A = -F\cos 45° + T\cos\alpha\cos\beta = 39{,}51$ n;

$Z_A = -Z_B - T\sin\alpha + P + F\cos 45° = 78{,}28$ N.

Answer: X_A = 101,09 N, Y_A = 39,51 N, Z_A = 78,28 N, X_B = 16,33 N, Z_B = −28,28 N, T = 156,56 N.

The minus sign means that the direction $\overline{Z_B}$ is opposite to the one shown in Figure 11.3.

11.3 Calculation-graphic and control tasks

S4. Calculation of support reactions of a spatial structure

To a weightless rigid plate ABCD (P. 60–64) are applied forces F_1, F_2 and a pair of forces with moment M acting in the plane of the plate. The supports of the plate are: in p. *A* – spatial hinge, in p. *B* – cylindrical hinge and in p. C – rod OC.

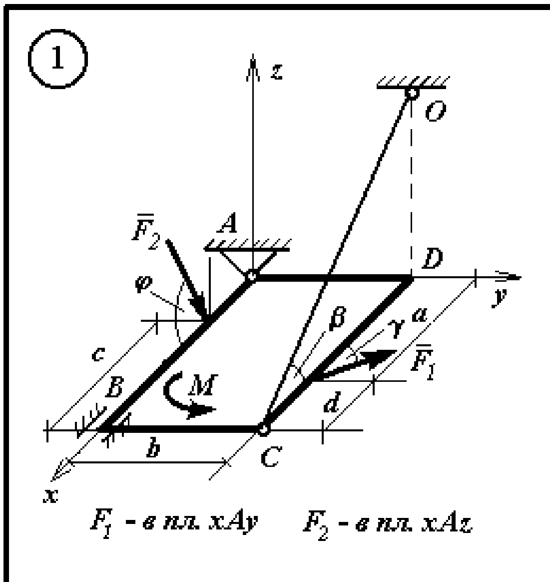
The belonging of the forces F_1, F_2 to the planes is indicated in the figures by variants, and their direction and the direction of the OC rod are determined by the angles γ, β and φ. The geometric dimensions of the plate and the position of the points of application of forces are indicated by dimensions *a*, *b*, *c*, *d*.

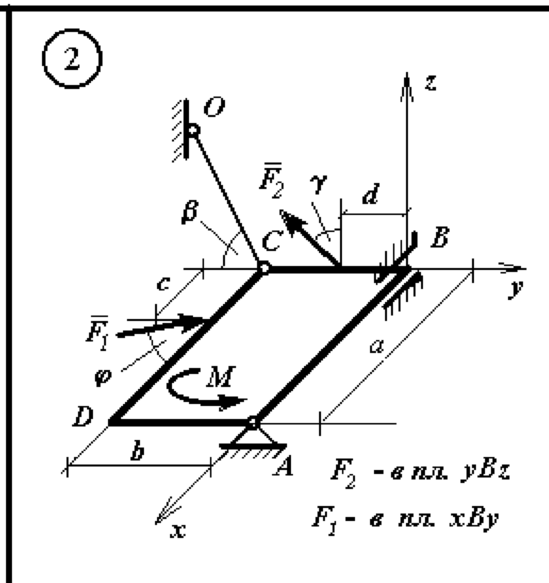
The data for the calculations are given in Table 11.1, where the forces are given in N, the moment in N·m, the angles in degrees, and the dimensions in meters.

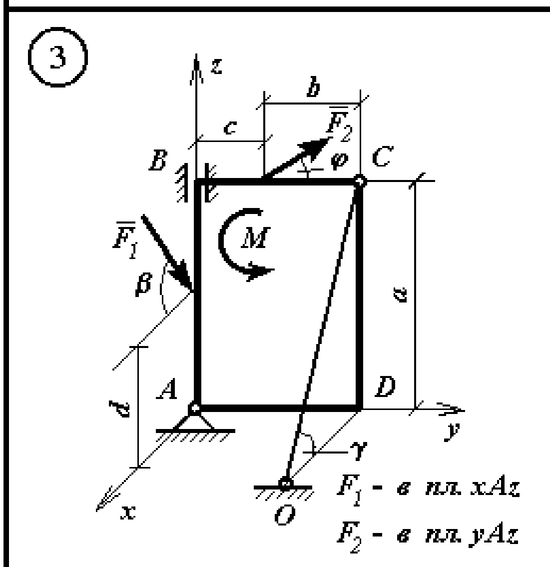
Find the modules of the reactions of the supports at points A, B, C at the equilibrium of the plate.

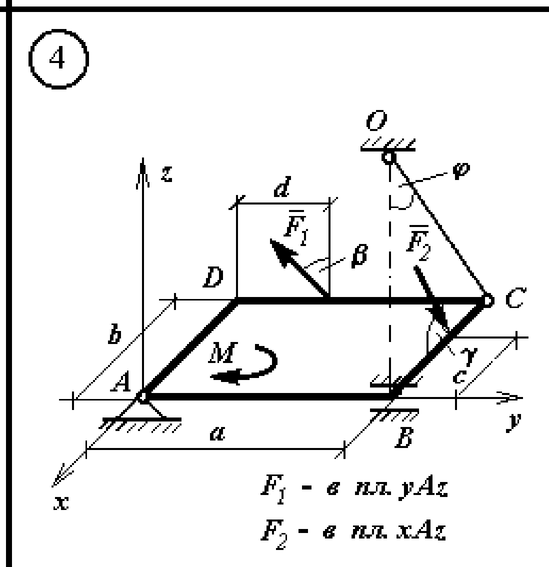
Table 11. 1

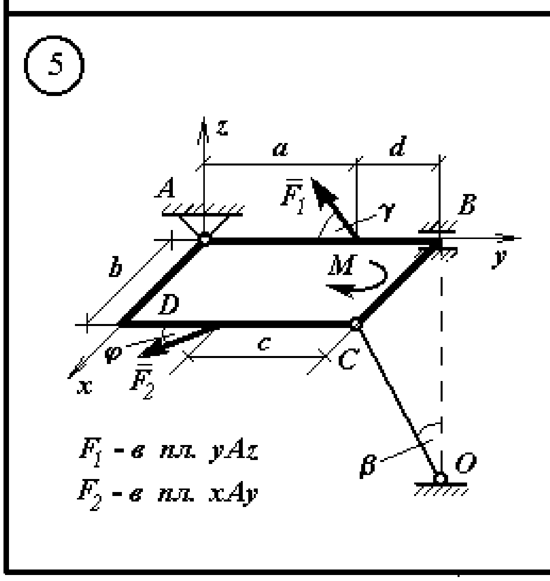
Task	F_1	F_2	M	a	b	c	d	γ	β	φ
1	15	5	22	6	4	2	3	30	60	45
2	6	8	18	8	5	3	2	60	45	30
3	12	14	14	10	3	1	4	45	30	60
4	8	3	10	12	5	3	3	45	30	60
5	7	8	15	9	8	4	2	30	60	45
6	11	10	19	14	6	2	3	60	30	45
7	4	12	10	10	8	4	3	30	45	60
8	10	7	14	16	7	5	2	60	30	30
9	8	5	16	11	5	3	3	30	45	60
0	9	6	17	15	8	4	2	60	45	30

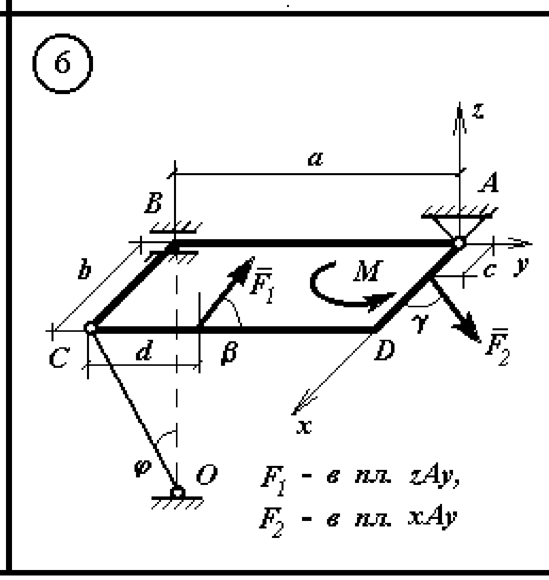

1
z
O
A
D
y
F̄2
φ
β
γ
a
c
B
M
F̄1
d
b
C
x
F_1 - в пл. xAy
F_2 - в пл. xAz


2
O
z
F̄2
γ
d
β
C
B
c
y
F̄1
a
φ
M
D
b
A
x
F_2 - в пл. yBz
F_1 - в пл. xBy

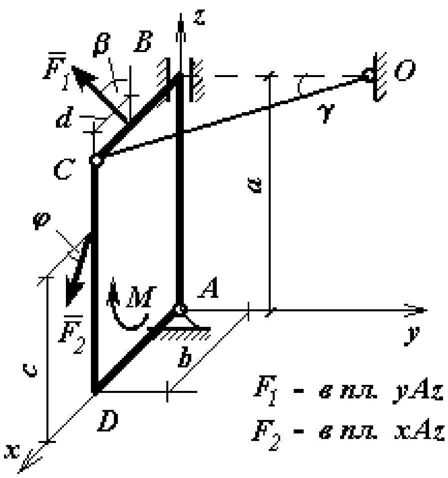

3
z
b
c
F̄2
B
φ
C
F̄1
M
β
a
A
D
d
y
γ
x
O
F_1 - в пл. xAz
F_2 - в пл. yAz


4
O
z
φ
d
F̄1
β
F̄2
D
C
b
M
γ
A
c
y
a
B
x
F_1 - в пл. yAz
F_2 - в пл. xAz


5
z
a
d
A
F̄1
γ
B
y
b
M
D
φ
c
C
x
F̄2
β
O
F_1 - в пл. yAz
F_2 - в пл. xAy

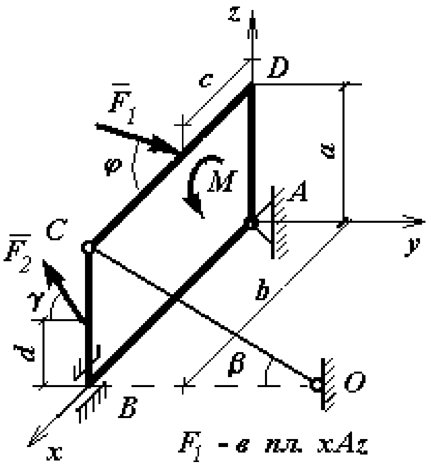

6
z
a
A
B
b
F̄1
M
c
y
C
d
β
D
γ
F̄2
x
φ
O
F_1 - в пл. zAy,
F_2 - в пл. xAy

7



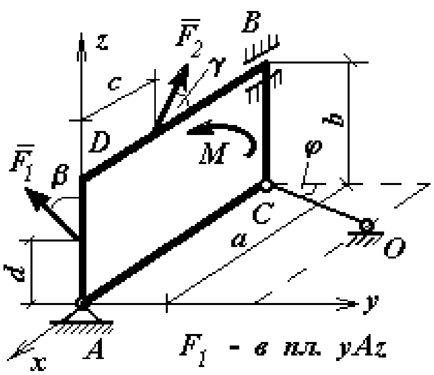
F_1 - в пл. yAz
F_2 - в пл. xAz

8



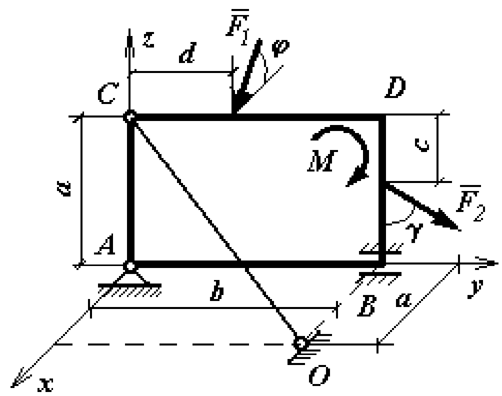
F_1 - в пл. xAz
F_2 - в пл. yAz

9



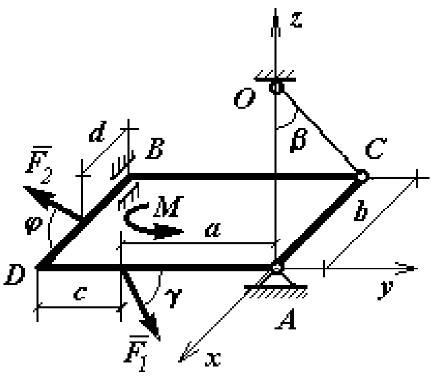
F_1 - в пл. yAz
F_2 - в пл. xAz

10



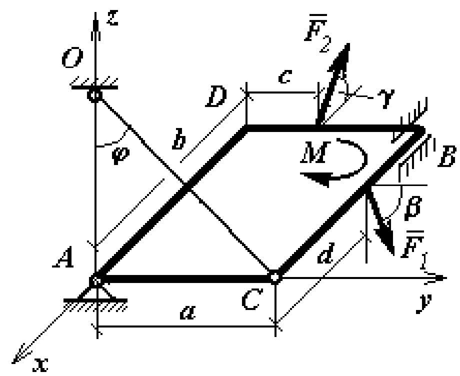
F_1 - в пл. xAz, F_2 - в пл. yAz

11



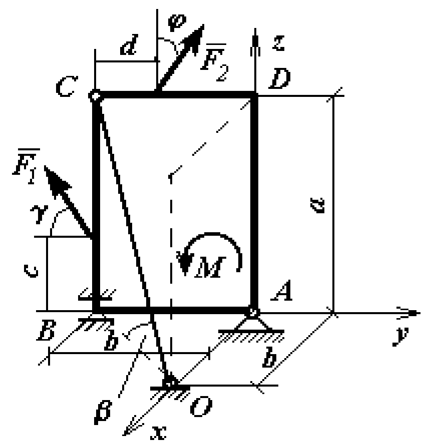
F_1 - в пл. yAz, F_2 - в пл. xAz

12



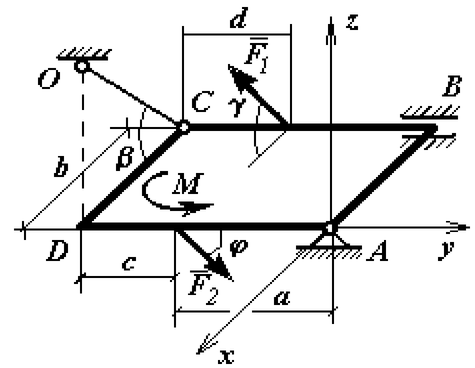
F_2 - в пл. xAz F_1 - в пл. yAz

(13)



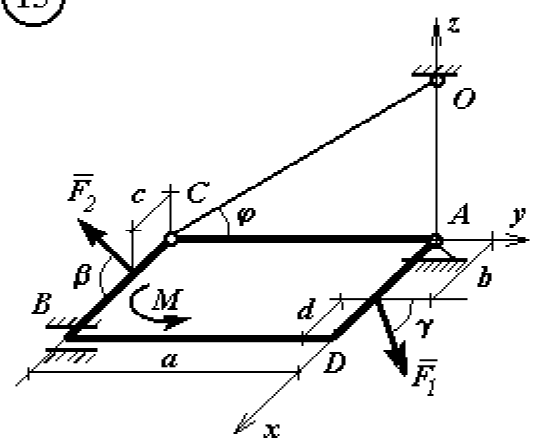
F_1 - в пл. yAz, F_2 - в пл. xAz

(14)



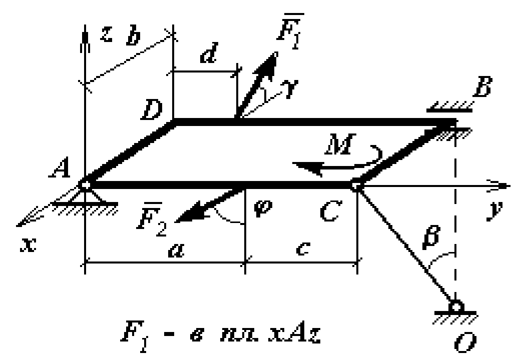
F_1 - в пл. xAz, F_2 - в пл. yAz

(15)



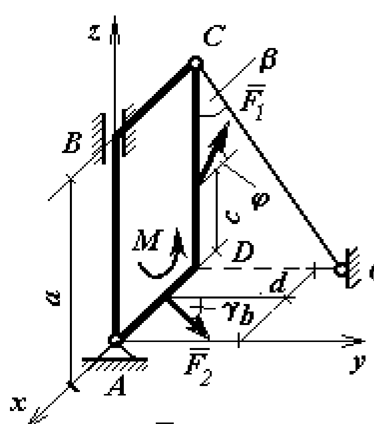
F_2 - в пл. xAz F_1 - в пл. yAz

(16)



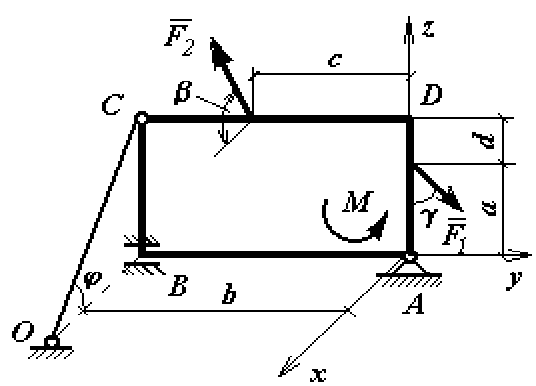
F_1 - в пл. xAz
F_2 - в пл. yAz

(17)



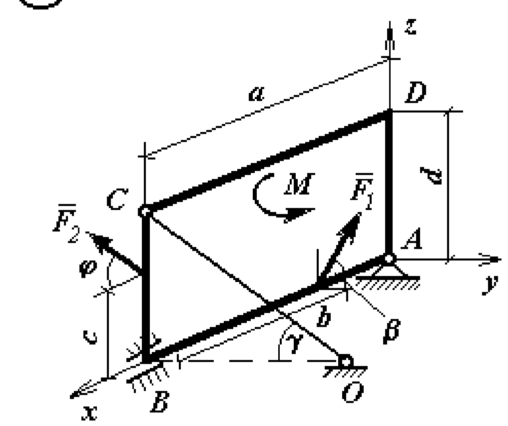
F_1 - в пл. xAz
F_2 - в пл. yAz

(18)



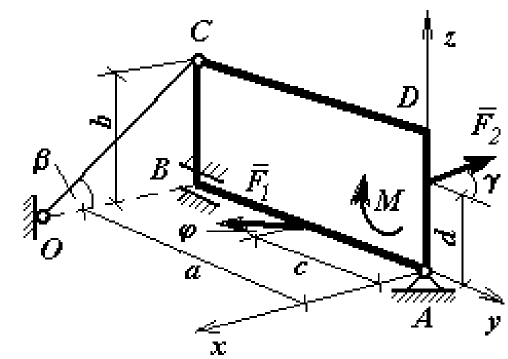
F_1 - в пл. yAz, F_2 - в пл. xAz

19



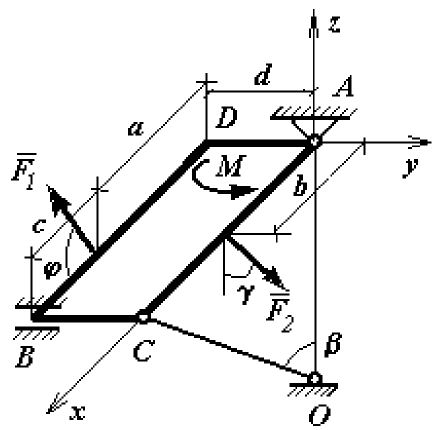
$\bar{F}_1$ - в пл. yAz $\bar{F}_2$ - в пл. xAz

20



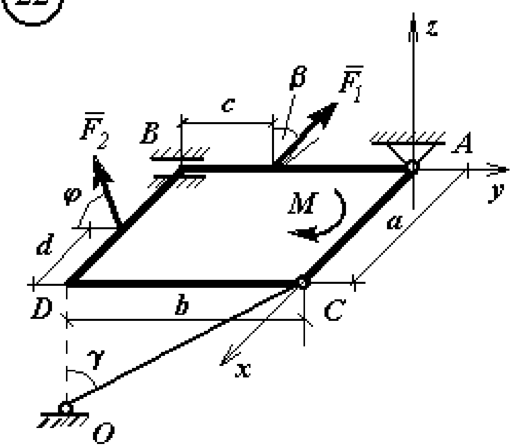
F_1 - в пл. xAy, F_2 - в пл. yAz

21



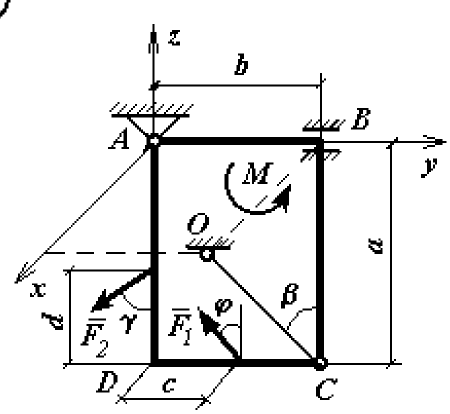
F_1 - в пл. xAz, F_2 - в пл. yAz

22



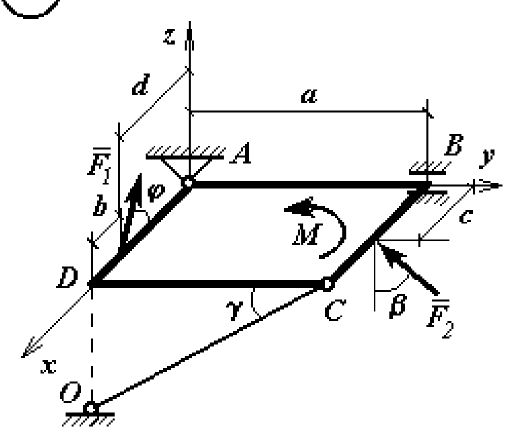
F_1 - в пл. xAz, F_2 - в пл. yAz

23



F_1 - в пл. yAz, F_2 - в пл. xAz

24



F_1 - в пл. xAz, F_2 - пл. yAz

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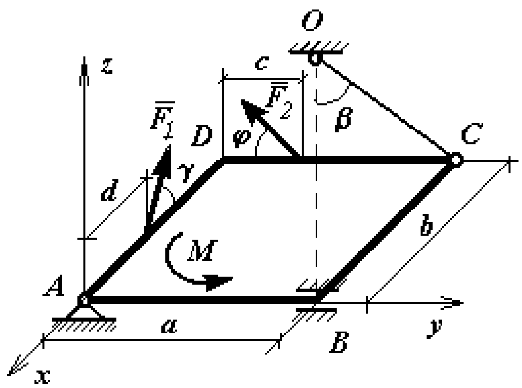

F_1 - в пл. xAz, F_2 - в пл. yAz

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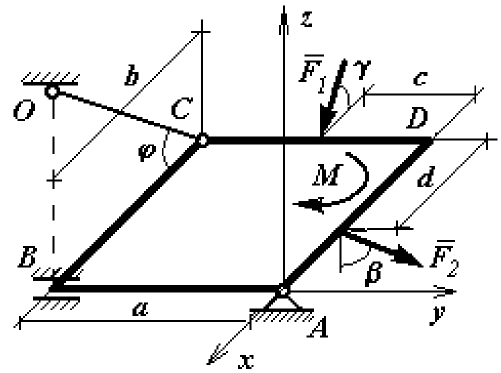

F_1 - в пл. xAz, F_2 - в пл. yAz

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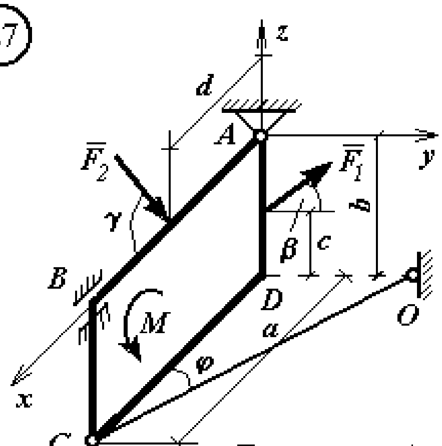
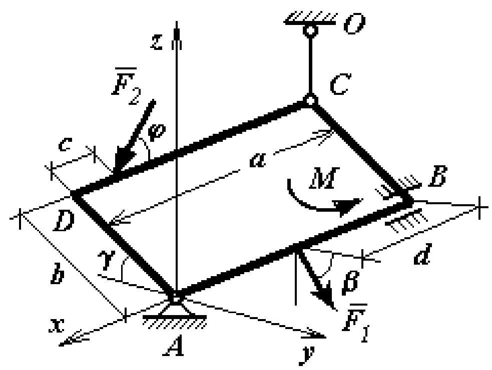


F_1 - в пл. yAz

F_2 - в пл. xAz

28



F_1 - в пл. yAz, F_2 - в пл. xAz

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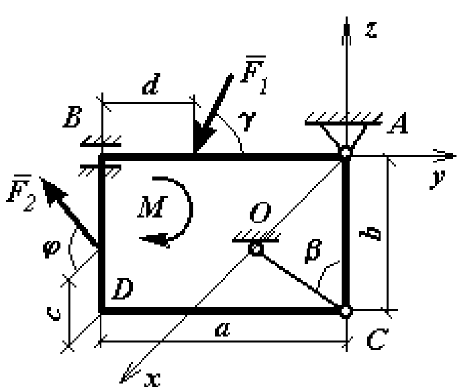
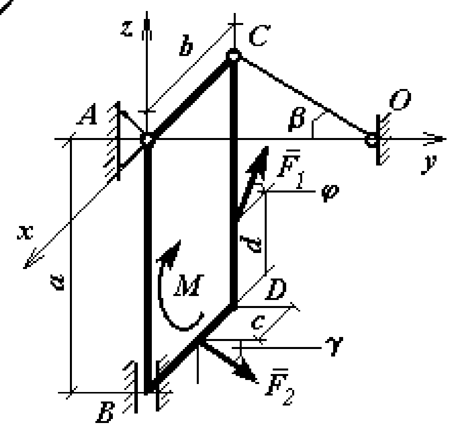


F_1 - в пл. yAz, F_2 - в пл. xAz

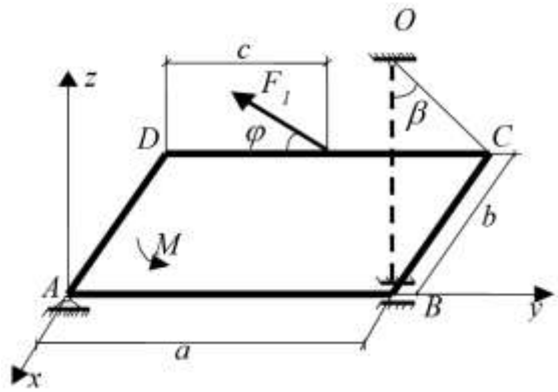
30



F_1 - в пл. xAz, F_2 - в пл. yAz

11.4 Example of task execution

The weightless plate ABCD shown in Fig. 11.4 is in equilibrium under the action of a force F_1 = 20 N and a pair of forces with a moment M = 30 N·m acting in the plane of the plate. The force F_1 is in a plane parallel to the *yAz* plane and acts at an angle φ to the *y-axis*. The dimensions of the plate and the position of the point of application of the force F_1 are determined by the values: a = 6 m, b = 8 m and c = 2 m.



F_1 – in the plane of yAz

Figure 11.4

The plate is supported by supports: in p. A – by a spherical hinge, in p. B – by a cylindrical hinge, and in p. C – by the OC rod, which lies in a plane parallel to the *xAz* coordinate plane and makes an angle β = 60 ° with the vertical.

Find the reactions of the supports at points A, B and C.

The solution: Figure 11.5 shows the forces and reactions acting on the plate. At support A, the reaction $\overline{R_A}$ is decomposed into components $\overline{X_A}$, $\overline{Y_A}$, $\overline{Z_A}$, at support B, the reaction $\overline{R_B}$ is decomposed into $\overline{X_B}$ and $\overline{Y_B}$, at point C, the reaction $\overline{R_C}$ is directed along the OC rod.

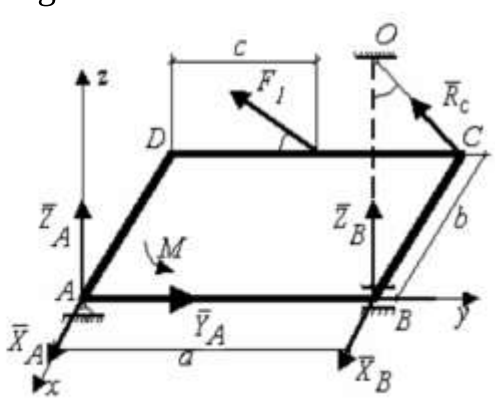


Figure 11.5

The equilibrium equation for the spatial arbitrary system of forces considered in the problem is as follows:

$$\left.\begin{aligned}
&\sum F_X = 0;\ X_A + X_B + R_C \sin\beta = 0;\\
&\sum F_Y = 0;\ Y_A - F_1 \cos\phi = 0;\\
&\sum F_Z = 0;\ Z_A + Z_B + F_1 \sin\phi + R_C \cos\beta = 0;\\
&\sum M_X(\overline{F}) = 0;\ F_1 c \sin\phi + Z_B a + R_C a \cos\beta = 0;\\
&\sum M_Y(\overline{F}) = 0;\ F_1 b \sin\phi + R_C b \cos\beta = 0;\\
&\sum M_Z(\overline{F}) = 0;\ M - X_B a + F_1 b \cos\phi - R_C a \sin\beta = 0.
\end{aligned}\right\}$$

We now solve the equations in the following sequence:

$$\left.\begin{aligned}
&Y_A = F_1 \cos\varphi;\\
&R_C = -\frac{F_1 b \sin\varphi}{b\cos\beta};\\
&X_B = \frac{M + F_1 b\cos\varphi - R_C a \sin\beta}{a};\\
&X_A = -X_B - R_C \sin\beta;\\
&Z_B = -\frac{F_1 c \sin\varphi + R_C a \cos\beta}{a};\\
&Z_A = -Z_B - F_1 \sin\varphi - R_C \cos\beta.
\end{aligned}\right\}$$

Substituting the data, we have:

$$\left.\begin{aligned}
&Y_A = 17{,}32N;\\
&R_C = -20N;\\
&X_B = 10{,}77N;\\
&X_A = 6{,}55N;\\
&Z_B = 6{,}67N;\\
&Z_A = -6{,}67N.
\end{aligned}\right\}$$

Let's find the reaction modules R_A and R_B using the formulas:

$$R_A = \sqrt{X_A^2 + Y_A^2 + Z_A^2} = 15{,}98N,$$
$$R_B = \sqrt{X_B^2 + Z_B^2} = 12{,}67N.$$

Questions for self-testing

1. What is called a moment of force about an axis?
2. The rule of signs of the moment of force relative to the axis?
3. In which cases the moment of force relative to the axis is zero?
4. What is called an arbitrary spatial system of forces?
5. Conditions of equilibrium of an arbitrary spatial system of forces.

12 FRICTION OF SOLID BODIES

12.1 Sliding friction

The force interaction when two material objects come into contact is associated with the concept of linkages. Linkages, limiting the behavior of the body, change its state.

Consider the interaction of a plane and a solid body at p. M (Fig. 12.1).

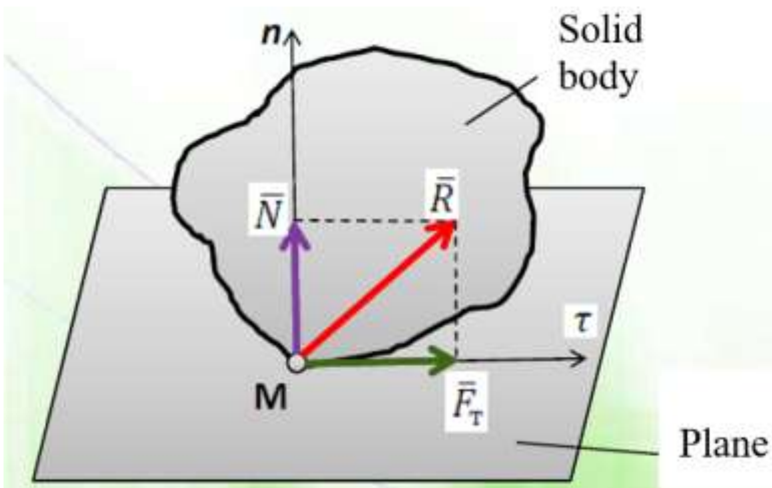


Figure 12.1

$\overline{R}$ – plane reaction (linkages reaction);
$\overline{N}$ – normal reaction (normal component of the force);
$\overline{F_T}$ – friction force (tangential component of the force $\overline{R}$).

If the friction force is small, it is not taken into account. In this case, the surface is called perfectly smooth and only the normal component is taken into account.

However, in real life, when two material bodies come into contact, friction is most often taken into account. **Friction** is a physical phenomenon accompanied by the destruction of contact surfaces, heating of bodies, electrification, etc.

In theoretical mechanics, friction is taken into account based on the Amonton-Coulomb laws of sliding friction at rest.

The sliding friction force is the friction force that acts on a body when it slides on a supporting surface.

The resting friction force is the friction force that occurs before the sliding starts in the presence of forces trying to move the body.

The **Amonton-Coulomb laws** apply to dry friction (without lubrication) and are formulated as follows:

1. When one body tries to move along the surface of another, a friction force $\overline{F_T}$ occurs in the plane of contact of the bodies, the value of which can vary from 0 to the maximum limiting friction force $\overline{F_{T\,\max}}$.

$$F_T = 0 \ldots F_{T\max}; \qquad F_T \le F_{T\max}$$

2. The friction force is directed in the opposite direction to the possible direction of movement or sliding (Fig. 12.2).

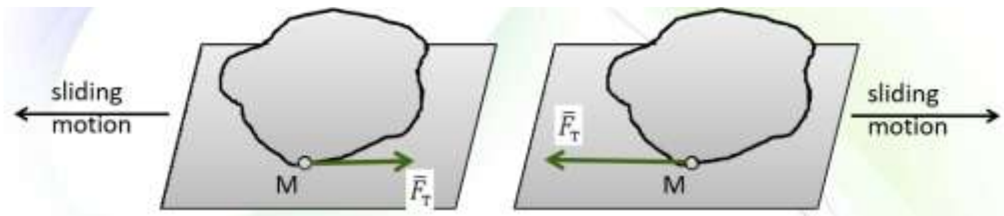


Figure 12.2

3. The value of the maximum maximum friction force is equal to the product of the friction coefficient and the normal reaction

$$F_{T\max} = f \cdot N,$$

where f is the static friction coefficient (dimensionless value).

4. The magnitude $\overline{F_{T\,\max}}$ does not depend on the size of the surface area of the interaction.Thus, at equilibrium, the resting friction force

$$F_T \le F_{T\max} \text{ або } F_T \le f \cdot N.$$

5. The friction coefficient f depends on the material and condition of the friction surface.

6. The sliding friction force is less than the resting friction force. In reference books, f is denoted as the sliding friction coefficient, f_0 as the resting friction coefficient.

7. It is considered that the friction force does not depend on the sliding speed.

During movement, the friction force is equal to the product of the dynamic coefficient of friction by the normal reaction

$$F_T = f_Д \cdot N.$$

The angle of friction is the largest angle φ between the limiting force of the rough linkage reaction $\overline{R}^{max}$ and the normal reaction $\overline{N}$ (Fig. 12.3).

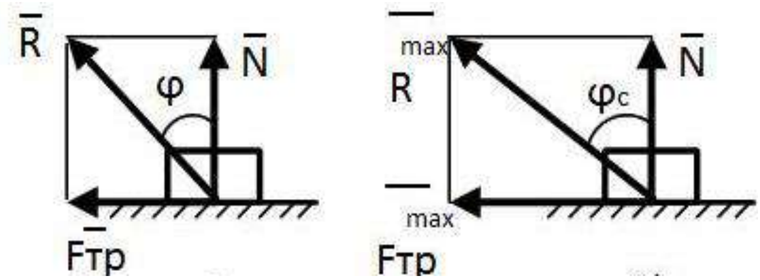


Figure 12.3

$$tg\varphi = \frac{F_T}{N} = (F_T = f \cdot N) = \frac{f \cdot N}{N} = f \ .$$

The cone of friction is the surface described by the total reaction when it is rotated around the normal reaction (Fig. 12.4).

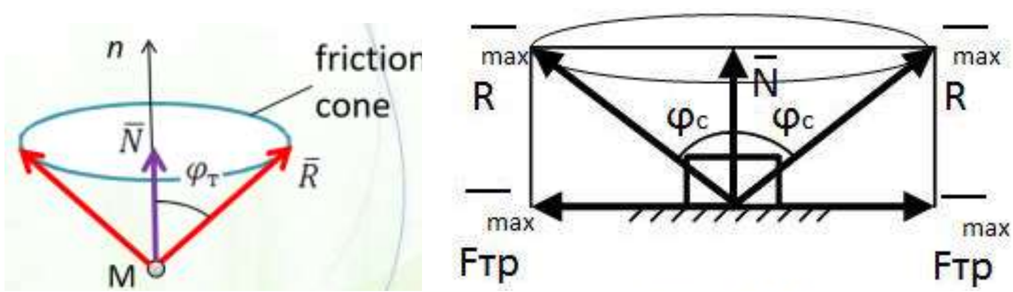


Figure 12.4

If the equivalent of the active forces acting on a body is inside the friction cone, then no increase in the modulus of this equivalent can disturb the body's balance.

This phenomenon is called self-braking and is widely used in industry, in particular in lifting mechanisms.

12.2 Rolling friction

The resistance that occurs when one body rolls on the surface of another is called **rolling friction.**

Consider a cylinder at rest on a horizontal plane. Let's show the forces that arise (Fig. 12.5).

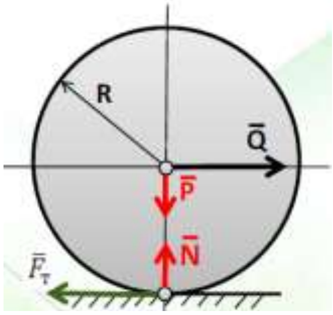


Figure 12.5

$\overline{Q}$ – an active force that tries to move the cylinder out of place;
$\overline{P}$ – weight force;
$\overline{N}$ – normal reaction;
$\overline{F_T}$ – friction force;
R – cylinder radius.

The cylinder is at rest. The active force $\overline{Q}$ and the frictional force $\overline{F_T}$ form a force pair that can cause rolling.

The body (cylinder) does not move, because in fact the contact of the body with the horizontal plane does not occur at point C (Fig. 12.6), but along some plane (due to the deformations of the roller and the horizontal support surface, they touch each other along some contact area). Thus, the normal reaction is actually shifted towards the active force by a certain amount δ.

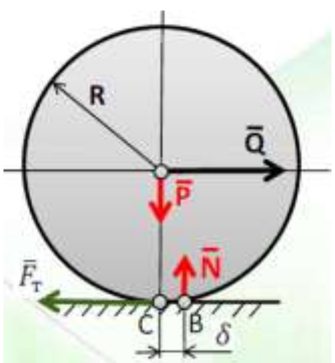


Figure 12.6

In the state of equilibrium we have:

$$\sum_{i=1}^{n} M_C\left(\overline{F_i}\right) = 0;$$

$$-Q \cdot R + N \cdot \delta = 0;$$

$$Q_{\lim} = \frac{N \cdot \delta}{R}.$$

When $Q < Q_{\lim}$, the body is at rest, and when $Q > Q_{\lim}$, rolling begins.

The value δ is called the rolling friction coefficient and has units of length.

The product of the normal reaction to the rolling friction coefficient is called **the rolling limit moments**

$$M_{\lim} = N \cdot \delta .$$

Typically, the rolling friction force is much lower than the sliding friction force.

Questions for self-testing

1. What is called friction?
2. What is called sliding friction?

3. What is the sliding friction force?
4. What is the angle of friction? What is the tangent of the angle of friction?
5. What are the units of measurement of the sliding friction coefficient?
6. What is called rolling friction?
7. What is the rolling friction moment?
8. What are the units of measurement of the coefficient of rolling friction?

12.3 Example of solving a problem with friction forces

Determine the limiting force Q, acting at an angle of 30º, after which the body will move (Fig. 12.7). The weight of the body is 10 N, the coefficient of friction is 0,6.

Solution

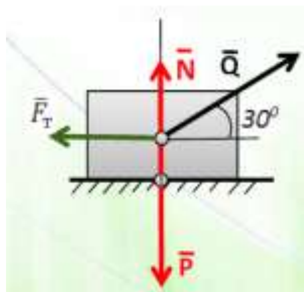


Figure 12.7

We show all the forces acting on the body. Since this problem is a flat convergent system of forces, we can write two equations of equilibrium:

$$\sum_{i=1}^{n} F_{ix} = 0; \quad -F_T + Q \cdot cos30° = 0 \quad \rightarrow \quad F_T = Q \cdot cos30°.$$

$$\sum_{i=1}^{n} F_{iy} = 0; \ N - P + Q \cdot sin30° = 0 \ \rightarrow \ N = P - Q \cdot sin30°$$

$$Q \cdot cos30° \leq (P - Q \cdot sin30°) \cdot f;$$

$$Q \cdot cos30° + Q \cdot sin30° \cdot f \leq P \cdot f;$$

$$F_T \leq F_{T\,max}$$

$$F_{T\,max} = f \cdot N$$

$$Q \leq \frac{P \cdot f}{cos30° + sin30° \cdot f}$$

$$Q \leq 5{,}2\ H.$$

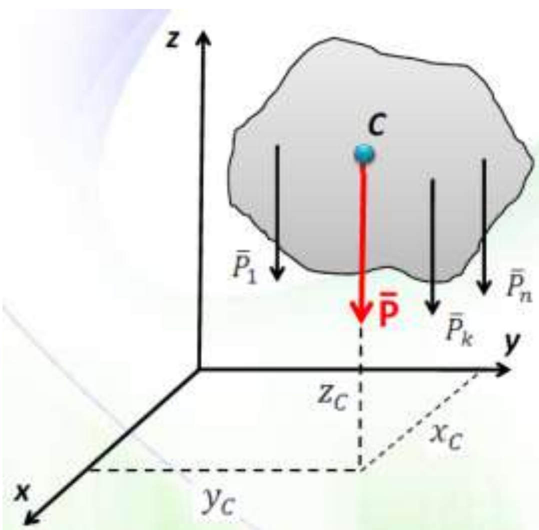
13 CENTER OF GRAVITY OF A SOLID BODY

13.1 The concept of the body's center of gravity, coordinates of the center of gravity

Any body placed on the Earth's surface is subject to the forces of the weight of each part of that body. The lines of action of these forces intersect in the center of the Earth. Because the size of the bodies is small enough, we can consider that they form a spatial system of parallel forces (Fig. 13.1).

The force of gravity or the weight of a solid body is an equal force determined by the sum of the forces of the weight of all parts of the body

$$P = \sum_{i=1}^{n} P_i$$



p. C – center of gravity of the body;
x_C, y_C, z_C – coordinates of the center of gravity.

Figure 13.1

The center of gravity of a solid body is the point invariably associated with this body through which the line of action of the equal force of weight of the body parts passes at any position of the body in space, or it is the point of application of the body's weight.

The following formulas can be used to determine the coordinates of a body's center of gravity:

$$x_C = \frac{\sum_{k=1}^{n} x_k P_k}{\sum_{k=1}^{n} P_k} = \frac{\sum_{k=1}^{n} x_k P_k}{P},$$

$$y_C = \frac{\sum_{k=1}^{n} y_k P_k}{\sum_{k=1}^{n} P_k} = \frac{\sum_{k=1}^{n} y_k P_k}{P}, \qquad (13.1)$$

$$z_C = \frac{\sum_{k=1}^{n} z_k P_k}{\sum_{k=1}^{n} P_k} = \frac{\sum_{k=1}^{n} z_k P_k}{P}.$$

where P_k – weight of a separate body part;
x_k, y_k, z_k – coordinates of the body part;
P – weight of the whole body.

The center of gravity of a homogeneous solid body

Homogeneous bodies include bodies whose specific gravity is constant in volume $\rho = const$. Then $P = \rho V$, $P_k = \rho V_k$, where V – whole body volume; V_k – volume of the body particle.

Substituting these values into the formulas (13.1)

$$x_C = \frac{\sum_{k=1}^{n} x_k P_k}{P} = \frac{\sum_{k=1}^{n} x_k \rho V_k}{\rho V} = \frac{\rho \sum_{k=1}^{n} x_k V_k}{\rho V} = \frac{\sum_{k=1}^{n} x_k V_k}{V},$$

we obtain

$$x_C = \frac{\sum_{k=1}^{n} x_k V_k}{V};$$

$$y_C = \frac{\sum_{k=1}^{n} y_k V_k}{V}; \qquad (13.2)$$

$$z_C = \frac{\sum_{k=1}^{n} z_k V_k}{V}.$$

Equation (13.2) allows us to determine the coordinates of the center of gravity of a three-dimensional body.

Center of gravity of a homogeneous plane body

The weight of a homogeneous plane body and the weight of its individual parts can be determined by the formulas

$$P=\rho S, P_k=\rho S_k,$$

where ρ – weight per unit of body area;

S – whole body area;

S_k – the area of each part of the body.

Thus, taking into account the above and formulas (13.1), we obtain

$$x_C = \frac{\sum_{k=1}^{n} x_k S_k}{S};$$

$$y_C = \frac{\sum_{k=1}^{n} y_k S_k}{S}. \qquad (13.3)$$

Equation (13.3) allows us to determine the coordinates of the center of gravity of a plane body.

Center of gravity of a homogeneous linear body

Let ρ be the weight of a unit length of a homogeneous linear body. Then its weight and the weight of its part are equal to

$$P=\rho L, P_k=\rho l_k,$$

where L – body length;

l_k – length of each body element.

Substituting these values into formulas (13.1), we obtain:

$$x_C = \frac{\sum_{k=1}^{n} x_k l_k}{L};$$

$$y_C = \frac{\sum_{k=1}^{n} y_k l_k}{L}; \qquad (13.4)$$

$$z_C = \frac{\sum_{k=1}^{n} z_k l_k}{L}.$$

Equation (13.4) allows you to determine the coordinates of the center of gravity of a linear body (for example, a rod structure, a curved line etc.).

13.2 Methods for determining the center of gravity of a solid body

The symmetry method

If a body has a plane, an axis, or a center of symmetry, then the center of gravity of the body is located either in the plane of symmetry, on the axis of symmetry, or in the center of symmetry (Figure 13.2).

It follows that the center of gravity of regular geometric bodies (circle, disk, sphere, rhombus, rectangle) is located in their geometric centers.

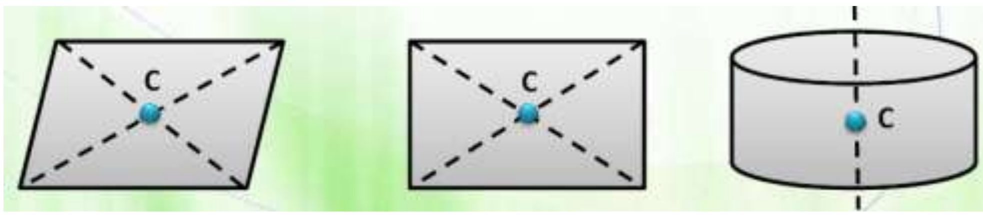


Figure 13.2

Breakdown method

If a homogeneous solid can be divided into parts for which the position of their centers of gravity is known in advance (13.3), then the coordinates of the center of gravity of the whole body are determined by the corresponding formulas given above.

In this case, the number of terms in each of the sums will be equal to the number of parts into which the body is divided.

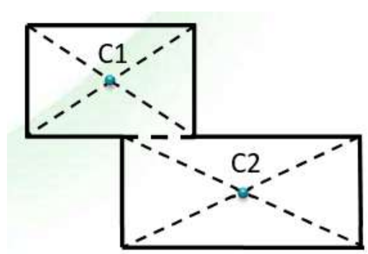


Figure 13.3

An example of solving the problem

Determine the coordinates of the center of gravity of the homogeneous plane shown in Fig. 13.4. All dimensions are in centimeters.

Solution. Draw the *x* and *y* axes and divide the plate into three rectangles (the dividing lines are shown in the figure). Calculate the coordinates of the centers of gravity of each of the rectangles and their areas. Enter the data in the table.

Table 13.1

D.n.	S_k	x_k	y_k	$S_k x_k$	$S_k y_k$
1	4	-1	1	-4	4
2	20	1	5	20	100
3	12	5	9	60	108

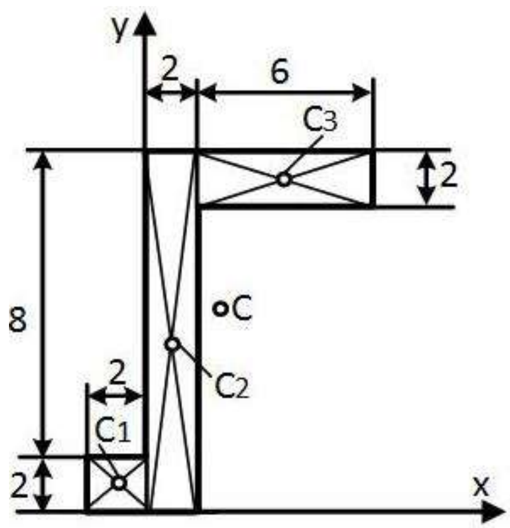


Figure 13.4

Area of the whole plate $S=S_1+S_2+S_3=36\ cm^2$.
Substituting the calculated values into formulas (13.3), we obtain:

$$x_C = \frac{x_1S_1+x_2S_2+x_3S_3}{S} = \frac{-4+20+60}{36} = 2{,}11\ см,$$

$$y_C = \frac{y_1S_1+y_2S_2+y_3S_3}{S} = \frac{4+100+108}{36} = 5{,}89\ см.$$

The resulting position of the center of gravity C is shown in Figure 13.4.

Answer: $x_C = 2{,}11\ cm$, $y_C = 5{,}89\ cm$. The point C was outside the plate.

Addition method (negative weight)

The addition method is applied to bodies that have cut-out parts (holes, cavities) (Figure 13.5).

When calculating, keep in mind that cut parts (holes, cavities) have negative areas or volumes.

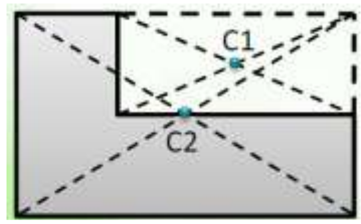


Figure 13.5

An example of solving the problem

Determine the position of the center of gravity of a circular plate of radius *R* with a hole of radius *r* (13.6). The distance C_1C_2 = a.

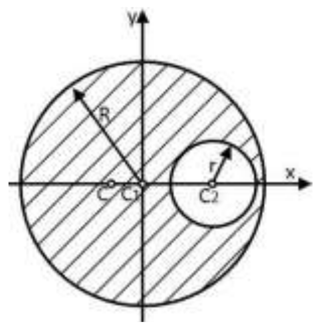


Figure 13.6

Solution. The center of gravity of the plate lies on the line C_1C_2, since this line is the axis of symmetry. Let's draw the coordinate axes. To find the coordinate x_C, add the area of the plate to a complete circle (part 1), and then subtract the area of the cut out circular hole (part 2) from the resulting area. In this case, the area of part 2 (the hole) is taken with a negative sign. Then

$$S_1 = \pi R^2,\ S_2 = -\pi r^2,\ x_1 = 0,\ x_2 = a,$$

$$S = S_1 + S_2 = \pi(R^2 - r^2).$$

Substituting the calculated values into formulas (13.3), we obtain

$$x_C = \frac{x_1S_1 + x_2S_2}{S} = -\frac{ar^2}{R^2 - r^2},$$

$$y_C = 0.$$

Answer: $x_C = -\dfrac{ar^2}{R^2 - r^2}, y_C = 0$. As can be seen in Figure 13.6, the center of gravity C is left of point C_1.

There are also experimental methods for determining the position of the center of gravity of bodies, such as **the method of suspension, the method of weighing**.

Questions for self-testing

1. What is the weight of a solid?
2. What is called the center of gravity of a solid?
3. What formulas are used to determine the coordinates of the center of gravity of a solid?
4. What are the methods of determining the position of the center of gravity of the body?
5. What is the method of symmetry?
6. What is the essence of the method of breaking?
7. Explain the features of the method of addition (negative weight).

13.3 Calculation-graphic and control tasks

S5 Center of gravity of the plate

Determine the center of gravity of the plate (Fig. 13.7). The data for the calculation in Table 13.2 are given in meters.

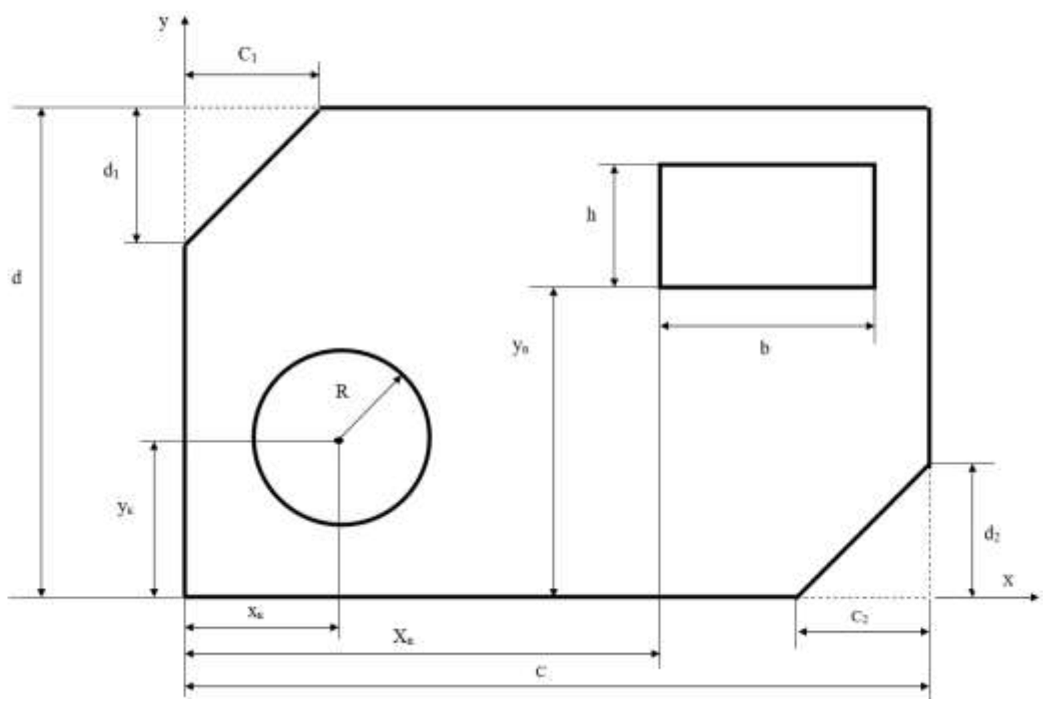


Figure 13.7

Table 13.2

№ task	***C***	***D***	C_1	C_2	d_1	d_2	$x_к$	$y_к$	X_n	y_n	***h***	***b***	***R***
1	0.2	0.7	-	0.1	-	0.2	0.02	0.04	0.1	0.4	0.2	0.05	0.01
2	0.3	0.2	0.1	-	0.05	-	0.02	0.02	0.15	0.1	0.05	0.1	0.01
3	0.4	0.3	-	0.15	-	0.3	0.3	0.25	0.05	0.05	0.1	0.2	0.02
4	0.5	0.4	0.2	-	0.2	-	0.1	0.1	0.2	0.2	0.1	0.2	0.05
5	0.6	0.5	-	0.2	-	0.1	0.3	0.4	0.1	0.2	0.2	0.1	0.1
6	0.7	0.6	0.3	-	0.4	-	0.5	0.4	0	0	0.3	0.1	0.1
7	0.8	0.7	-	0.3	-	0.4	0.6	0.5	0.1	0.2	0.2	0.2	0.1
8	0.7	0.6	0.2	-	0.1	-	0.3	0.2	0.4	0.4	0.3	0.1	0.2
9	0.6	0.5	-	0.2	-	0.25	0.2	0.2	0.3	0.3	0.2	0.1	0.15
10	0.5	0.4	0.1	-	0.05	-	0.3	0.3	0	0	0.15	0.2	0.05
11	0.4	0.3	-	0.2	-	0.1	0.1	0.1	0.2	0.25	0.1	0.1	0.1
12	0.3	0.5	0.1	-	0.2	-	0.2	0.3	0	0	0.1	0.2	0.1
13	0.2	0.4	-	0.05	-	0.2	0.1	0.1	0.1	0.2	0.05	0.1	0.05
14	0.8	1.0	0.4	-	0.05	-	0.6	0.7	0.1	0.1	0.2	0.3	0.1
15	0.7	0.9	-	0.3	-	0.2	0.5	0.7	0.5	-	0.4	0.2	0.1
16	0.6	0.8	0.2	-	0.3	-	0.2	0.3	0.5	0.5	0.1	0.1	0.1
17	0.5	0.7	-	0.2	-	0.4	0.3	0.5	0	0	0.2	0.3	0.1
18	0.4	0.6	0.1	-	0.2	-	0.2	0.2	0.3	0.1	0.2	0.1	0.2
19	0.3	0.5	-	0.1	-	0.2	0.2	0.2	0	0	0.1	0.2	0.1
20	0.4	0.4	0.05	-	0.1	-	0.1	0.1	0.3	0.3	0.1	0.1	0.1
21	0.9	0.7	-	0.3	-	0.1	0.5	0.4	0	0	0.2	0.3	0.1
22	0.8	0.6	0.3	-	0.1	-	0.1	0.1	0.6	0.5	0.1	0.2	0.1
23	0.7	0.5	-	0.3	-	0.1	0.5	0.4	0	0	0.2	0.2	0.1
24	0.6	0.4	0.2	-	0.1	-	0.1	0.1	0.4	0.3	0.1	0.1	0.2
25	0.5	0.3	-	0.1	-	0.05	0.4	0.2	0	0	0	0	0.1
26	0.4	0.2	0.1	-	0.05	-	0.1	0.1	0.3	0.25	0.05	0.1	0.2
27	0.3	0.5	-	0.05	-	0.1	0.2	0.4	0	0	0.1	0.1	0.2
28	0.8	0.6	0.3	-	0.2	-	0.1	0.1	0.5	0.4	0.2	0.3	0.1
29	0.7	0.5	-	0.3	-	0.1	0.5	0.4	0	0	0.3	0.3	0.1
30	0.6	0.4	0.2	-	0.1	-	0.1	0.1	0.5	0.3	0.1	0.1	0.2

13.4 Example of task execution

To find the center of gravity of the cross section shown in Fig. 13.8. The dimensions are given in millimeters.

The center of gravity of the cross section is found from the equations

$$X_c = \frac{\sum_{k=2}^{n} S_k \cdot X_k}{\sum_{k=1}^{n} S_k}, Y_c = \frac{\sum_{k=2}^{n} S_k \cdot Y_k}{\sum_{k=1}^{n} S_k}, \tag{13.5}$$

where S_k – the area of each figure;

X_k, Y_k – coordinates of the center of gravity of each figure.

Поперечний переріз розглядаємо як прямокутник 1 розмірами 100×80мм, з якого вирізали трикутник 2, прямокутник 3 та коло 4.

Визначимо площі та координати центрів ваги тіл 1, 2, 3, 4.

The cross-section is considered as a rectangle 1 with dimensions 100×80 mm from which triangle 2, rectangle 3 and circle 4 are cut.

Determine the areas and coordinates of the centers of gravity of bodies 1, 2, 3, 4.

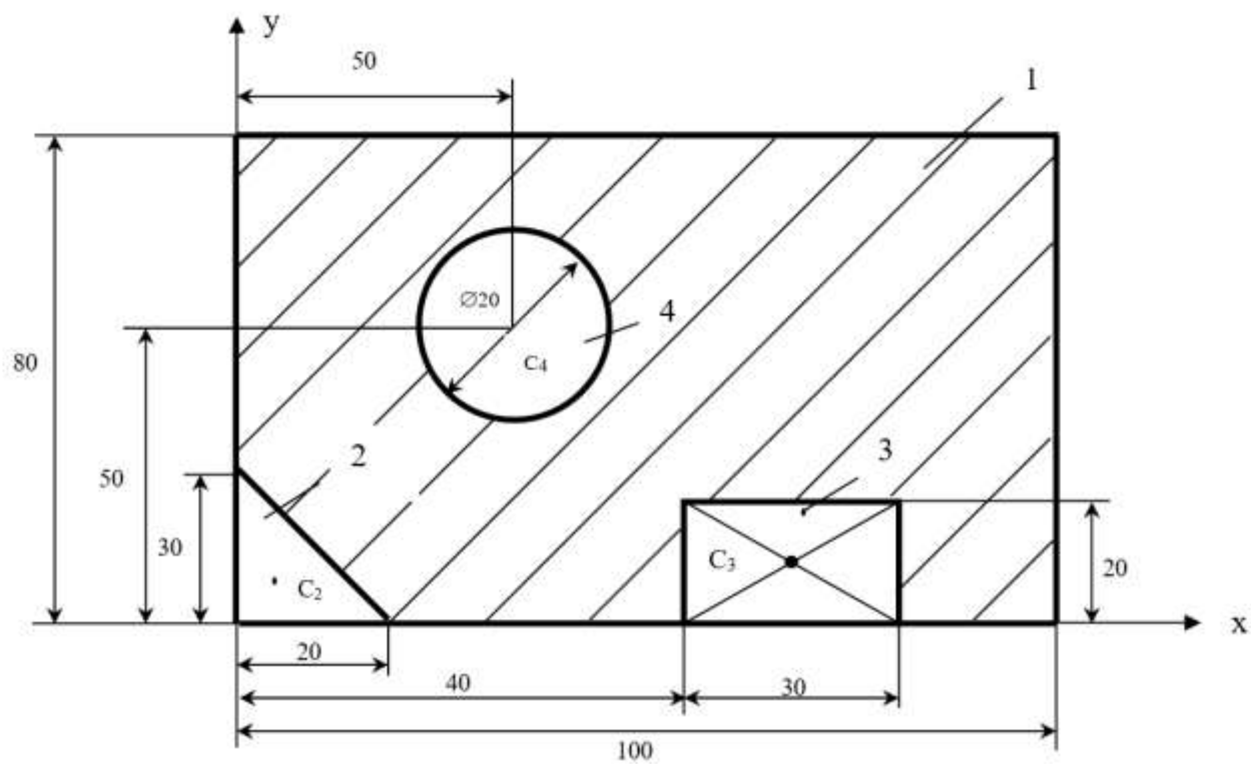


Figure 13.8

Body 1:

$$S_1 = 80 \times 100 = 8 \cdot 10^3 mm^2$$

$$x_1 = 50mm, y_1 = 40mm$$

Body 2:

$$S_2 = 0{,}5 \cdot 20 \cdot 30 = 300mm^2$$

$$x_2 = \frac{1}{3} \cdot 20 = \frac{20}{3} mm; y_2 = \frac{1}{3} \cdot 30 = 10mm$$

Body 3:

$$S_3 = 30 \cdot 20 = 600mm^2$$

$$x_3 = 55mm; y_3 = 10mm$$

Body 4:

$$S_4 = \frac{\pi d^2}{4} = \frac{3{,}14 \cdot 20^2}{4} = 314mm^2$$

$$x_4 = 50mm; y_4 = 50mm$$

Based on the expressions (13.5), we find the center of gravity of the cross section, given that the bodies 2, 3, 4 were cut out of the rectangle. That is, the areas of bodies 2, 3, 4 must be taken into account with a negative sign.

$$X_c = \frac{S_1 \cdot x_1 - S_2 \cdot x_2 - S_3 \cdot x_3 - S_4 \cdot x_4}{S_1 - S_2 - S_3 - S_4} =$$

$$= \frac{8 \cdot 10^3 \cdot 50 - 300 \cdot \frac{20}{3} - 600 \cdot 55 - 314 \cdot 50}{8 \cdot 10^3 - 300 - 600 - 314} = 51{,}47mm$$

$$Y_c = \frac{S_1 \cdot y_1 - S_2 \cdot y_2 - S_3 \cdot y_3 - S_4 \cdot y_4}{S_1 - S_2 - S_3 - S_4} =$$

$$= \frac{8 \cdot 10^3 \cdot 40 - 300 \cdot 10 - 600 \cdot 10 - 314 \cdot 50}{8 \cdot 10^3 - 300 - 600 - 314} = 43{,}52mm$$

Answer: $X_c = 51{,}47mm; Y_c = 43{,}52mm.$

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Олександр Володимирович Грушко

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Навчальний посібник

Рукопис оформила *І. Кириця*

Редактор *В. Дружиніна*

Оригінал-макет підготувала *Т. Старічек*

Підписано до видання 22.11.2024 р.
Гарнітура Times New Roman.
Зам. № Р2024-182

Видавець та виготовлювач
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Свідоцтво суб’єкта видавничої справи
серія ДК № 3516 від 01.07.2009 р.