

Michael Köhler · Sven Jenne  
Kurt Pötter · Harald Zenner

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# Load Assumption for Fatigue Design of Structures and Components

Counting Methods, Safety Aspects,  
Practical Application

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# Preface

The strength assessment for structural components constitutes a comparison between the occurring stress and allowable stress (the capacity to withstand loads). The capacity of a structural component to withstand loads must be higher than the stress which occurs in operation, including a certain safety factor. For the certification of fatigue life under variable load amplitude, the use of simple characteristic values, such as static strength characteristics, is not sufficient. For this purpose, characteristic functions are necessary for describing the load and capacity to withstand loads. For the load, this function is a frequency distribution of the amplitudes, that is, a load spectrum or load matrix. For the capacity to withstand loads, the function can be an S-N curve for the structural component, for instance. In principle, the comparison can be performed with forces, bending or torsion moments, nominal stresses, local stresses, local elastic strains, or on the basis of fracture-mechanical characteristics. For the assessment of the capacity to withstand loads further methods are also applied: the local concept on the basis of local elastic-plastic strains and the fracture-mechanics concept on the basis of fracture-mechanical characteristics.

The subject of this book is the load on structural components, structures, and systems. The load assumption necessary for the strength assessment is of the same importance as the determination of the capacity to withstand loads. Many cases of damage encountered in practice are the result of an erroneous load assumption. Without a reliable load assumption, designing for ensuring fatigue life under variable stress amplitude is not possible. This, however, is an essential prerequisite for lightweight construction. In this book, the counting methods by means of which measured load-time functions can be transformed to frequency distributions (spectra, matrices) are discussed and evaluated. Special emphasis is thereby placed on the rainflow counting method. Furthermore, the procedure for deriving the load assumption necessary for certifying the fatigue strength under variable stress amplitude is described.

This book is intended for use by engineers in industry and research who work with load assumptions for dimensioning structural components subject to fatigue load or—stated more generally—with the subject designated nowadays as structural integrity. It is addressed especially to design engineers as well as test experts.

A special intention of the authors was to consistently consider questions relevant to practice, to describe new and recent developments, and to offer recommendations for practical application.

The previous edition of this book was published in German in 2012. Many specialised German terms which are well established in the technical field which is the subject of the present treatment cannot be translated exactly into English. In fact, the problem already begins with the German term for the technological field which is the topic and thus the title of this book, “Betriebsfestigkeit” (German). Numerous similar terms do of course exist in English. Examples include “fatigue life under variable stress amplitude” or “in-service integrity”. However, none of these terms completely or exactly describe the field of engineering which is designated by the term “Betriebsfestigkeit”. Nevertheless, the authors have taken great care to employ standardised designations for the definition of the technical terms.

The content of the present English translation corresponds very closely to that of the original German version of the book, “Zählverfahren und Lastannahme in der Betriebsfestigkeit”, published by Springer-Verlag in 2012 [Köhl 12].<sup>1</sup> In addition to a few minor corrections and alterations, the list of references has been updated. Furthermore, the quality of many figures has been improved. For the sake of transparency and better understanding, the English version has not been subdivided into two parts on “counting methods” and “load assumptions”. Correspondingly, the assignment of a few text sections to the chapters has also been slightly altered.

Special thanks should be expressed to Dr. W. Fischer, Chairman of the FVA Working Group “*Lastkollektive*” (Load Spectra) for many years, for his continuing support on this subject, including his support for this book.

For the support received in the preparation of this book, the authors wish to express their special thanks to the Institute of Plant Engineering and Fatigue Analysis at Clausthal University of Technology, Prof. Dr. A. Esderts, Ms K. Friedrichs, Dr. R. Masendorf, Dr. K. Hinkelmann, Dr. H. Mauch, and Dr. C. Müller.

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<sup>1</sup>See references Chap. 1.

# Notes Regarding the Translation

This English translation has been kept close to the original German book “Zählverfahren und Lastannahme in der Betriebsfestigkeit”, Springer Verlag (2012), and has been carefully prepared. The authors hope that any ambiguity due to difficulty in translating technical terms and paraphrases will kindly be accepted. As far as possible, the authors have adopted a common English notation. Nevertheless, the original German notation of mathematical symbols, indices, and formulas had to be maintained in some cases.

In this book, some quotations of German documents have been translated into English. Although this has been done with great care, the authors of the original documents have not checked the translations for their approval.

Everyone is expected to decide for himself whether the examples of practical application given in this book meet his particular requirements and to observe appropriate care in their application. Neither the publisher nor the editor nor the authors shall be liable to the purchaser nor to any other person or entity with respect to any liability, loss, or damage caused or alleged to have been caused directly or indirectly by this book.

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# Abbreviations

ASTM	American Society for Testing and Materials
CG	Calculation Group
DVS	Deutscher Verband für Schweißtechnik (German Welding Association)
FEM	Finite Element Method
FKM	Forschungskuratorium Maschinenbau (Mechanical Engineering Research Forum (part of the VDMA))
FVA	Forschungsvereinigung Antriebstechnik (German Research Association for Drive Technology)
HCF	High cycle fatigue
LIF	Load-input function
LTF	Load-time function
STF	Stress-time function
TGL	Technical Standards for Quality Specifications and Terms of Delivery, Gütevorschriften und Lieferbedingungen, German Democratic Republic 1955–1990
UIC	Union internationale des chemins de fer (engl.: International Union of Railways)
VDI	Verein Deutscher Ingenieure (The Association of German Engineers)
VDMA	Verband Deutscher Maschinen-und Anlagenbau (German Mechanical Engineering Industry Association)

# Symbols<sup>2</sup>

## Loads, Service Loads, Load-Time Functions

$A_i$	Operational states or operating processes
$f$	Frequency (Hz)
$F$	Load, force
$F_O$	Overload
$F_x, F_y, F_z$	Force in longitudinal, lateral, and vertical direction
$M_x, M_y, M_z$	Moments about the axes in longitudinal, lateral, and vertical direction
$M_1, M_2$	Measuring points for bending stresses of railway axle, Fig. 2.6
$n$	Number of revolutions
$Q1, Q2$	Vertical forces (railway wheelset), Fig. 2.6
$R$	Stress ratio, ratio of minimum and maximum value of a load cycle, $R = (S_m - S_a)/(S_m + S_a)$
$S$	Nominal stress
$S(t)$	Stress-time function STF
$S_a$	Nominal stress amplitude of a cycle
$S_m$	Nominal mean stress
$S_{aD}$	Endurance limit for nominal stress amplitude, Fig. 1.2
$\hat{S}_a$	Maximum of nominal stress amplitude of a spectrum, Fig. 4.3
$\hat{S}_{aM}$	Maximum spectrum amplitude for measurement
$\hat{S}_{aN}$	Maximum spectrum amplitude for a defined lifetime
$S_{ai(hi)}$	Nominal stress amplitude of step $i$ of a spectrum
$S_{ai(Ni)}$	Nominal stress amplitude of S-N curve level $i$
$t, T$	Time, duration
$t_i$	Time at level, dwell time in a class I, Fig. 7.2
$t_M$	Duration of measurement

---

<sup>2</sup>In some cases, if the symbols are applied only on one occasion, they are explained immediately in the text.

$t_N$	Total design life
$\Delta t$	Time difference
$T_a$	Driving, torsional moment, torque, amplitude
T1, T2, T3	Torsional moments (rolling stand), Fig. 2.10
$\Delta T$	Load range of T, Fig. 2.12
Y1, Y2	Lateral forces (railway), Fig. 2.6

## Counting Methods, Load Spectra, Matrices

a	Constant in the formula by Rice
a, b	Levels
$a_i, b_i$	Classes for two-parameter time-at-level
B	Partial Spectra, s. Sect. 10.2
C	Crest factor, $C = x_{\max}/x_{\text{eff}}$ , Eq. (2.6)
e	Extrapolation factor, $e = H_N/H_M$ , Fig. 4.11
E	Incremental factor for extrapolation, Eq. (4.9)
h	Step frequency, Fig. 4.4
hi	Step frequency of cycles of step i
H	Cumulative frequency of the cycles (ISO 12110)
$H_M$	Cumulative frequency of cycles for measurement, Fig. 4.11
$H_N$	Cumulative frequency of cycles for useful life, Fig. 4.11
$H_0$	Block length
$H_0$	Number of zero or mean-load crossings of the spectrum, formula by Rice
i	Ordinal number, class number, horizons
I	Irregularity factor, $I = n_0/n_1$
$n_0$	Number of zero crossings in one direction
$n_1$	Number of maxima
p	p-value, measure of the spectrum fullness according to Gassner
S	Nominal stress
$S_a$	Nominal stress amplitude of a cycle
$\hat{S}_a$	Maximum of nominal stress amplitude of a spectrum, Fig. 4.3
T	Period, duration
$\Delta t_i$	Dwell time in a class i
v	Fullness degree of a spectrum
x	Ordinal number
x	Normalised stress amplitude, $x = \frac{S_{ai}}{S}$ , Fig. 4.3
$x_{\text{eff}}$	Effective or RMS value (root mean square), $x_{\text{eff}} = \sqrt{x^2}$ , Eq. (2.3)
$x_{\max}$	Maximum of $x(t)$
$\bar{x}$	Arithmetical mean value, Eq. (2.1)
$\overline{x^2}$	Mean square value, Eq. (2.2)
$\nu$	Parameter to describe the shape of the spectrum, Fig. 4.5 and Eq. (4.7)
$\sigma$	Standard deviation
$\sigma^2$	Variance, Eq. (2.4)

### Analytical Fatigue-Life Estimate

$d_i$	Fatigue damage per cycle, $d_i = 1/N_i$ , Eq. (9.1)
$e$	Extrapolation factor, $e = H_N/H_M$ , Eq. (4.8)
$D, D_i$	Damage sums, damage sum for the horizon I, Fig. 9.3
$D_A$	Damage sum for special spectrum A
$D_B$	Damage sum until failure (stress)
$D_{B50}$	Mean value of damage sum $D_B$
$D_{calc}$	Calculated damage sum
$D_{CG}$	Damage sum of a computation group
$D_{eff}$	Effective damage sum, Table 9.5
$D_{exp}$	Damage sum determined experimentally
$D_{F50}$	Mean value of damage sum (strength)
$D_{max}$	Maximum of $D$
$D_{50}$	Mean value of damage sum (log)
$D_{10}, D_{90}$	Damage sum for $P = 10$ and $90\%$
$k^*$	Slope of the deflected S-N curve in the modifications
$K_t$	Stress-concentration factor
$\hat{N}$	Number of cycles until failure at variable stress amplitude, Gassner curve
$\hat{N}_{calc}$	$\hat{N}$ calculated
$\hat{N}_{exp}$	$\hat{N}$ determined experimentally
$S$	Nominal stress
$S_a$	Nominal stress amplitude of a cycle
$\hat{S}_a$	Maximum of nominal stress amplitude of a spectrum, Fig. 4.3
$z$	Number of repetitions of a special spectrum until failure, Eq. (9.4)

### Experimental Test Results—Static

$R_m$	Tensile strength
$R_{p0.2}$	0.2 % yield limit

### Experimental Test Results—Constant Amplitude

$f$	Frequency (Hz)
$i$	Ordinal number, class number, horizons
$k$	Slope of the S-N curve
$K_t$	Stress-concentration factor
$m$	Slope of the S-N curve for crack propagation
$N$	Number of load cycles until failure, S-N curve, Wöhler curve

$N_D$	Number of cycles at the deflection point S-N curve
$N_i$	Number of cycles on horizon $i$
$N_{50}$	Mean value of the cycles $N_i$ (log)
$R$	Stress ratio, ratio of minimum and maximum value of a load cycle, $R = (S_m - S_a)/(S_m + S_a)$
$S$	Nominal stress
$S_a$	Nominal stress amplitude of a cycle
$S_{aD}$	Endurance limit for nominal stress amplitude, Fig. 1.2
$S_m$	Nominal mean stress

## Experimental Test Results—Variable Amplitude

$D_{\text{exp}}$	Damage sum for experiment calculated with a specific hypothesis
$f$	Frequency (Hz)
$i$	Ordinal number, class number, horizons
$\hat{k}$	Slope of $\hat{S} - \hat{N}$ curve
$\hat{N}$	Numbers of cycles until failure at variable stress amplitude, Gassner curve
$\hat{N}_{\text{exp}}$	$\hat{N}$ determined experimentally
$S$	Nominal stress
$\hat{S}_a$	Maximum of nominal stress amplitude of a spectrum

## Statistics, Safety, Reliability

$j$	Ordinal number
$j_{C,n}$	Safety factor for confidence probability
$j_S$	Safety factor (reliability concept)
$\hat{j}_N$	Safety factor referred to cycles (reliability concept)
$\hat{j}_S$	Safety factor referred to stress (reliability concept)
$k$	Sampling size
$m$	Difference of mean values
$n_S$	Number of standard deviations in the data pooling
$p$	Distribution density
$P$	Probability
$P_a$	Probability of failure for capacity to withstand loads
$P_e$	Probability of occurrence of stress
$P_{\bar{u}}$	Probability of survival, $P_{\bar{u}} = 1 - P_a$
$P_A$	Overall probability of failure, for example Fig. 11.7 and Eq. (11.7)
$s$	Standard deviation, for example Eq. (9.14)
$s_B$	Standard deviation of the stress



$s_F$	Standard deviation of the strength
$s_{CG}$	Standard deviation of the computation group
$s_{DB}$	Standard deviation of the damage sum (stress), Fig. 11.9
$s_{DF}$	Standard deviation of the damage sum (strength), Fig. 11.9
$s_{s,N}$	Standard deviation of the standard deviations in data pooling in the N direction
$s_{s,S}$	Standard deviation of the standard deviations in data pooling in the S direction
$s_{50,N}$	Mean value of the standard deviation in data pooling in the N direction
$s_{50,S}$	Mean value of the standard deviation in data pooling in the S direction
$s_{90,S}$	90 % $P_{\bar{u}}$ value of the standard deviations in data pooling in the S direction
S	Safety factor
S	Nominal stress
$S_{aB}$	Amplitude of fatigue load, nominal stress, Figs. 11.4 and 11.5
$S_{aF}$	Amplitude of fatigue strength, nominal stress, Figs. 11.4 and 11.5
$S_{aF50}$	Mean value of $S_{aF}$ (log)
T	Range of scatter
$T_D$	Range of scatter for the damage sum, Eq. (9.15)
$T_{D,CG}$	$T_D$ for a computation group CG
u	Quantile for the probability of failure
$u_0$	Relative safety range, for example Eq. (11.16)
z	Difference value (reliability concept), Eq. (11.3)
v	Scattering ratio, $v = s_B/s_F$ (reliability concept), Fig. 11.8
$\sigma$	Standard deviation
$\sigma^2$	Variance, Eq. (2.4)

# Chapter 1

## Introduction

The purpose of the present textbook is to describe and discuss the measurement, recording, and processing of load-time signals and the derivation of the necessary load assumption for dimensioning of structural components and constructions with the required fatigue strength under variable load-amplitude.

For a better understanding of the essential content of this book, which was originally published in German, a brief definition of the important terms and concepts is necessary. First of all, it must be pointed out that no adequate English term exists for the field of engineering treated in the present book, which is designated by the term “Betriebsfestigkeit” in German. Instead, terms such as “operational stability”, “endurance strength”, “fatigue strength”, “in-service integrity”, or “service strength” are employed for the purpose. However, each of these terms covers only a limited portion of the engineering field which is designated by the German word “Betriebsfestigkeit”.

In German usage, the terms “load” and “stress or strain” are employed for distinguishing between the cause and effect of a “force”. This differentiation does not exist in English. In this context, the “load” is considered to be caused by external “forces or couples of forces which act on a part or structure”. As a result, local stresses or strains occur as the “effect” in the structural component.

In this connection, differences also exist between the definitions of the “loading capacity” and “strength”. The term “loading capacity” designates the ability of a structural component or of a structure to withstand external loads, whereas the term “strength” is referred to the local strength of the material with respect to a “local stress”.

The measurement of the service loads under real operational conditions provides the most effective basis for a reliable load assumption. To an increasing extent, a load assumption is also derived from a computational simulation at present. For this purpose, a machine or a vehicle is considered as a system of individual springs, masses and damping elements, which are connected by kinematic compliances. This system is capable of oscillation, and the stress at the critical sites of failure in a construction is derived from the dynamics of the overall system. Such a simulation

is well suited for analysing the basic behaviour of an oscillating system and can indicate possible design approaches for decreasing service stresses or loads; thus, it also offers a potential solution for light-weight construction. For instance, the simulation can also be applied in individual machine design if the operational experience hitherto gained with a new design is not sufficient. A further application is the determination of overloads, which can occur during special load cases such as those typical of technical damage case, but which are usually not accessible to measurement. A correct estimate of the limits is vital for the simulation. For this purpose, comprehensive experience with the comparison between simulation and measurement over a wide range is necessary. Even a small error in the load assumption can drastically affect the service life of a construction. As indicated by the developments in this field during recent years, the stresses and loads generated under specific operating conditions can be approximated with increasing accuracy by simulation, and this trend will continue in the future. Examples in this context include especially automotive construction, the wheel-rail system, and wind turbines.

For the requirements of fatigue strength under variable stress amplitude, that is, for the purpose of dimensioning components and structures for a specific fatigue life, the magnitude of the stress amplitudes and the frequency of the stresses are especially important. The frequency content of the stress-time function (STF), the order in which the events occur, and the shape of a load cycle are in general neglected for this purpose. As a result, the amount of information included in the STF can be reduced, for instance, to the magnitude and frequency of the maxima and minima, which occur. Various counting methods are available for data reduction of this kind. For the application of counting methods, therefore, a check is necessary to determine whether or not the prerequisites for neglecting the aforementioned items are satisfied in the specific case of application. Thus, the neglect of the frequency content may not be permissible if the behaviour of the material and of the structural component is time-dependent. For example, this may be the case in the creep range at high temperature, during fatigue in corrosive media, or with elastomers, even under normal ambient conditions. A significant effect of the order in which loading occurs on the allowable fatigue life can be demonstrated in the laboratory, for instance, in the high-low and low-high two-stage test or during application of excess loads at different instants. On the other hand, however, experience gained for many years with the occurrence of a very large number of load cycles and with extensive mixing of small and large amplitudes indicates that the order in which loading occurs does not affect the fatigue life. In the case of multiaxial, non-proportional stress, it is known that the shape of a load cycle (triangular, sinusoidal, trapezoidal) affects the fatigue life and the endurance limit [Soci00, Liu01, Sons04].

The purpose of counting methods in fatigue-strength designing is to determine the frequency distribution of the amplitudes for arbitrary load-time functions. The frequency distribution of the amplitudes, or the stress matrix, is a decisive quantity for the lifetime of structural components subjected to fatigue stress. In comparison

with the stress-time functions, the counting method constitutes a data reduction, which is necessary for the fatigue life time assessment.

First, a short overview is given on the main contents of the book.

In Chap. 2, stress-time-functions (STF) are shown for characteristic service loads. Fatigue stresses occur during the operation of a machine plant, an automobile or an aeroplane. A distinction can be made between basic stresses caused by operational processes, additional stresses from the environment, and resonant oscillations. A distinction must also be made between deterministic and stochastic STFs. In machine facilities and means of transport, rare operational states can occur and may result in overload. The causes and various protective measures are discussed.

In Chaps. 3–8 the most important one- and two-parameter counting methods are described in detail and assessed with respect to their applicability. The two-parameter rainflow counting method has now become established world-wide for most measurements under operational conditions. On the other hand, however, a graphical representation of spectra can be quite useful because of the easier visualisation. Consequently, the procedure for deriving spectra from matrices is described. In order to clearly illustrate the different results obtained with the individual counting methods, the matrices and spectra are mutually compared for four examples of stress-time functions. The present status of counting methods for multiaxial stresses and loads as well as the associated problems is discussed in detail.

Moreover, the one- and two-parameter time-at-level counting methods are described, and specific applications are indicated.

Various criteria are available for the selection of counting methods. Recommendations are given for the presentation of the results. With a number of practical examples, it is shown that the correct application of counting methods always requires a critical examination.

In Germany counting methods were first applied shortly before the introduction of fatigue strength assessment under variable stress amplitude by E. Gassner [Gass41]. Thus, frequency distributions for stresses resulted from measurements performed on agricultural machines in 1932 and in 1936 by W. Kloth and T. Stroppel [Klot32, Klot36]. Since that time, numerous counting methods have been published for dealing with the requirements of fatigue strength. These counting methods have undergone a historical development, especially in the aviation industry. In this process, developments in measuring technology and data processing were especially decisive. High-performance computers and data-recording media are now available for such purposes. In the past, on-line counting during the measurement was necessary because of limited storage capacity. Today, however, large volumes of data can be recorded as transient signals, sometimes for hours at a time. Hence, the subsequent data reduction with the application of counting methods is now essentially an instrument for determining characteristic functions for the fatigue-life estimate, besides other methods of general signal analysis.

In Chaps. 9–12, the analytical fatigue-life estimate, the design and dimensioning of load spectra, as well as the safety and reliability concepts are described. Load assumptions in special fields like automobiles, rail vehicles, cranes, and bicycles are considered.

The analytical fatigue-life estimate is considered to the extent necessary for deriving the load assumption. The discussion is restricted to the nominal stress concept.

The measurement of forces, moments, pressures, local strains, etc. under realistic operating conditions still constitutes the basis for design and dimensioning spectra, as considered in Chap. 10. For this purpose, recommendations are offered for performing and evaluating the measurements. The individual steps from the measurement under operational conditions all the way to the design and dimensioning spectrum are described and discussed.

In a more general context, safety aspects of component designing and dimensioning are considered in Chap. 11. Conclusions concerning the reliability of structural components are always based on a comparison between loads, forces and moments and the strength. The conventional safety concept involves the use of a safety factor, which is based especially on experience. In the case of the reliability concept, the scatter of the loading capacity and of the stress (load) is also considered. If the probability of failure is taken as basis, a quantitative and transparent conclusion can be derived for the behaviour of the structural component in the field. On the other hand, however, the accuracy with which the scatter is known may be limited, and the knowledge of the distribution-density function may not be sufficient at very low probability of failure. The importance of testing in connection with the strength assessment as well as cases of damage which have always accompanied and will continue to accompany technical development are discussed in detail. Moreover, several critical aspects of the safety and reliability concept are treated for the example of the so-called safety factor. This consideration is also justified by the fact that the characteristic parameters employed for determining the loading capacity and the stress are only estimated values, and the quality of the model employed for the strength calculation is often limited. For dynamic stresses with variable amplitude, which are particularly characteristic for the entire field of transport-vehicle construction, a reliable load assumption is especially difficult, since the progressing damaging processes, which ultimately result in failure, are extremely complex. Finally, the different approaches employed in various fields of application are described, and safety-relevant measures, which are implemented during product development and during operation, are indicated and discussed.

In Chap. 12, the load assumption is described for various specialised fields, in which a distinction can always be made between a load assumption which is defined by a series of technical rules (such as railway-vehicle construction) and one which is not defined and is the manufacturer's own, autonomous responsibility (such as automotive construction). References are provided for specific fields of application, which are of historical as well as current interest, see also Chap. 13 in this context.

In Chap. 13, “Additional references on load assumptions in various fields of application”, several important textbooks on the subject of fatigue strength are listed; in these books, special emphasis is also placed on the load assumption.

## Concluding Remarks

An important characteristic value for the comparison of measurements is the damage sum. This value is obtained by means of a damage accumulation calculation with respect to a real or fictitious S-N curve. An S-N curve is a plot of the allowable load amplitude versus the number of cycles to failure, see Chap. 9. If a fictitious S-N curve is employed, the damage sum does not provide any information on the actual failure of the structural component; in this case, it is useful only for a comparative analysis of measured data. In former times, several counting methods were applied in practice. Today, however, rainflow counting has become the established method for analytical fatigue-life prediction and generation of load-time functions, for instance, for the test-bench control of a fatigue-strength test. Furthermore, it is the only counting method with a physical background. For the graphical representation of stresses in the form of load spectra, one-parameter range-pair counting and level-crossing counting are also employed, for instance, as described in Sect. 8.2, since spectra indicate the specific features more clearly for a stress comparison than do load matrices. This applies to the following applications, for example:

- Checking of measurements (plausibility, for instance, with respect to the extreme values, the average value, and the frequency)
- Qualitative and quantitative comparison of measurements for assessing different operational conditions (shape of the spectrum, damage sum)
- Fatigue-life prediction by calculation
- Separation of causes of stresses as far as possible, for instance, by forming characteristic partial spectra
- Generation of load-time functions for performing fatigue-strength tests

As a matter of principle, there are several possibilities of counting events for a continuous load-time function:

- The measured value attains an inversion point (maximum, minimum).
- The measured value passes through a range, that is, it passes from a minimum to the next maximum and vice-versa.
- The measured value exhibits hysteresis in the positive and negative directions.
- The measured value crosses or exceeds a predetermined horizon in the positive or negative direction.
- The measured value is determined for equidistant instants in time.
- The measured value is determined at instants which are predetermined by a different quantity, such as the rotational speed or angle of rotation.

The characteristic quantities for the capacity to withstand loads are the amplitude and the mean value of a load cycle, or the maximum and minimum. A distinction is made between one-parameter counting methods, where the amplitude or level alone is considered, and two-parameter counting methods, where the pair of values, amplitude and mean value, or maximum and minimum, is considered.

The counting methods can decisively affect the spectrum and thus the result of the analytical fatigue-life prediction. As dictated by the type of load-time function involved, the results of the different counting methods differ to various extents from one another. In this context, the evaluation of a load-time function depends decisively on mean-load variations. From a fatigue-strength standpoint, preference must be given to that counting method which best describes the damage, which is decisive for the fatigue life. In correspondence with the present state of the art, rainflow counting [Euli94, Euli97], is the method which has become internationally established. Basic remarks on the application of counting methods are frequently mentioned in the literature. References include the following: [Buxb86, Schü92, Seeg96, Rice97, Gude99, Haib02, Rada07, Schi09].

In this book, the one- and two-parameter time-at-level and level-distribution counting methods are also considered, in addition to the classical counting methods for estimating the fatigue strength. With these counting methods, the dwell time in every single class is counted, rather than the frequency with which a maximum attains a class, for instance. These counting methods are frequently applied in the field of transmission-gear and engine-component designing. The same applies to the two-parameter time-at-level method. A new feature is the proposal for applying a relative time at level method, which is applicable as a one-parameter as well as a two-parameter version. As an extension of the level-distribution counting method, the class frequency depends on a further reference value, such as the rotational speed, rather than on the dwell time. It is especially well suited for the counting of stress-time functions on rotating components.

The application of the counting methods is explained for multiaxial loads and stresses. For complex local stresses several components of forces and moments in different directions with variable amplitudes, with a phase shift or a different frequency, the limitations of the currently available methods become evident. The development of appropriate counting methods for such applications is still the object of current research.

In the field of fatigue strength under variable stress amplitude, where a defined design life (duration, mileage, number of process sequences, etc.) must be ensured, it is not possible to specify an exact instant for load assumptions from a historical standpoint. For the example of postal carriages, it is known that maintenance work was performed at regular intervals, and that chassis components were replaced accordingly. Since the beginning of experimental component testing, the importance of load assumptions has steadily increased, as shown with the use of two examples.

In the Series of Technical Rules for designing of structural components with the required fatigue strength, load assumptions constitute an exception. This is understandable, since the operation of the components can vary tremendously. If, for example, a gear unit with a specific power rating is installed in a drive system, the

dynamics of the overall system is decisive for the fatigue life of the gears (tooth profile, tooth root). This in turn depends on the starting and stopping processes, on the working process, on the spring-mass system of the drive train, and on the effective damping of vibration. Hence, in the normal case, the manufacturer will derive a load assumption, which is based on measurements under operational conditions and which is usually also based on decades of operational experience. From the operational measurements as starting point, various steps are necessary for deriving the load assumption:

- Determination of the total design life
- Distinction among various operational states
- Weighting of the operational states (operational profile)
- Extrapolation of the partial spectra
- Consideration of overloads associated with special events
- Derivation of an overall spectrum or a corresponding matrix

Since the loads and the capacity of structural components to withstand loads are subject to scatter, a partial safety factor must be considered for both the load and the capacity to withstand loads. For the load, this implies an estimate of the respective shares with which the individual operational states contribute to the damage, for example. If this share is known, a further estimate is necessary for determining which realistic, worst case operational profile must be taken as basis. Ultimately, the risk in terms of the consequences associated with a case of failure is decisive for the partial safety factors to be employed.

Fatigue-strength designing originated in the aviation and automotive industries, but has been gaining increased importance in nearly all fields of technology during recent decades. Examples include plant and machine designing, all the way to heavy machinery construction, power-plant, naval, and bridge construction, as well as wind turbines, railway vehicles, off-shore platforms, cranes, pipelines, agricultural machines, and sporting equipment. Critical zones are stress concentrations in components and structures, and particularly in machine components. Joints such as welds are of special importance. Endurance-strength designing is by no means the usual case today.

The basis for fatigue-strength or fatigue life designing is the assumption of a limited design life, as illustrated in Fig. 1.1. In Fig. 1.2, endurance-limit designing is compared with fatigue-strength designing for nominal stresses.

Since amplitudes which exceed the endurance limit are permissible for fatigue-strength designing, components can be dimensioned with a smaller cross-section than for endurance-limit designing. Thus, the fatigue-strength approach constitutes an extremely effective measure for light-weight construction.

Designing of structural components for a limited design life and thus designing of light-weight constructions do not imply an increased risk. Ensuring sufficient safety and reliability is precisely the task of fatigue-strength or fatigue-life designing. This task includes the development of suitable designs, the selection of appropriate materials and joining methods, an analytical and, as a rule also an experimental proof of fatigue strength, monitoring of the manufacturing process, quality assurance for the products, as well as monitoring of service loads in many cases.





Fig. 1.1 Examples of useful life

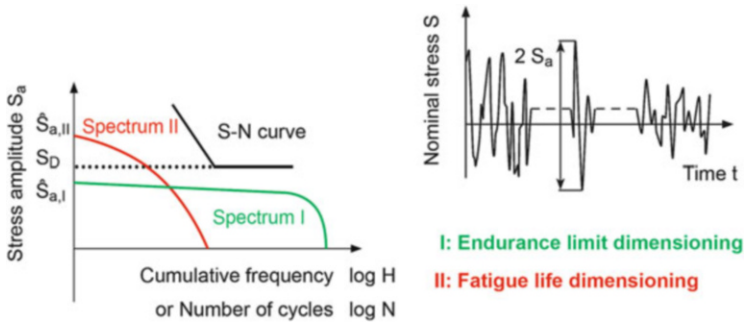


Fig. 1.2 Endurance limit and fatigue strength dimensioning of structural components

A reliable load assumption is a fundamental prerequisite for ensuring sufficient safety.

As far as is known at present, *Oberberggrat* (Senior Inspector of Mines) J. Albert performed the first component-fatigue tests around 1828 in Clausthal/Harz, today Lower Saxony, Germany [Albe37, Mann70, Hane89]. These tests were performed on chains, which were designated as ‘ropes’ at that time. With the application of weights, the chains were subjected to loads corresponding to hoisting loads and

dead weight. The chains moved over a drum (pulley), and the chain links were thus subjected to bending loads. A cranking device was employed for driving the chain in the forward and reverse directions. The test stand was designed and constructed to reproduce operational conditions directly. It should be recalled, however, that an endless chain is employed in the actual mining operations; this chain moves over a drum at the top and bottom of the mine shaft. J. Albert writes:

A calculation indicated that each chain link concerned had been subjected to about 93 120 bending processes at every site of articulation during the 14 725 times that it had to turn in the course of operations under the indicated severe pressure on both pulleys, the cage, and at the top of the mine shaft. In accordance with that hypothesis, therefore, the same success should always result under similar conditions.

A. Wöhler is considered to be the first engineer who performed systematic experimental investigations on fatigue strength. The reason for his investigations was the fracture of railway-vehicle shafts (axles). He published five papers between 1858 and 1870 [[Wöhl58](#), [Wöhl60](#), [Wöhl63](#), [Wöhl66](#), [Wöhl70](#)]. The first and second paper are concerned with the measurement of the maximal moment of bending and torsion under service conditions on the railway route Berlin—Breslau (Wrocław) and Frankfurt/Oder—Breslau (Wrocław). The measurements were carried out over a distance 22,000 km. The deflection of the axles was scratched on a zinc plate by a scribe by means of a compound lever system [[Schü96](#), [Zenn15](#), [Zenn17](#)]. As director of the central workshop of the Niederschlesisch-Märkisch Railways in Frankfurt/Oder, he conducted fatigue tests on test bars and structural components as of 1858; these investigations yielded fundamental information on fatigue strength, for instance, on the effects of the stress type, mean stress, and notches [[Wöhl70](#), [Mann70](#), [Hane89](#)]. Because of his work, the use of these results became an established procedure for the analytical proof of fatigue strength. Designing and dimensioning had previously been performed on the basis of static stresses. In conjunction with the load assumption, a less-known, but interesting feature is worthy of mention: By means of measurements on wheelset axles under operational conditions, Wöhler demonstrated that the maximal bending moments incurred during operation differ considerably from the static moments on which designing and dimensioning had hitherto been based. He had also developed a mechanical measuring set-up for this purpose [[Wöhl58](#), [Tros14](#), [Blau18](#), [Brau69](#)]. Thus, he provided a new basis for the proof of fatigue strength with respect to the loading capacity as well as the stresses.

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# Chapter 2

## Characteristic Service Stresses

A prerequisite for a reliable load assumption is an understanding of the essential and characteristic operational states of engineering constructions. Measurements or theoretical derivations of service stresses, for instance, with the application of numerical simulation models, constitute an essential instrument for this purpose. The service stresses are considered in the time and frequency ranges. The evaluation in the time range provides a means of controlling the measurements under operational conditions and is also useful for obtaining information on the effect of specific operational conditions and states on the value and variation of the stresses with time. From a visual standpoint, this is possible only for a limited period of time. The frequency distribution of amplitudes (load spectrum or load matrix) constitutes the basis for the fatigue-life estimate for structural components and thus also for the analytical and experimental proof of fatigue life. The frequency analysis, which is discussed only for one example, can provide important information on the cause of high stress amplitudes and thus is often useful for explaining cases of damage. A frequency analysis, for example the power density spectrum, cannot contribute directly to the fatigue-life estimate, because the number of cycles of the oscillations is lost in the process [Sons89].

### 2.1 Stress-Time Functions

Very different types of stress-time functions can occur in practice. Static and dynamic stresses are distinguished. In the case of static stresses, the system concerned may be at rest or subject to quickly and monotonically changing stresses, Fig. 2.1, whereas dynamic stresses can be subdivided into oscillating stresses and impact stresses, Fig. 2.2.

For the certification of fatigue strength under variable stress amplitude, the shape of the load cycle can in general be neglected (with the exception of creep and corrosion as well as multiaxial, non-proportional stresses). During measurements

Fig. 2.1 Static stress

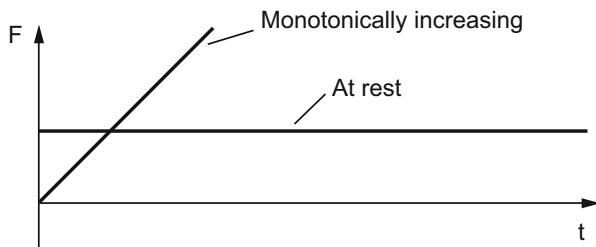
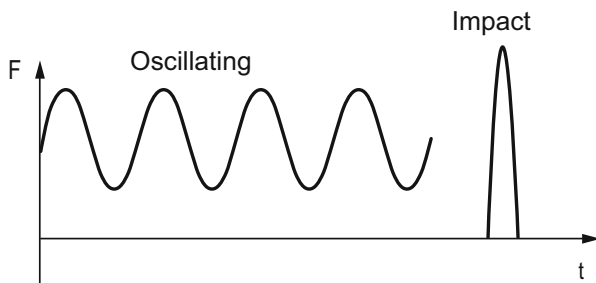


Fig. 2.2 Dynamic stress



under operational conditions, stress-time functions with variable amplitude as well as variable mean load occur almost without exception, Fig. 2.3.

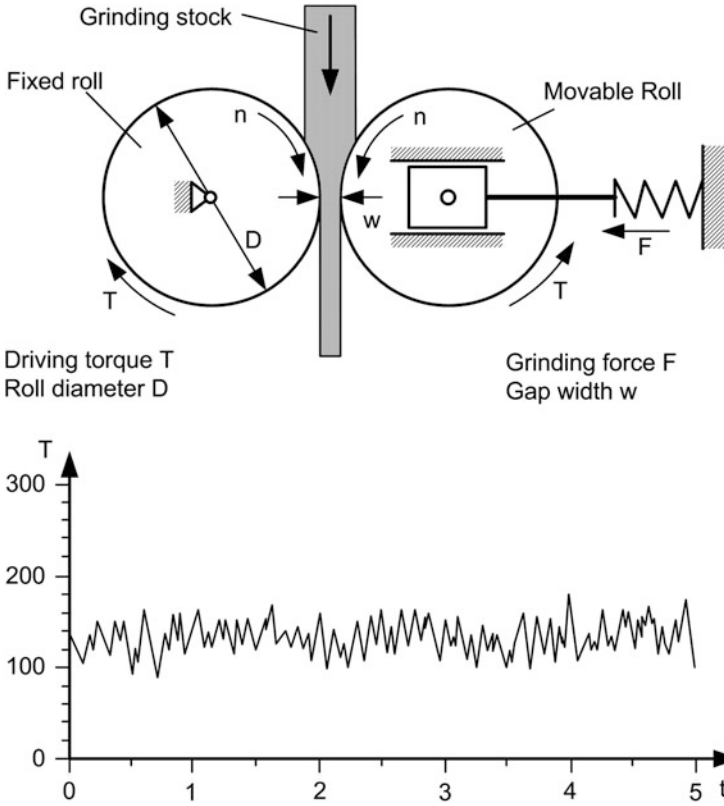
For assessing the durability of components, a measurement of the loads which will occur during the service life is crucial. However, the quality of load data thus obtained may be compromised by measurement anomalies, such as spikes. For correcting such signals, the application of spike-removal algorithms is proposed [Corn14].

A characteristic distinction among stress-time functions depends on whether or not mean-load variations occur, Fig. 2.4.

A typical case of mean-load variation is the variation of local stresses on the lower side of an aeroplane wing during travel on the runway and during flight. In the case of travel on the runway, oscillations occur in the compression range (bending loads due to the weight of the wing as well as stresses due to the condition of the runway and travel manoeuvres by the pilot). During flight, oscillations occur in the tensile range (bending moment due to buoyant force, as well as stresses due to turbulence and flying manoeuvres). A similar characteristic of a stress time function can occur on the driving shaft of a reversing rolling stand.

Operating processes can take place continuously as well as discontinuously, Fig. 2.5. As can be seen from the pressing of a fender, constant amplitude can occur in some cases for a limited period of time during which a specific structural component is produced.

In Fig. 2.6, the variations determined as a function of the time at measuring points on wheel-set axles of a train during travel over switch points are plotted in the upper section [Jenn04]. The wheel set under investigation was equipped with appropriate measuring devices; thus, the bending stress was measured with the use



**Fig. 2.3** Stress-time function with variable amplitude, drive moment  $T$  of a high-pressure grinding-rod mill during grinding of cement clinker (measurement of  $T$  between the motor and transmission gear)

of strain gauges in a full-bridge circuit. The vertical forces  $Q_1$  and  $Q_2$  at the wheel-contact point of the left and right wheels are plotted in the middle section of the figure. The lateral forces  $Y_1$  and  $Y_2$  at the wheel-contact point of the wheel set are plotted in the bottom section.

Each complete load cycle of the bending measuring points  $M_1$  and  $M_2$  corresponds to one revolution of the wheel. If the plane of the bending measurement is situated in the horizontal position, the measured signals exhibit zero crossing (neutral zone). If the measuring planes are positioned vertically, a maximum is observed, and a 180-degree rotation of the wheel-set axle results in a minimum. During interference-free running on a straight track, a sinusoidal oscillation occurs at a frequency equal to the rotational speed of the wheel. In this manner, the rotational speed or frequency of the wheel set can be determined.

The forces which act on the rail are represented in the negative sense for rail vehicles. Consequently, the  $Q$ -forces (vertical forces) have a negative offset, which results from the static load of the train.

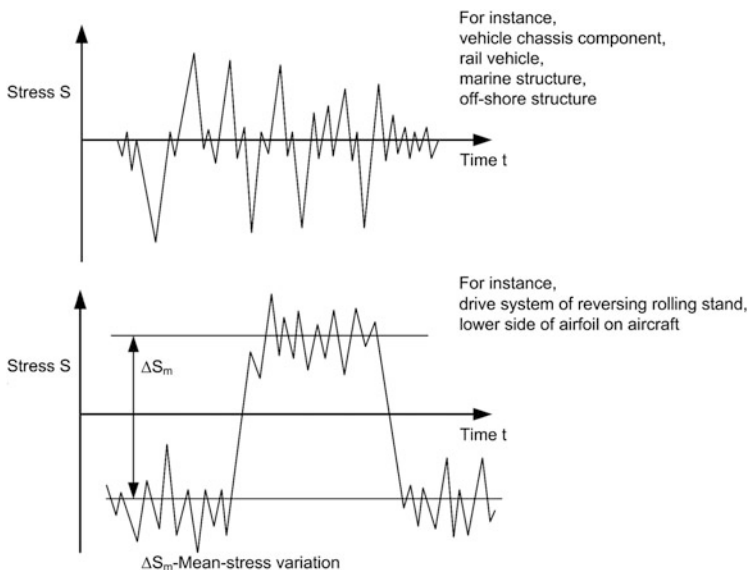


Fig. 2.4 Stress-time functions without and with mean-stress variation (schematic)

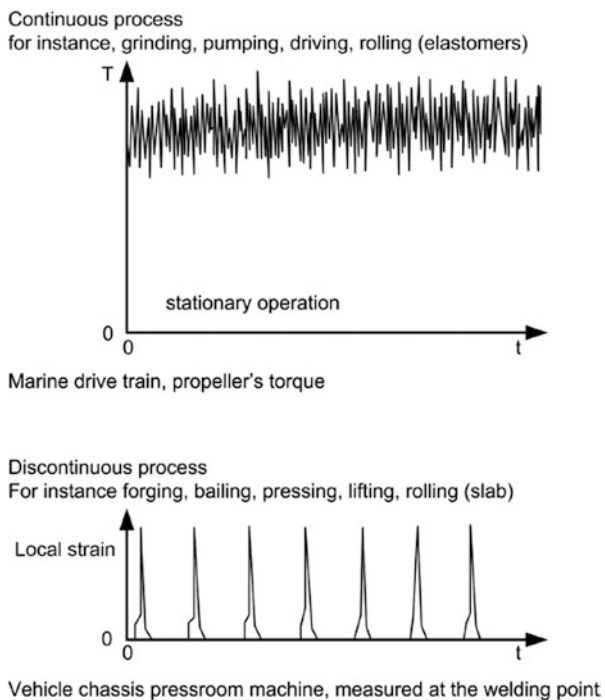
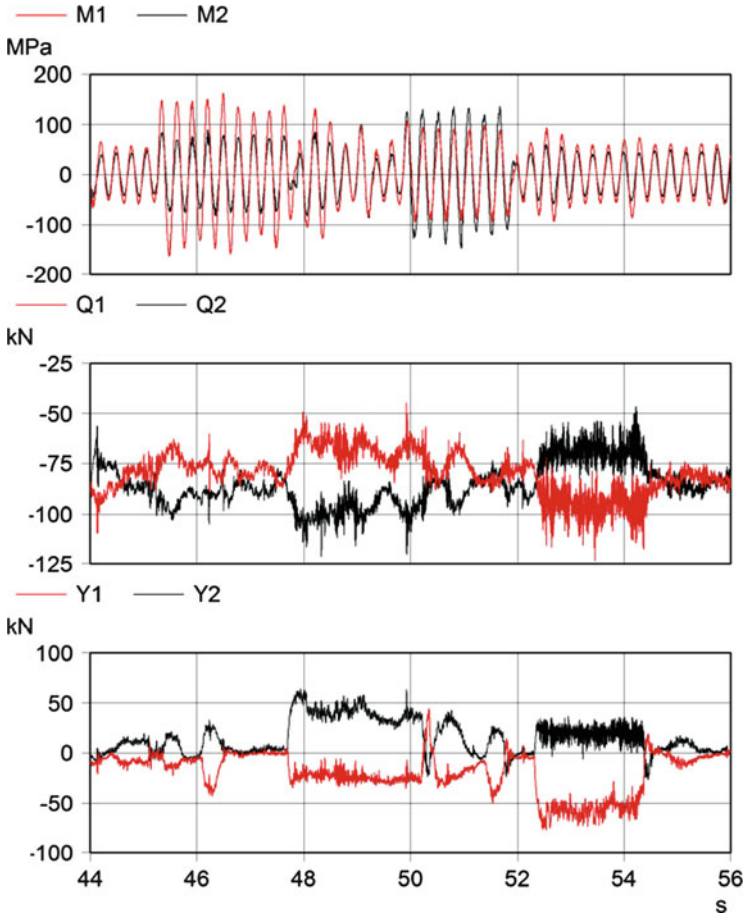


Fig. 2.5 Continuous and discontinuous stress-time functions



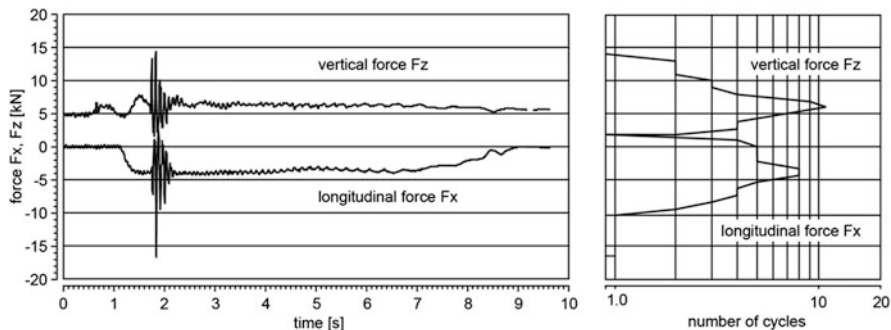


**Fig. 2.6** Variation with time for bending of wheel-set axle, vertical and lateral forces during travel over a switch point

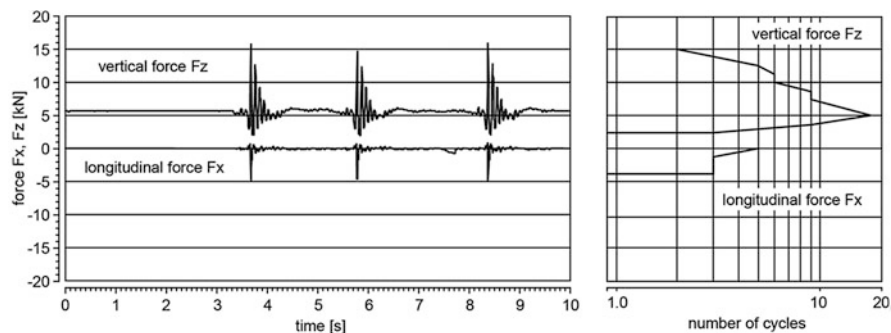
The forces at the wheel-contact point of the left and right wheels exhibit mutually opposing tendencies. During travel on a curve, the load on the outer wheel results in relief of the inner wheel because of the rocking motion of the rail vehicle.

In the present example, the variation-with-time curves were plotted during travel over a switch point. In the zone of the switch point, severe bending stresses occur on the wheel-set axle as a result of the very rapid quasistatic and dynamic transverse acceleration.

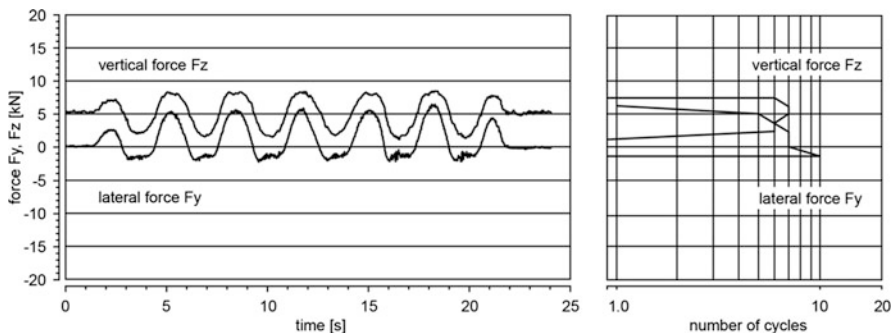
In Figs. 2.7, 2.8, and 2.9, the variations of the wheel forces with time are plotted for a motor vehicle during various driving manoeuvres. The wheel forces were determined with the use of wheel force transducers, which indicate the translational



**Fig. 2.7** Variations with time and spectra (level-crossing count) for vertical force and longitudinal force on the wheel of a motor vehicle upon rolling over three potholes



**Fig. 2.8** Variations with time and spectra (level-crossing count) for vertical force and longitudinal force on the wheel during a braking manoeuvre over a bumpy roadway



**Fig. 2.9** Variations with time and spectra (level-crossing count) for vertical force and transverse force on the wheel during slalom driving

forces acting on the wheel (longitudinal force  $F_x$ , transverse force  $F_y$ , vertical force  $F_z$ ) and the associated moments ( $M_x$ ,  $M_y$ ,  $M_z$ ) at each instant of time.

In Fig. 2.7, the variations of the vertical force and the longitudinal force with time are plotted for the process of rolling over three potholes. At the beginning of the manoeuvre, the vertical force corresponds to the static wheel load which is caused by the weight. The longitudinal force is nearly equal to zero. During the process of rolling over a pothole, the wheel is initially relieved from load because of the damping force of the suspension. Upon impact of the wheel against the rear edge of the pothole, a shock pulse acts through the centre of the wheel and generates pronounced longitudinal and vertical forces. The vertical force oscillates at the wheel frequency of 12–15 Hz, but this oscillation is very effectively damped.

The variations of the vertical force and of the longitudinal force with time during a braking manoeuvre are plotted in Fig. 2.8.

At the beginning of the manoeuvre, the vertical force corresponds to the static wheel load, which is caused by the weight. As soon as the braking manoeuvre begins, the negative longitudinal force increases and nearly attains the absolute value of the static wheel load because of the friction at the wheel-contact point (coefficient of friction:  $\mu \approx 1$ ). During the braking process, the wheel encounters a bumpy site in the roadway; this results in dynamic excitation in both the vertical and longitudinal directions. Oscillation occurs in the vertical and longitudinal directions. This oscillation is very effectively damped and decays after a few load cycles.

During slalom driving, the maximal transverse forces attain the absolute value of the static wheel load, Fig. 2.9. In correspondence with the position of the centre of gravity and because of the effective transverse acceleration, the wheel-contact force increases or decreases in proportion to the transverse force.

## 2.2 Causes of Stress

Fatigue stresses occur during the operation of a machine plant, a means of transport, as well as in stationary structures (such as bridges). The following cases can be distinguished:

- Basic stresses caused by operational processes as well as open- and closed-loop control processes,
- additional stresses from the environment, and
- resonant oscillations.

In the following, examples of basic and additional stresses as well as stresses associated with resonant oscillations are considered for an automobile, an aeroplane, a roll stand, and a crane facility:

### Basic Stresses

Automobile      acceleration, braking, travel on curves (driving manoeuvres)

Aeroplane	Ground-air-ground load alternations and flying manoeuvres
Roll stand	Air-gap moment in electric motors and rolling moment of working rolls
Crane facility	Raising and lowering of loads

### **Additional Stresses**

Automobile	Variations in roadway quality
Aeroplane	Turbulence
Roll stand	Chattering oscillation during slippage
Crane facility	Lateral forces due to track deviation

### **Resonant Oscillations**

Automobile	Components mounted on engines: excitation due to engine vibration
Aeroplane	Oscillation of jet engines
Roll stand	Resonant frequencies in the drive train
Crane facility	Vertical load oscillations due to elasticity of the ropes

In many cases, a static stress is superimposed on the fatigue stress, for instance, because of the dead weight (maximal axle weight of a motor vehicle), because of initial stress (screw connections) or because of residual stresses (welded joints). These static stresses can also vary during operation (changes in the payload, relief of residual stresses).

Not only mechanical forces and moments cause stresses. The occurrence of temperature gradients also causes stress if thermal expansion is hindered. The resulting stresses may be quasistatic, or they may also vary with time.

In many cases, resonant oscillations are also decisive for fatigue behaviour. As a matter of principle, machine facilities and means of transport can always be represented as systems of masses and springs (elasticity) which are capable of oscillation. The drive train of a roll stand is taken as an example for illustrating this principle, Fig. 2.10 [Brun91]. The moving (rotating) masses include the rotor, clutches, drive shafts, gears, spindles, and rolls. In the idealised case, the masses can be regarded as rigid bodies, and these masses can be considered to be connected by springs with zero mass. For unimpeded multiple-mass oscillators, the number of resonant frequencies corresponds to the number of masses. In this model, a resonant frequency of zero is also possible.

If a constant moment were introduced as load-input function on the working rolls, a constant torque over the drive train would result if the rotor of the electric motor were blocked. In this case, only the branching by the pinion gear and the transmission ratios of the gears would have to be considered.

However, if a sudden load-input function were imposed, all resonant frequencies would be excited, and different stress-time functions would occur over the drive train. In Fig. 2.10, corresponding stress-time functions are plotted for three points of the drive train during the entry of a block. These functions differ with respect to the

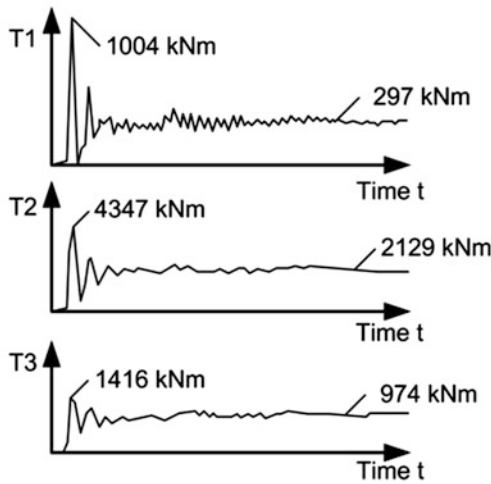
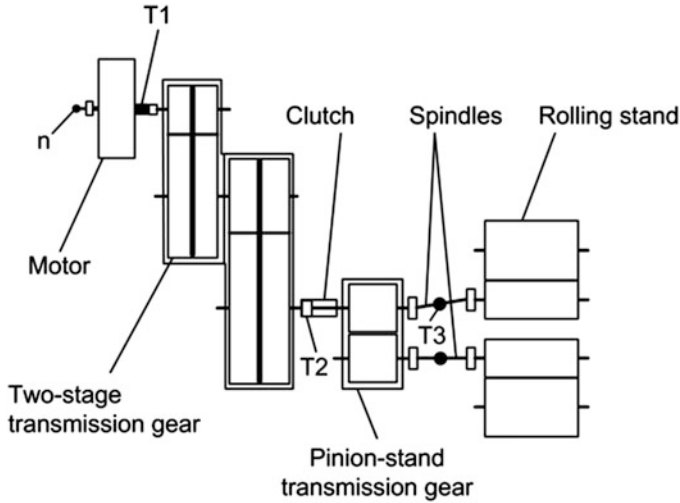


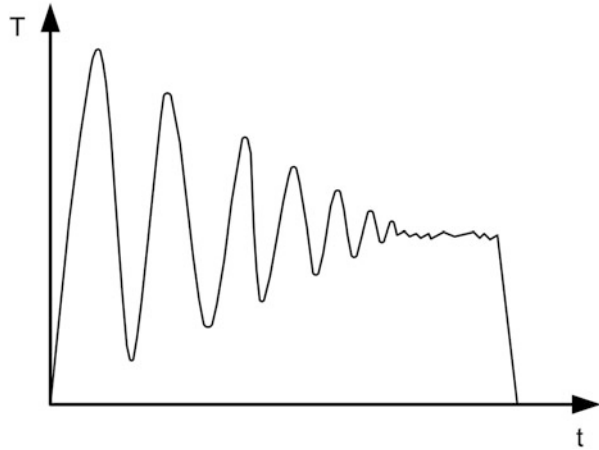
Fig. 2.10 Drive train of a rolling stand with three measuring points, torques for a first pass

magnitude of the moments, the ratio of the maximum to the stationary rolling moment, and the frequencies in the stationary range [Brun91].

In Fig. 2.11, an idealised stress-time function is plotted for the torque on the drive shaft during a first pass. The sudden stress, which occurs upon entry of the rolling stock into the air gap between the working rolls, is represented by the load-input function, which excites the oscillations of the system.

Pronounced damping occurs as a result of the rolling process, and a constant moment is thus established. After the passage of the rolling stock, this moment

**Fig. 2.11** Idealised stress-time function for the first reduction pass (rolling stand)



decreases to zero. The stress-time function can be decomposed into two excitations: the rolling process and a damped oscillation, Fig. 2.18.

Especially high stresses can occur in oscillating systems as a result of periodic excitation and low damping if the frequency of the excitation corresponds to a resonant frequency of the system, that is, if resonance occurs.

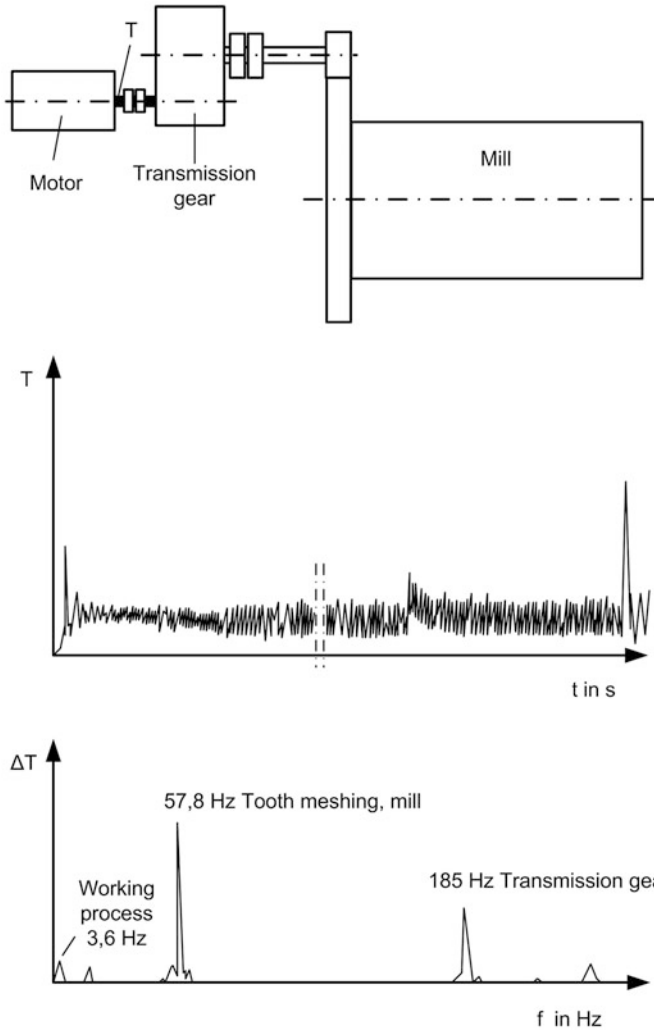
In Fig. 2.12, the stress-time function and the frequency spectrum are plotted for a ball mill. The stress-time function is a section of the starting process with a star-delta connection for the electrical power supply to the motor. The frequency analysis over an extended period of time indicates that high amplitudes can be ascribed to specific tooth-meshing frequencies (pinion/gear rim of ball mill and gear transmission).

Especially starting and stopping processes can result in high stresses. In Figs. 2.13, 2.14, and 2.15, the torque is plotted for the drive train of a ship, an elastomer roll, and a bowl-mill crusher. For measurements on such systems under operational conditions, care must be taken to ensure that the measurements are performed under conditions which yield the maxima which are relevant for designing and dimensioning.

The effects of unfavourable roadway conditions and of driving manoeuvres on the spring excursion at the front of an automobile are clearly illustrated in Figs. 2.16 and 2.17. The resulting path-versus-time curves differ drastically in maximal amplitudes, irregularity, directional sense, and frequency.

### 2.3 Deterministic and Stochastic Stress-Time Functions

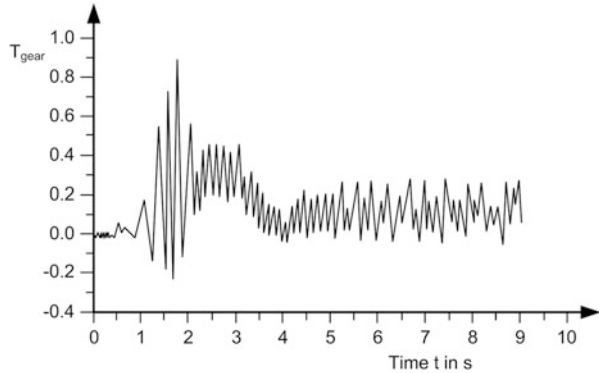
In the case of deterministic stress-time functions, the stress-time variation obeys a definite law. Thus, the magnitude of the stress and the instant at which it occurs can be indicated unambiguously. This is the case during a normal reduction pass, as



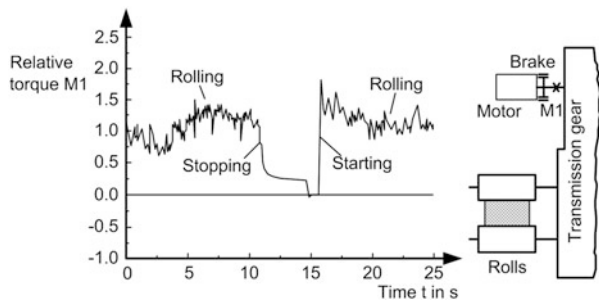
**Fig. 2.12** Stress-time function and frequency analysis for a ball mill, motor moment or double amplitude

shown in Fig. 2.11. The torsional moment  $T$  of the drive consists of the rolling moment, which remains constant during the rolling process, and a damped oscillation, Fig. 2.18. The rolling moment can be derived from the parameters of the rolling process (decrease in the cross-section of the rolling stock, temperature, rotational speed, etc.). A sudden impact occurs during entry of the block between the working rolls and excites oscillations in the drive train. The resulting frequency depends on the masses and stiffness values of the drive train. Damping is caused especially by the rolling stock.

**Fig. 2.13** Stress-time function for the drive moment of a ship upon starting



**Fig. 2.14** Stress-time function for the drive train of an elastomer roll upon stopping and starting



Stochastic stress-time functions cannot be described explicitly by mathematical equations. A description is possible only with the application of statistics and probability calculus. Stresses of this kind are also designated as random stresses. However, the statistical properties can be described by definite values. A prerequisite for a description is a stationary process.

Typical stochastic stress-time functions are caused by stresses from the environment, by the effects of turbulence on aircraft, of rough seas on ships and offshore constructions, or of roadway conditions on motor vehicles. Furthermore, stochastic stress-time functions result from grinding processes, for instance, Fig. 2.3.

**Characterisation of Stochastic Signals** Stochastic signals can be characterised by average properties [Buxb79, Buxb92]:

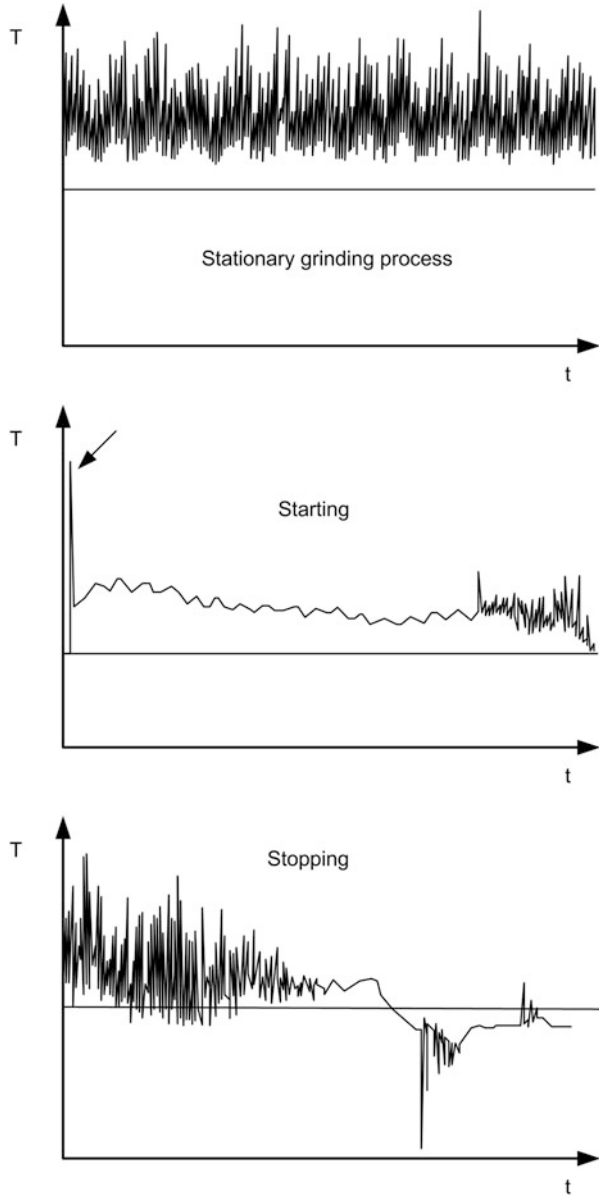
- arithmetical mean value

$$\bar{x} = \frac{1}{T} \int_0^T x(t) dt \tag{2.1}$$

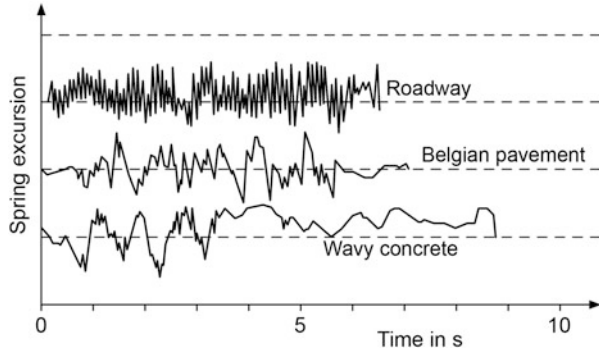
$\bar{x} \rightarrow$  Expected value, 1<sup>st</sup> order, for  $T \rightarrow \infty$



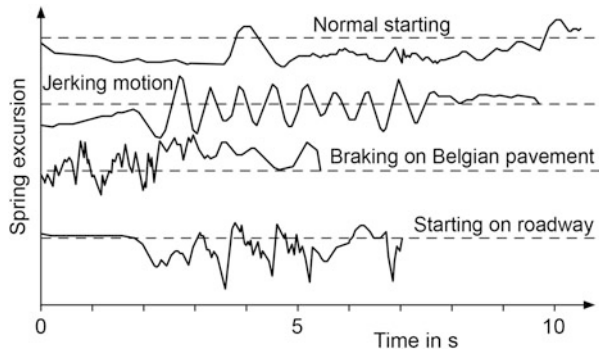
**Fig. 2.15** Stress-time functions for the drive train of a bowl-mill crusher



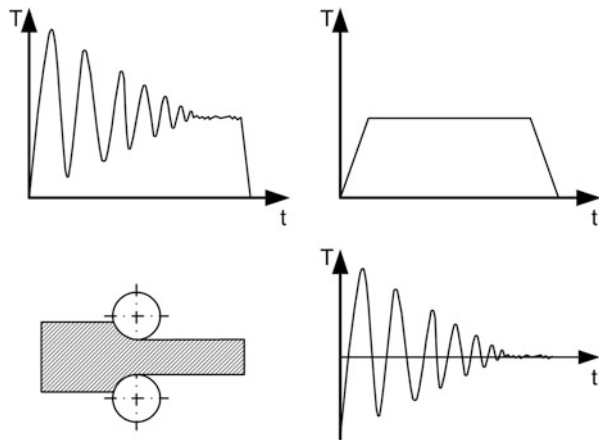
**Fig. 2.16** Measurement of the spring deflection at the front of a motor vehicle, for the stress during travel over a bad roadway



**Fig. 2.17** Measurement of the spring deflection at the front of a motor vehicle, for the stress during a manoeuvre



**Fig. 2.18** Stress-time function for a first pass, decomposition of the excitations



- mean-square value

$$\overline{x^2} = \frac{1}{T} \int_0^T x^2(t) dt \quad (2.2)$$

$\overline{x^2} \rightarrow \text{Expected value, 2}^{nd} \text{ order, for } T \rightarrow \infty$

- Effective or RMS value (root-mean-square value)

$$x_{eff} = \sqrt{\overline{x^2}} \quad (2.3)$$

- Variance

$$\sigma^2 = \frac{1}{T} \int_0^T (x(t) - \bar{x})^2 dt \quad (2.4)$$

$\sigma$  – Standard deviation

The relationship between the mean-square value, the arithmetical mean value, and the variance is given by:

$$\overline{x^2} = \bar{x}^2 + \sigma^2$$

For  $\bar{x} = 0 \rightarrow \overline{x^2} = \sigma^2$  (2.5)

The crest factor describes the ratio between the maximum and the effective value of a signal:

$$C = \frac{x_{\max}}{x_{eff}} \quad (2.6)$$

$x_{\max}$  – Maximum of  $x(t)$

In Sect. 4.3, the irregularity factor will be explained and employed for describing the variation of the mean value of the load time history of a signal, Fig. 5.1. This factor represents the ratio between the number of zero or mean-load crossings in one direction and the number of maxima. The procedure for treating minima is analogous.

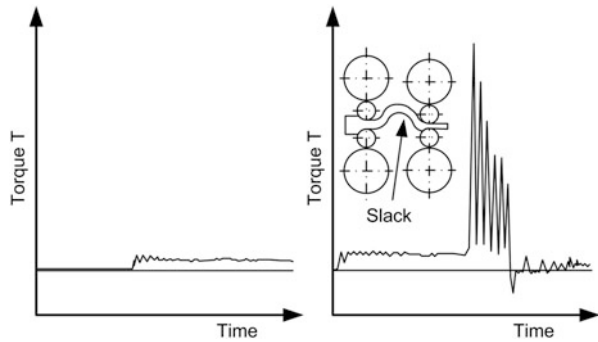
## 2.4 Special Events and Misuse

In machine facilities and means of transport, rare operational states can occur and can result in overload. In Fig. 2.19, the variation of the driving moment on a roll stand is plotted for a normal pass (*left*), that is, for the passage of the rolling stock through the roll opening, and for a blockage process due to sheet-metal tripling (loop formation) (*right*). In Fig. 2.20, a normal reduction pass is plotted for a cogging mill (*left*). A reduction pass with superimposed chatter oscillations is also indicated (*right*); these oscillations can result from relative motion between the rolling stock and the working rolls.

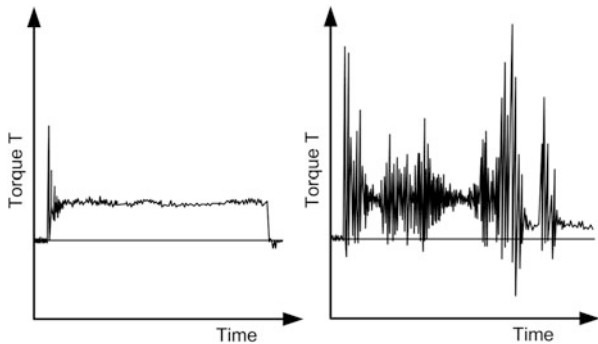
Excess loads can occur in airplanes as a result of extreme turbulence. In a crane they can result from overloading; in wind turbines they can occur because of a short-circuit in the power grid, and in an automobile they can result from travel over a threshold with a heavy payload. Excess loads on a railway vehicle can be caused by a derailment. If a multitude of special events encountered in practice are considered, these events can be assigned to three causes: operational interference, incorrect operation, and misuse.

Excess loads due to special events can cause fracture of a structural component, permanent distortion, crack initiation, propagation of an existing crack, or local plastic deformation with residual stresses. In designing and dimensioning for

**Fig. 2.19** Normal reduction pass (*left*) and blocking process due to sheet tripling (*right*)



**Fig. 2.20** Normal reduction pass (*left*) and reduction pass with superimposed rattling oscillations (*right*)



ensuring sufficient fatigue strength, excess loads should first be considered as static loads. In a second step, the manner in which the fatigue process is affected by the excess load, either once or repeatedly, must be examined. In many cases, the severity of overloads during special events and the frequency of occurrence of such excessive loads during the design life of a component are extremely difficult to determine. Measurements such as those plotted in Fig. 2.19 constitute an absolute exception for obvious reasons. An estimate may be possible by means of an analytical simulation, although the accuracy is often limited.

With the assumption that locally limited plasticity occurs as a result of excessive loads tests have shown that a higher tolerance toward excess load can be assumed for a limited number of excess loads (for instance 100) [Hars96, Jung93]. However, this assumption applies only to metallic materials of sufficiently high ductility. As a result of plasticity, strain hardening occurs together with the generation of residual stresses. In this case, the directional sense of the excess load is a decisive factor. During tensile overloading, local stress redistribution occurs at a notch and results in the generation of compressive residual stresses, which may favourably affect the fatigue behaviour. A similar result applies to incipient cracks in structures where crack propagation may be delayed after the occurrence of excess loads [Rada07, Schi09].

Of course, various means are available for protecting against excess loads such as mechanical impact (protection of helical springs against coil contact), slipping clutches for protecting drive trains against blocking processes, predetermined breaking points between primary and secondary units, emergency electronic shut-down, etc. Continuous monitoring of structural components and systems by means of load measurements during operation is still an exception, even though such measurements usually do not present any technical problem. In cases of damage, questions concerning the magnitude of the loads, which have occurred still cannot be answered satisfactorily.

In the automotive industry, a more specific definition of loads due to special events and misuse has become established [Hauk04, Spor03]. In analogy with the Law Relating to Product Liability, a distinction is made between use of the motor vehicle for the intended purpose and use which is not in accordance with the intended purpose. By way of departure from events of misuse, special events are by definition still included under the term “use of the motor vehicle for the intended purpose” in this case. These events involve extremely rare, severe loads, which, however, must be tolerated by the motor vehicle without permanent damage and without limitation of the planned design life. Because of the low repetition rate of the events in the course of testing, a separate confidence test is often performed.

If the motor vehicle is employed in a manner, which is not in accordance with the intended purpose, hazards to drivers, passengers, and pedestrians must still be avoided. Consequently, events of misuse are also considered in designing and dimensioning the motor vehicle. The decisive requirement is the avoidance of hidden damage. Impairment of the functional safety and reliability or of the remaining fatigue life of automotive components after misuse must be unambiguously recognised by the driver and kept safely under his control. For this purpose,

the “deformation before fracture” principle is applied. The individual chassis components along the path of a load, for instance, wheel, wheel support, chassis steering assembly, auxiliary frame, and vehicle body, are dimensioned in such a way that a permanent deformation initially occurs only in defined structural components in the event of loads due to misuse. The explicit use of ductile materials in the chassis zone allows the chain of damage to be defined in such a way that the driver receives an unambiguous warning response without the risk of accidental failure due to the fracture of the component concerned. Examples of such causes include a skewed position of the steering wheel or dents and bulges on the wheel rim or tire damage. At the same time, the resulting repair costs are kept as low as possible by appropriate designing of the damage chain.

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# Chapter 3

## Description of the Counting Methods

In the following, the most important counting methods applied for fatigue strength calculation are described and assessed. Recommendations are given for application.

### 3.1 Basic Principles

A stress-time function is employed as example for the explanation, see Fig. 3.1.

#### 3.1.1 Classes

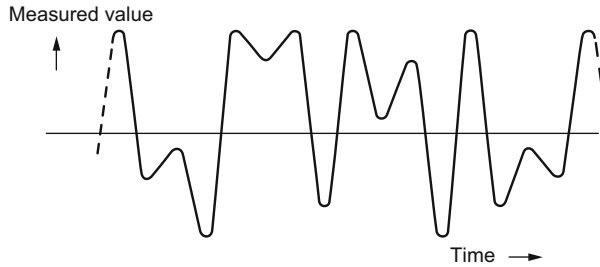
A prerequisite for counting is the subdivision of the range of measurement into classes of equal size. The classes must be numbered consecutively in the direction of positive measured value. For the counting examples, eight classes of equal size are selected, Fig. 3.2. As a rule, values which are situated on a level class border are assigned to the higher of the two classes.<sup>1</sup>

For the following examples, the zero line of the measured value is located on the 4th level, that is, between classes 4 and 5.

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<sup>1</sup>Results of various counts are mutually comparable only if such conventions agree.

**Fig. 3.1** Example of a stress-time function



**Fig. 3.2** Classification of counting



### 3.1.2 Range of Restoration and Class Width

For all counting methods, oscillations which occur within a class are not recognised and are thus automatically suppressed. However, if such an oscillation of low amplitude were to exceed the adjacent level, a count would be triggered.

For preventing random effects of this kind, and especially for immediately filtering out small oscillations which would not cause damage and which frequently result from noise or hum in the electronic measuring system, a range of restoration, also designated as hysteresis, is introduced.

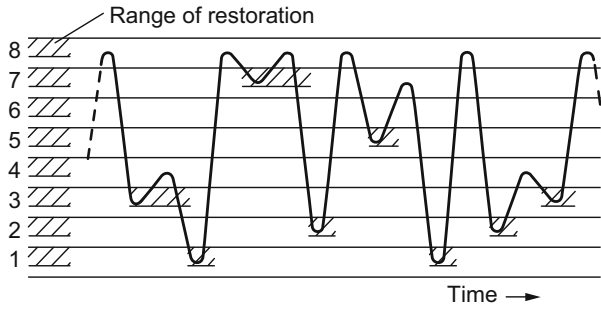
Introducing a range of restoration ensures that counting does not begin until an oscillation exceeds a defined value of the amplitude. The range of restoration can be level-oriented or extreme-value oriented, Figs. 3.3 and 3.4. For counting methods with which crossing of levels is evaluated, the range of restoration is directly bound to the levels, Fig. 3.3. If a count is performed at a level, the next count cannot occur until the stress-time function has crossed the lower or upper level in the direction opposite to that of counting and then crosses the level once again.

For counting methods which operate with ranges, the range of restoration can be oriented with respect to the extrema, that is, a further count does not occur until the counting range is larger than the range of restoration, Fig. 3.4.

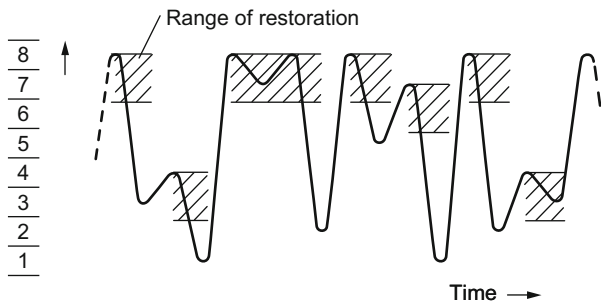
With computer-aided recording and evaluation of measured data, the selection of a higher number of classes does not present any difficulty. Nevertheless, it should be ensured that the range of measurement and the number of classes are defined in a reasonable manner. The selection of the range of measurement should correspond to the expected maximal load. For the subdivision of the range of measurement into



**Fig. 3.3** Range of restoration, level-oriented



**Fig. 3.4** Range of restoration, extreme-value oriented



classes, the expected accuracy of the measurements should be considered. Sixty-four classes are often employed for historical reasons. In the past, on-line counting methods operated with 8, 16, 32, or a multiple of classes, as dictated by the state of the art. For the stress-time function employed as an example here, only eight classes are employed, for the sake of clarity. It must be ensured that the ratio of the class width to the accuracy of measurement makes sense. With 128 classes, a class width describes less than 1% of the range of measurement.

The range of restoration can be defined for one or several class widths. It must be adapted to match the load-time function. In accordance with [Gude99], a value of 2.5% of the range of measurement or less is recommended for the range of restoration.

The treatment of counting methods in the literature is not standardised. Consequently, individual publications may deviate from the descriptions presented here.

Important publications on counting methods are the following:

Schijve 1963	[Schi63]	Schütz 1992	[Schü92]
Buxbaum 1966	[Buxb66]	Murakami 1992	[Mura92]
DIN 1969	[DIN69]	Seeger 1996	[Seeg96]
Kowalewski 1969	[Kowa69]	Rice 1997	[Rice97]
Günther 1973	[Günt73]	Gudehus 1999	[Gude99]
TGL 1977	[TGL77]	ASTM 2005	[ASTM05]
De Jonge 1982	[Jong82]	Lee 2005	[Lee05]

(continued)

Gassner 1983	[Gass83]	Haibach 2006	[Haib06]
Potter 1989	[Pott89]	Schijve 2009	[Schi09]
Buxbaum 1992	[Buxb92]	ISO 2013	ISO13a], ISO13b]

## 3.2 Standards

The counting methods described in the following for the cycle counting of load-time functions are described in standards and series of technical rules. In particular, ASTM Standard E 1049–85, “Standard Practices for Cycle Counting in Fatigue Analysis”, should receive special mention [ASTM05]. This standard includes level-crossing counting, peak counting, range counting, range-pair counting, as well as rainflow counting, with a reference to ISO 12110 [ISO13b]. The French AFNOR Standard A03–406 describes a four-point algorithm for rainflow counting.

The use of DIN 45667, published in 1969, is not recommended, since this standard is now obsolete and does not include any reference to fatigue strength under variable stress amplitude, [Buxb92].

## 3.3 One-Parameter Counting Methods

In the following, four one-parameter counting methods—peak counting, level-crossing counting, range counting, and range-pair counting—are described and assessed.

### 3.3.1 Peak Counting

#### Description of the Method

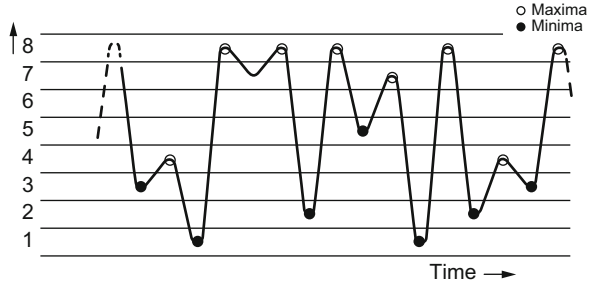
Peak counting yields the frequency distribution of the extreme values (reversing points) of a stress-time function. In general, only the maxima are counted. The spectrum is plotted as a cumulative frequency.

#### Description of the Counting Algorithm

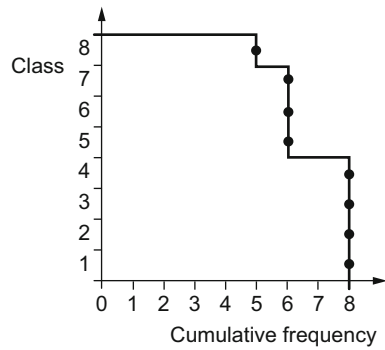
Counting begins at an arbitrary peak value of the measuring signal. The maxima are counted in the respective classes, Fig. 3.5. The resulting stress spectrum is shown in Fig. 3.6.

Three further versions of this method exist:

**Fig. 3.5** Stress-time function, peak counting



**Fig. 3.6** Result of peak counting (maxima only)



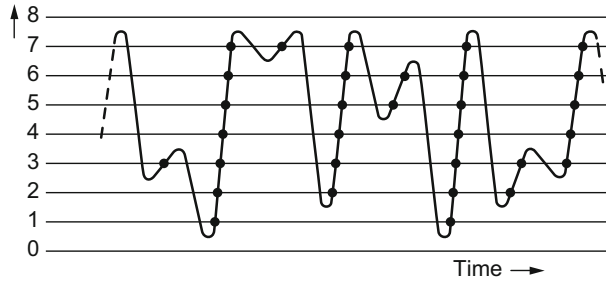
- Version I: The minima are counted. In this case, the result of the count differs from that obtained by counting the maxima, Fig. 3.6.
- Version II: The positive peaks above and the negative peaks below the basic stress are counted (*peaks and troughs counting*). Counting then begins at a “zero crossing” of the stress (in this case, between classes 4 and 5).
- Version III: Only extreme values between zero crossings are counted (*mean-crossing-peak-counting*), [Schi63].

All versions indicated here yield different results for typical stress-time functions from measurements under operational conditions.

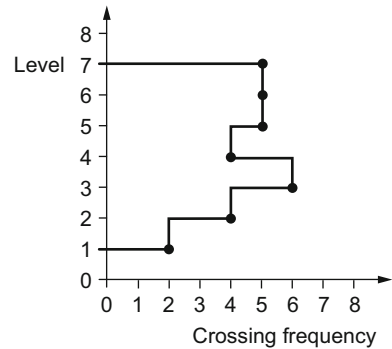
**Comment**

In general, the oscillation ranges represented in the spectrum are larger than those which actually occur in the stress-time function. This counting method is not recommended for analytical fatigue-life predictions. Peak counting is probably the oldest and certainly the simplest counting method [Schi63].

**Fig. 3.7** Stress-time function, level-crossing counting



**Fig. 3.8** Result of level-crossing counting



### 3.3.2 Level-Crossing Counting

#### Description of the Method

The level-crossing counting method yields the crossing frequency of levels as result. The frequencies are cumulative frequencies.

#### Description of the Counting Algorithm

The zero level is located below the lowest measured value, Fig. 3.7. The level crossings are counted on the positive (increasing) slopes of the stress-time function, Fig. 3.8.

Version I: Counting on negative (decreasing) slopes is also possible.

#### Comment

In level-crossing counting, the absolute value of the measured quantity is recorded, but the information on the amplitude and mean value of the individual oscillation is lost. Hence, this method is especially well suited for obtaining a quick survey of the measured maxima and minima, but not for determining the amplitudes.

With an irregularity factor  $I < 1$  (not all cycles have a mean-value crossing<sup>2</sup>), oscillation ranges determined in the load spectrum are larger than those which are actually present, see Sects. 4.3 and 5.1.

The crossing frequency corresponds to the cumulative frequency  $H$  for *all cycles* only if at least one level is crossed by all cycles. This is not the case for the stress-time function selected as an example, ( $H = 6$  for 8 cycles, Figs. 3.8 and 3.12). The number of peak values for two neighbouring classes cannot be derived from the number of level crossings for both levels [Schi63].

In the past, the level-crossing counting method was frequently applied for fatigue-life predictions. As dictated by the type of stress-time function, this method leads to shorter calculated lifetimes than other methods, for instance, range-pair counting. On the basis of present knowledge, level-crossing counting is not suited for fatigue-life predictions. If calculated and experimentally determined fatigue lives are compared for realistic stress-time functions, the scatter is significantly greater with the use of spectra from the level-crossing count than with spectra from the range-pair count, for instance, [Euli94], see Sect. 9.3.

For stress-time functions with variable mean loads, for instance different loading conditions, spectra can be generated for the respective individual mean loads; that is, partial spectra can be obtained [Li] [Schü76, Fisc80, Gude99].

Regular and irregular extrema (inversion points) can be distinguished for stress-time functions. Regular extrema occur in the case of maxima in the positive range and minima in the negative range. Irregular extrema occur in the case of maxima in the negative range and minima in the positive range. As shown in [Euli06], every irregular extremum “cancels” a regular extremum in the same class. The lower the irregularity factor, see Sect. 5.1, for a stress-time function is, the more regular peaks are deleted. With the level-crossing count, therefore, stress-time functions with a low irregularity factor are not assessed conservatively with respect to damage.

Level-crossing counting spectra are very clear-cut and informative, since the absolute values of the extrema as well as the associated frequency distribution are indicated. They are especially well suited for a comparison of spectra (for instance, various routes of travel, various drivers, various payloads, etc.), as well as for checking measurements under operational conditions. (Are the measured maxima and minima realistic? Could a zero-point drift be present?)

---

<sup>2</sup>If the mean load is equal to zero, one often speaks of zero crossings.

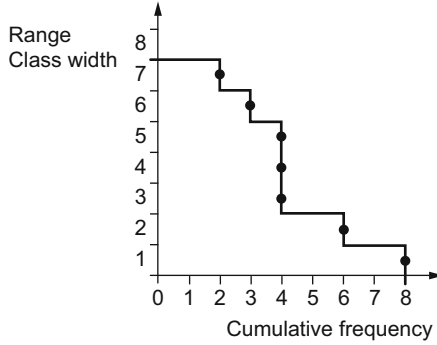


Fig. 3.9 Result of range counting

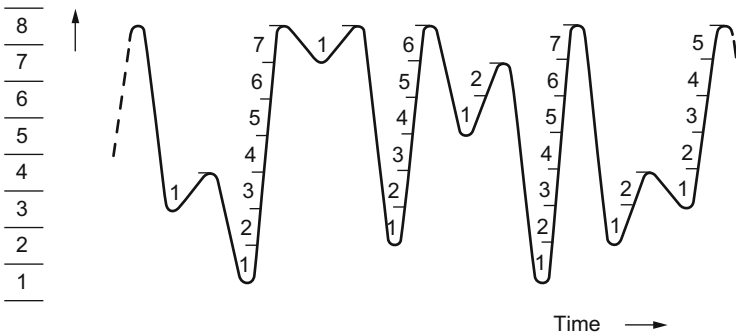


Fig. 3.10 Stress-time function, range counting

### 3.3.3 Range Counting

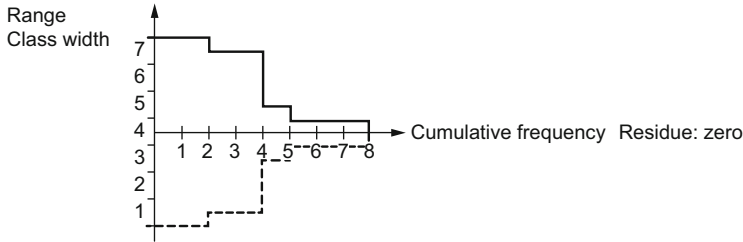
#### Description of the Method

The differences between two consecutive extreme values (reversal points) of the signal are counted as ranges. The result of range counting (range is a double amplitude), is plotted as a cumulative frequency curve, Fig. 3.9.

#### Description of the Counting Algorithm

Counting is performed on the positive slope of a cycle, Fig. 3.10. The size of the range is measured in class widths. Since the range size is determined by the preceding extreme value (in this case, the minimum), the resulting levels are variable.

Version I: Counting can also be performed on the negative slopes.



**Fig. 3.11** Result of range-pair counting

**Comment**

The results of range counting on the positive slopes differ from those obtained by counting on the negative slope. The absolute value of the peak is lost.

Range counting yields a stress spectrum, which is too narrow, since all ranges are referred to a fictitious level. This method is not recommended, either for a comparison of spectra or for estimating the fatigue life.

**3.3.4 Range-Pair Counting**

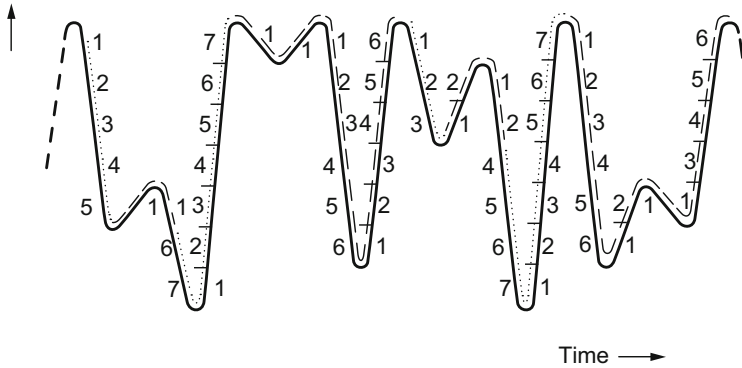
**Description of the Method**

By means of range-pair counting, the frequency of oscillation ranges is determined. A range pair consists of a positive and a negative slope of equal size and mean stress. The corresponding slopes can be immediately consecutive or at a larger distance from one another, as dictated by the shape of the stress-time curve. Non-sequential counting is thus involved here. The result of counting is plotted as a cumulative frequency curve, Fig. 3.11.

**Description of the Counting Algorithm**

Counting begins at an arbitrary minimum or maximum of the signal. The following increasing or decreasing slope of the signal, respectively, is subdivided into classes and numbered consecutively. The next maximum or minimum is the new zero point for counting. Ranges of equal size and mean stress on the positive and negative slope are combined to form a range pair, Fig. 3.12.

If ranges cannot be combined immediately to form a pair, the slope which is already present is “stored in memory” until the matching counterslope occurs. Ranges which are not combined to form pairs constitute the residue.



**Fig. 3.12** Stress-time function, range-pair counting

### Comment

The spectrum obtained by range-pair counting is often employed for predicting the fatigue life, since the amplitudes and the associated frequencies are correctly determined by this method as the decisive values for fatigue damage. Since exclusively stress ranges are counted, however, the absolute value of the signal (maximum, minimum, mean value) is lost.

As shown in a subsequent section, range-pair counting differs from rainflow counting because an additional parameter, the mean value of the stress cycle (or the maximum and minimum), is counted with the latter method.

For an analytical fatigue-life prediction, the rainflow matrix is often transformed to a range-pair counting spectrum. For this purpose, two possibilities exist:

- Neglect of the mean stresses, see Sect. 4.3, “Derivation of spectra from matrices”
- Performing an amplitude transformation for evaluating the effect of mean stresses [Haib06], see Sect. 9.1

Neglect of the mean stress can result in erroneous interpretations in the case of stress-time functions with large variations in mean load.

### Note

The application of range-pair counting is incorrect if stress ranges with different mean stresses are combined.



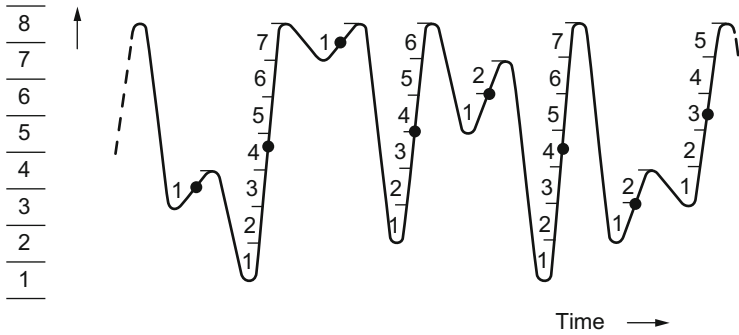


Fig. 3.13 Stress-time function, range-mean counting

**Development**

B.J. Lambie [[Lamb55](#)].

**3.4 Two-Parameter Counting Methods**

In the following, four two-parameter counting methods—range-mean counting, transition counting, range-pair-mean counting, and rainflow counting—are described and assessed.

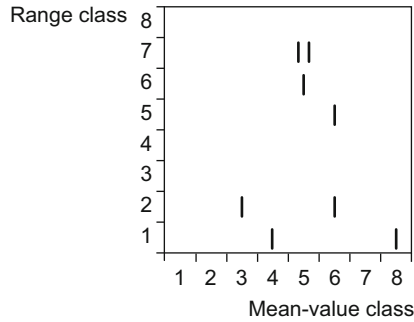
**3.4.1 Range-Mean Counting**

**Description of the Method**

As an extension of the one-parameter range-counting method RC, the respective mean value is also counted during range-mean counting, Fig. 3.13. The result of the count is a frequency matrix for ranges and mean values, Fig. 3.14. This frequency matrix can be transformed to the “start-class—target-class” form (see “Transition matrix”).

**Description of the Counting Algorithm**

The counting algorithm is identical to that for range counting; in addition, the mean load is recorded for each range.



**Fig. 3.14** Result of range-mean counting

### Comment

Range-mean counting is no longer in use.

### Development

A. Teichmann [[Teic41](#)].

## 3.4.2 Transition Counting

Transition counting is also designated as counting in a transition matrix TM, as well as correlation matrix, or Markov matrix.<sup>3</sup>

### Description of the Method

The positive and negative ranges are consecutively entered into a matrix, Fig. 3.15.

### Description of the Counting Algorithm

Counting proceeds from an extreme value as starting point. For the slope to be counted, the start value and target value are determined and entered into the corresponding class of the result matrix. In this process, increasing (positive) slopes

<sup>3</sup>A Markov matrix is the transition matrix of a Markov sequence of inversion points. Transition matrices are known from the field of stochastics for representation of the transition probability of Markov chains.

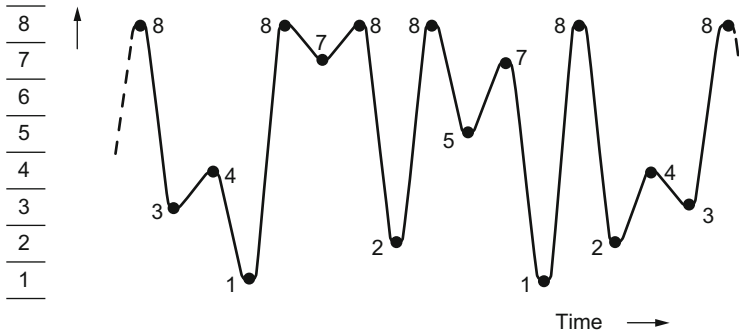


Fig. 3.15 Stress-time function, transition matrix

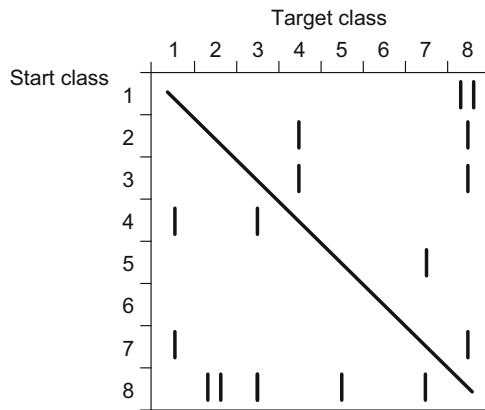


Fig. 3.16 Result of counting, transition matrix

are situated above the diagonal (of the matrix), and decreasing slopes are situated below. In conformance with the definition, the diagonal is not occupied, Fig. 3.16.

**Comment**

For the experienced user, the transition matrix provides a clear-cut survey of the essential contents of the stress-time function. The smallest slopes (half cycles) are located close to the diagonal; the largest slopes are located at the greatest perpendicular distance from the diagonal. Variations in the mean value result in a shift along the direction of the diagonal. The irregularity of a stress-time function is immediately visible, see ‘transition matrix’ for  $I = 0.99$  and  $I = 0.7$  in Chap. 5, “Comparison of the counting methods for exemplary stress-time functions”, see also Sect. 5.1.

The initial publication on transition counting with a transition matrix had aroused great expectations, since this method is very simple and highly transparent. However, transition counting as a sequential counting method is not capable of detecting superimposed oscillations with different frequencies. This fact has proved to be a decided disadvantage for estimating the fatigue life. In Chap. 5, transition counting is compared with rainflow counting for a superimposed sinusoidal-sinusoidal oscillation, see Figs. 5.4 and 5.5. The amplitude of the uncounted cycle 1-8-1 is considerably larger than that of the counted cycles. Consequently, the transition counting method is no longer recommended for fatigue-life prediction.

With the exception of those from range-pair counting, the results of one-parameter counting, as well as the irregularity factor, can be taken from the transition matrix.

## Development

Aicher	[Aich73]	Günther	[Günt73]
Fischer	[Fisc74]	Hück	[Hück76]
Krüger	[Krüg85a, Krüg85b]		

### 3.4.3 Range-Pair-Mean Counting

#### Description of the Method

The range-pair-mean (or range-pair-range) counting method corresponds to the range-pair counting method; however, the mean value is also recorded. The positive and negative slope of equal size are counted as a range pair and thus as a cycle. These two slopes possess the same mean value, Fig. 3.17.

The result of the count is entered into a matrix in which the number of range pairs and the class to which the mean values belong are plotted. The result can also be plotted separately as a cumulative frequency curve for each of the respective, individual mean loads, Fig. 3.18. If the mean load is not considered, the result of the count is the same as that obtained by range-pair counting. In the frequency distribution, the oscillation ranges are plotted in the order of decreasing mean values, see Fig. 3.18.

#### Description of the Counting Algorithm

The counting algorithm corresponds to that for range-pair counting; in addition, however, the mean load is recorded for each range.

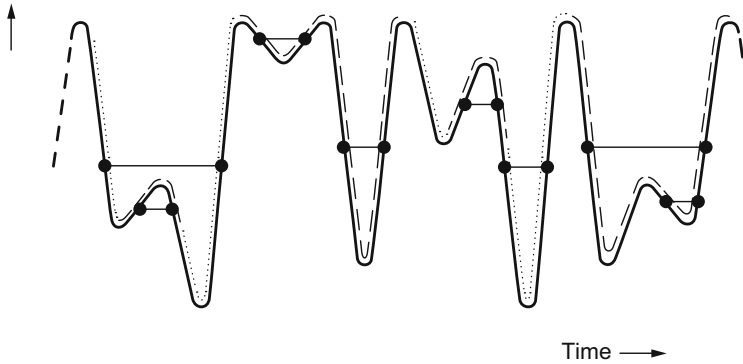


Fig. 3.17 Stress-time function, range-pair-mean counting

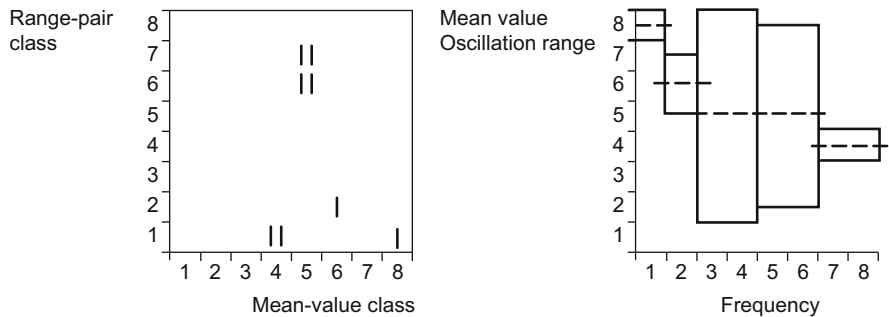


Fig. 3.18 Result of range-pair-mean counting, matrix and frequency distribution

**Comment**

Range-pair-mean counting can be applied for estimating the fatigue life. This method yields the same results as rainflow counting RFC [Jong82], or nearly identical results [Clor85]. It may be assumed that the two counting methods have originated independently from one another. The name “range pair range” was introduced by van Dijk [Dijk72].

**Development**

Range-pair-mean counting was described by de Jonge at an AGARD meeting in 1969.

De Jonge	[Jong69]	De Jonge	[Jong70]
Dijk van	[Dijk72]	De Jonge	[Jong80]
Clormann	[Clor85]	De Jonge	[Jong82]

### 3.4.4 Rainflow Counting

#### Description of the Method

During rainflow counting, the extrema are not counted sequentially as is the case with the transition matrix. Instead, the algorithm is selected in such a way that closed hystereses are recorded in each respective case. Hystereses which are not closed are recorded as a residue. The results of rainflow counting are entered in matrices.

#### Description of the Counting Algorithm

Various algorithms are available for rainflow counting. The “rainflow” form originally selected is illustrated in Fig. 3.19 [Mats68]. In this representation, the time axis must be imagined to have been rotated clockwise through an angle of  $90^\circ$ . Rain flows over the slopes and drops from one roof to the next. The following conditions apply:

The rain falls downward from inversion points (to the left and right of the vertical time axis), for instance, from b onto the straight line c–d, or from f onto the straight line g–h. Half cycles are counted if the water flows downward and attains a new inversion point (for instance, stress range a–b, b–c, or f–g), or if the water attains the point of arrival for the water which falls from an inversion point which is located above (plotted for the example of oscillation range c–b' and g–f').

Full cycles are formed from two half cycles of the same oscillation range and the same position (maximum, minimum), for instance, the cross-hatched areas a–d–e, b–c–b', f–g–f', e–h–i.

Unfortunately, this description fails to indicate the fact that the success of the rainflow counting method is based on the analogy with material mechanics. For

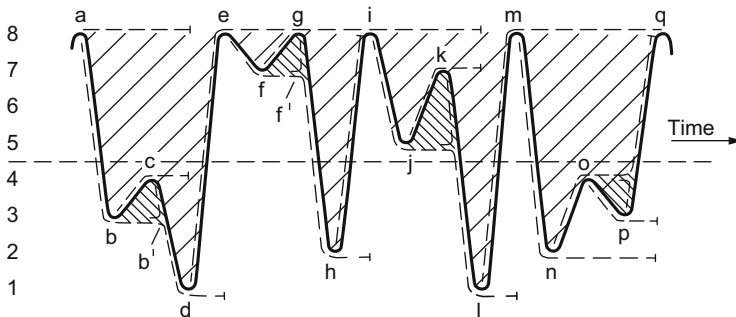
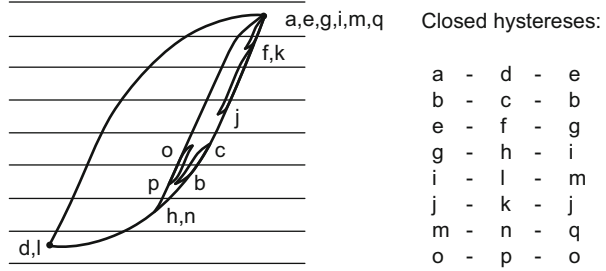
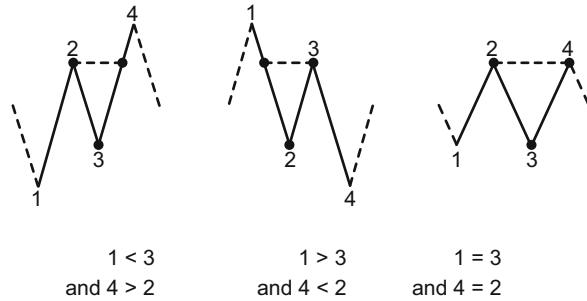


Fig. 3.19 Stress-time function, rainflow counting

**Fig. 3.20** Closed hystereses for the stress-time function taken as example



**Fig. 3.21** Logic of the four-point algorithm for recognising closed hystereses



better illustrating and visualising this situation, a consideration of the stress-strain curve in accordance with the local strain concept is appropriate [Berg85]. As shown in Fig. 3.20, standing stress-strain hystereses (that is a hysteresis starting with an increasing half cycle) result for the stress-time function taken as example (opposite: hanging, starting with a decreasing half cycle). These hystereses are located within the hystereses formed by the maximal extreme values.

In the past, various algorithms have been proposed for recognising closed hystereses, [Lang82, Down82, Böhm82, Heul84, Clor85, Glin87, Perr87, Anth97]. For this purpose, so-called three- and four-point algorithms are usually distinguished. That is, three or four consecutive extreme values are considered for forming closed hystereses, as the case may be, Fig. 3.21.

The various algorithms differ in the treatment of residues. A further difference concerns the manner in which initial and final states of the time signal are evaluated and how the order of the extrema affects the result of counting. Another difference depends on whether or not the algorithms define the position of the extreme value which occurs before counting. This implies that inversion points could be assigned to different cycles. Four-point algorithms are preferable to three-point algorithms. An assessment of algorithms is beyond the scope of the present work, however.

Finally, the question concerning the possibility of implementation in the program code, for instance, for real-time processing during on-line counting, also constitutes a distinguishing criterion.

Clormann and Seeger describe an algorithm with which the essential mechanisms of elastic-plastic material behaviour (Masing and memory behaviour) are

```

PROGRAM HCM
PARAMETER (UK=-3,OK=3)
INTEGER RES(OK-UK+1)
INTEGER HYMAT(UK:OK,UK:OK)

IR = 1
IZ = 0

1 READ(10,*,END=3) K

2 IF (IZ.GT.IR) THEN
  I = RES(IZ-1)
  J = RES(IZ)
  IF ((K-J)*(J-1).GE.0) THEN
    IZ = IZ-1
    GOTO 2
  ELSE IF (ABS(K-J).GE.ABS(J-1)) THEN
    HYMAT(J,1) = HYMAT(J,1)+1
    IZ = IZ-2
    GOTO 2
  END IF
ELSE IF (IZ.EQ.IR) THEN
  J = RES(IZ)
  IF ((K-J)*J.GE.0) THEN
    IZ = IZ-1
    GOTO 2
  ELSE IF (ABS(K).GT.ABS(J)) THEN
    IR = IR+1
  END IF
END IF
IZ = IZ+1
RES(IZ) = K
GOTO 1

3 WRITE(11,*) 'Residuum: '
WRITE(11,'(1X,33I4)') (RES(I),I=1,IZ)
WRITE(11,*) 'Hysteresisschleifenmatrix: '
DO 4 I=UK,OK
  WRITE(11,'(1X,33I4)') (HYMAT(I,J),J=UK,OK)
4 CONTINUE
STOP
END

```

Fig. 3.22 FORTRAN program for the algorithm of the hysteresis-counting method (HCM) [Clor85]

considered [Clor85]. In Fig. 3.22, the source code of the FORTRAN program for the Clormann-Seeger algorithm is given.



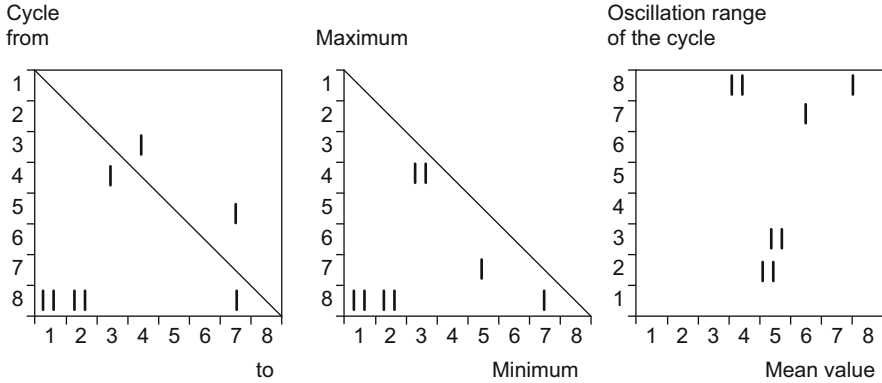


Fig. 3.23 Result of rainflow counting, three different matrix representations

**Limitation**

Because of the aforementioned effects, the results of counting with the different algorithms can deviate from the result indicated in Fig. 3.20.

**Comment**

The result of rainflow counting can be entered into matrices in various ways, Fig. 3.23:

- Full matrix: The cycles are entered into the matrix with their maxima and minima in correspondence with the direction of occurrence.
- Half matrix: The cycles are entered with their maxima and minima. The information on the direction of passage through the cycle, and thus also whether standing or hanging hystereses are involved, is lost.
- Full matrix: The cycles are entered into the matrix with their oscillation ranges and mean stresses. This type of entry corresponds to range-pair-mean counting. Without mean values, it corresponds to the result of range-pair counting.

Rainflow counting or range-pair-mean counting is currently regarded as the counting method by means of which the damage content of a stress-time function is best described. In analogy with the local strain concept, closed hystereses are considered. The area enclosed by a hysteresis in the stress-versus-strain plot can be interpreted as the energy, which is absorbed by a material element during a cycle. Energy is necessary for plastic deformation and for the formation of new surfaces (cracks). The damage process during fatigue of metallic materials can be associated with the absorption of energy.

The rainflow matrix is suited for performing manipulations such as amplitude suppression (omission) or truncation of high stresses in a simple manner. Omission is applied for eliminating cycles of low amplitude, which do not cause damage, for instance.

By recording the cycles in correspondence with oscillation range and mean value, rainflow counting offers the possibility of considering the effect of the mean stress on the damage for the analytical fatigue-life prediction. The so-called amplitude transformation converts all cycles of a rainflow matrix to damage equivalent cycles with the same mean stress or with the same stress ratio  $R$  [Haib06]. The Haigh diagram constitutes the basis for this procedure. The stress ratio  $R$  is the quotient of the minimum and maximum values of a load cycle.

One-parameter counting methods such as range-pair counting, level-crossing counting, peak counting, and the irregularity factor can be derived from the rainflow matrix. Rainflow counting is not restricted to cyclical elastic-plastic material behaviour; the method can also be applied to load-time functions for forces, couple of forces, nominal stresses, etc.

With the differentiation between standing and hanging hystereses, information can be obtained on the order of occurrence. For this purpose, one must distinguish whether a smaller intermediate cycle occurs before or after the occurrence of a larger extreme value [Berg85].

The stress-time function employed as an example here yields only closed cycles. If half cycles remain, one speaks of a so-called residue, see also Sect. 3.3.4. The largest possible residue for an arbitrarily long stress-time function is an augmenting or decaying series of half cycles, at the most. In the case of short stress-time functions, stresses, which are important for the fatigue-life estimate may possibly be recorded as a residue if the hystereses are not closed. For instance, no closed hysteresis is associated with a damped oscillation, see “Reduction-pass stress”, Sect. 5.4. Under some circumstances, for instance, with short stress-time functions, the residue must be taken into account for the fatigue-life prediction.

For stress-time functions with a large number of cycles (closed hystereses), for instance,  $N \geq 100,000$ , the residue can usually be neglected for the analytical fatigue-life prediction. For short stress-time functions, one should attempt to estimate the damage content of the residue. For instance, one-half the amount of damage for the full hysteresis can be assumed for a slope, or the time sequence can be counted a second time for resolving the residues. This consideration must be taken into account for the subsequent fatigue-life estimate.

## Development

The rainflow counting method was described in a Japanese publication by Matsuishi and Endo in 1968 [Mats68]. The term “pagoda-roof method” is also employed in the literature. A description of the method was published in the Journal of Materials (JMLSA) in 1972 [Dow172]. Numerous publications have appeared

since that time. In particular, new algorithms have been proposed for rainflow counting, and these have resulted in considerable simplifications.

Matsuishi	[Mats68]
Dowling	[Dowl72]
Endo	[Endo74]
Nowack	[Nowa76]
De Jonge	[Jong82]
Seeger	[Clor85]
Krüger	[Krüg85a, Krüg85b]
Perrett	[Perr87]
Rychlik	[Rych87]
Murakami	[Mura92]
Beste	[Best92]
Dressler	[Dres93]
Amzallag	[Amza94]
Johannesson	[Joha99]
Young-Li	[Youn05]
Schijve	[Schi09]
ISO 2013	[ISO13a, ISO13b]

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# Chapter 4

## Load Spectra and Matrices

As shown in Sect. 3.1, two-dimensional frequency distributions, so-called load spectra, are the results of one-parameter counting. The frequency of the amplitude or range (double amplitude) is thus represented graphically. The results of two-parameter counting are three-dimensional frequency matrices. With the use of a transition matrix, every half load cycle is recorded in a field of the matrix as a counting event, from a starting class all the way to a target class. From these min-max and max-min values, the amplitude of the stress as well as the mean value of each half cycle can be determined.

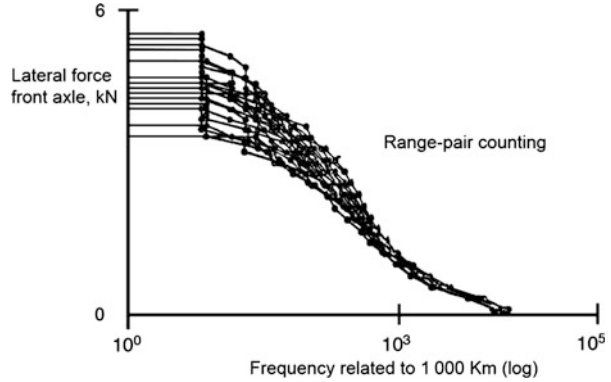
In this book, the two-parameter rainflow counting method is generally recommended for the evaluation and classification of measurements. This implies that a rainflow matrix should also constitute the basis for the analytical fatigue-life prediction. As a matter of principle, it is assumed that a mean value, that is, the mean stress, affects the allowable number of load cycles to failure. Proposals for performing the necessary calculations are presented in Chap. 9, “Analytical fatigue-life estimate”.

For the purpose of evaluation, however, plotting of spectra may be more informative than the use of matrices. This approach is especially useful for:

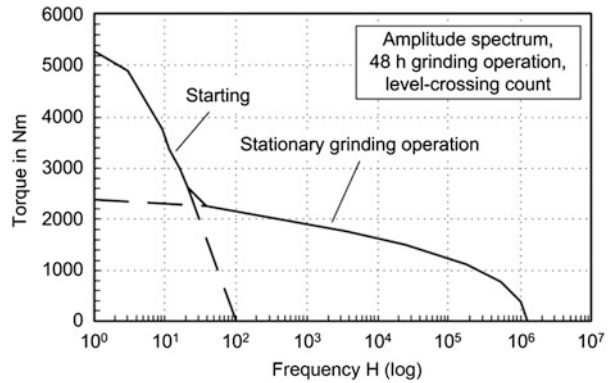
- a comparison of measurements (for instance, the effect of different drivers over the same route of travel), Fig. 4.1,
- a plausibility test on measurements,
- recognition of causes of stress, Fig. 4.2,
- appraisal of extrapolations, and
- appraisal of the damage in direct comparison with the S-N curve.

In Fig. 4.1, stress spectra are plotted for lateral forces acting on the front axle of motor vehicles for 14 drivers travelling on the same route. Only slight scatter occurs at low amplitudes, which result especially from travel on a straight roadway.

**Fig. 4.1** Stress spectra for lateral force at front axle of an automobile, 14 drivers on same regular roadway



**Fig. 4.2** Mixed spectrum for starting and stationary grinding operation of a bowl-mill crusher



However, pronounced scatter is observed at high amplitudes caused by the stress associated with driving manoeuvres during travel over curves.

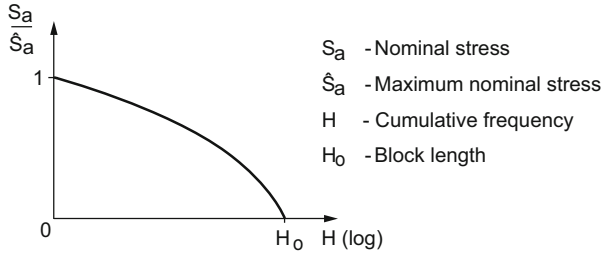
In many cases, causes of stress can be determined from the form of the spectrum. Thus, the spectrum plotted for a bowl-mill crusher (shaft torque), Fig. 4.2, can be attributed to starting processes and stationary grinding operation [Gehl92].

### 4.1 Description of Load Spectra

For representing a stress spectrum, the stress amplitude  $S_a$  is plotted as a function of the cumulative frequency  $H$ . If a spectrum is determined from measurements by counting methods, the frequency distributions are stepped with the same step spacing as that employed for counting. However, continuous distributions, that is, distributions with infinitesimal step spacing, can also be given. As a rule, these distributions must then be stepped in the course of an analytical fatigue-life estimate, for instance.

Spectra are usually plotted semilogarithmically, Fig. 4.3. A normalised plot of the stress is also possible.

**Fig. 4.3** Stress spectrum in normalised representation



In the field of fatigue strength under variable stress amplitude, many terms and concepts were coined decades ago by E. Gassner and co-workers at the LBF (*Laboratorium für Betriebsfestigkeit*) Darmstadt, Germany [Buxb92, Haib06]. These include the term ‘Betriebsfestigkeit’ itself. A further example is the use of the term ‘spectrum’ for the frequency distribution of amplitudes (load spectrum). A literal or word-for-word translation of these terms is often not possible. By way of departure from the international convention for plotting statistical functions, the coordinates for the frequency and feature are interchanged in German-language literature in this field. At international congresses, this situation can easily become a cause of confusion and of lively discussions. The use of the cumulative frequency rather than the step frequency and also the use of a logarithmic plot for the purpose require considerable experience for interpretation.

In the simplest case, a spectrum can be described by three quantities:

- the maximum nominal stress for the spectrum,  $\hat{S}_a$
- the block length,  $H_0$
- and the shape of the spectrum,  $\nu$ .

Further important quantities include the mean stress  $S_m$  or the stress ratio  $R$ . The irregularity factor  $I$  of the stress-time function, the effective value, and the crest factor are often indicated, too, see Sect. 2.3.

For the stresses which occur during oscillation processes, the basic principles known from the theory of continuous random processes can be applied under certain conditions [Buxb79]. The frequency distribution for the stationary Gaussian random process associated with the level-crossing count can be described by the following equation, as indicated by S. O. Rice, see [Buxb92]:

$$H(x) = H_0 \cdot \exp\left[-\frac{x^2}{2\sigma^2}\right] \tag{4.1}$$

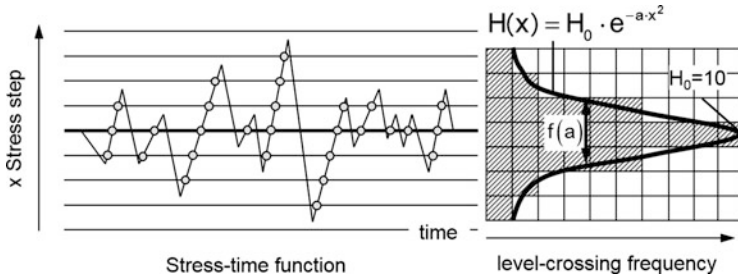
- $H(x)$  number of level crossings for horizon  $x$ ,
- $H_0$  number of zero or mean-load crossings, block length,
- $x$  in this case, normalised stress amplitude, see Fig. 4.3

With  $a = 0.5 \sigma^2$  Eq. (4.1) can be rewritten as

$$H(x) = H_0 \cdot \exp(-a \cdot x^2) \tag{4.2}$$

The stress-time function for a random process and the result of the level-crossing count are plotted on a linear scale in Fig. 4.4. For this random process with a stress





**Fig. 4.4** Stress-time function and spectrum for a level-crossing count as well as step frequency of a random process

ratio  $R = -1$  and an irregularity factor  $I = 1$  the level-crossing frequency is equal to the cumulative frequency  $H$ . With the parameter “ $a$ ” the highness of a LTF can be adapted.

In this context, the spectrum can be interpreted as an approximation to a Gaussian normal distribution. The amplitude spectrum is represented by the upper portion of the bell curve. The standard distribution which was empirically derived from results of measurements by E. Gassner in 1948 corresponds largely to this distribution shape [Haib06].

Equation (4.2) can be transformed to a power function. For

$$S_a = \widehat{S}_a$$

or  $x = 1$ , one obtains  $H = 1$ . Thus,

$$1 = H_0 \cdot \exp(-a) \tag{4.3}$$

With  $a = \ln H_0$ , Eq. (4.2) becomes

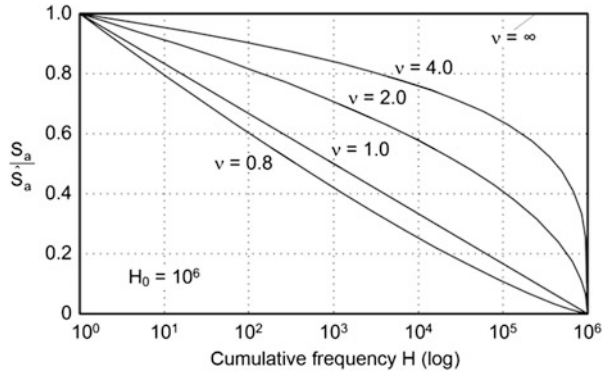
$$H(x) = H_0 \cdot \exp(-\ln H_0 \cdot x^2) \tag{4.4}$$

With  $\ln H(x) = \ln H_0 (1-x^2)$ , the following antilogarithmic form is obtained:

$$H(x) = H_0^{1-x^2} \tag{4.5}$$

In the field of fatigue strength under variable stress amplitude, this distribution is designated as a standard or normal distribution.

**Fig. 4.5** Related spectra for different shape parameters



For describing the shape of the spectrum, M. Hanke [Hank70] has proposed the substitution of the variable  $\nu$  for the exponent “2” of  $x$  in Eq. (4.5), see also [Buxb92, Hüeck88, Seeg96]:

$$H(x) = H_0^{1-x^\nu} \tag{4.6}$$

$$\text{For } \frac{S_a}{\hat{S}_a} = 1 \text{ becomes } H = 1$$

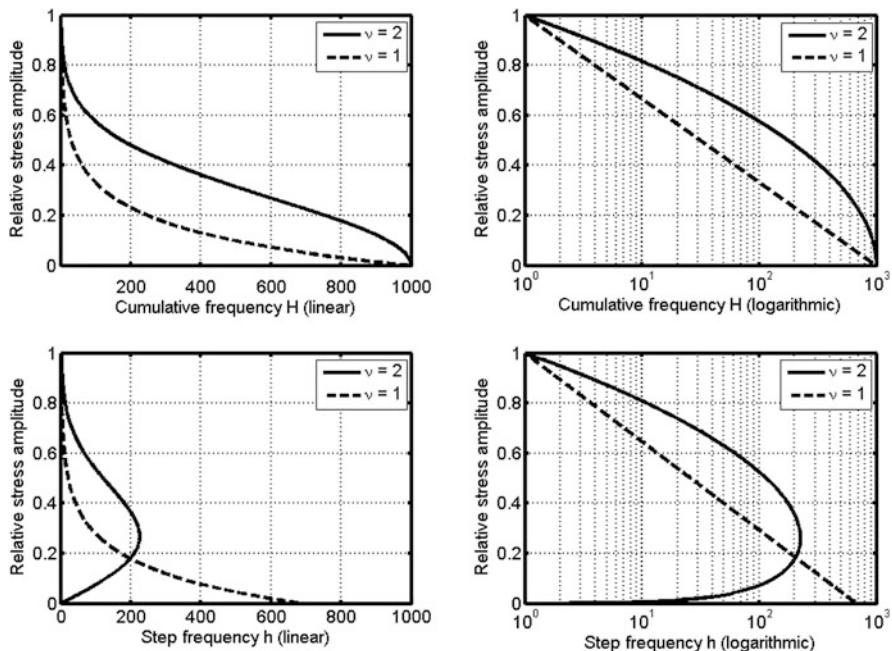
$$\text{For } \frac{S_a}{\hat{S}_a} = 0 \text{ becomes } H = H_0$$

With the shape parameter  $\nu$ , the following distribution shapes are obtained, see Fig. 4.5.

- $\nu = 0.8$  Concave distribution, interpretation as log-normal distribution possible: gusts of wind over an extended period [Buxb92]
- $\nu = 1.0$  Exponential distribution which forms a straight line in a semilogarithmic plot: This distribution can be interpreted as a mixture of different normal distributions, for instance, during the measurement of stresses due to a rough sea over an extended period
- $\nu = 2.0$  Standard or normal distribution which forms a parabola in a semilogarithmic plot: Frequency distribution from a stationary Gaussian random process; measurement of stresses due to a rough sea or measurement of the vertical forces on a motor vehicle on a specific roadway without driving manoeuvres
- $\nu = 4.0$  Convex distribution: Typical for crane and bridge construction
- $\nu \rightarrow \infty$  Rectangular spectrum (constant amplitude)

If an impact occurs only once,  $\nu = 0$ .

In Fig. 4.6, two distribution shapes,—the normal or standard distribution and the exponential or straight-line distribution, are plotted differently, that is, on a linear and on a semilogarithmic scale, and as a cumulative frequency  $H$  (probability) as well as a step frequency  $h$  (probability density). From these plots, it is obvious that



**Fig. 4.6** Cumulative and step frequency for two spectra, NV ( $\nu = 2$ ) and GV ( $\nu = 1$ ), in linear and semilogarithmic plots

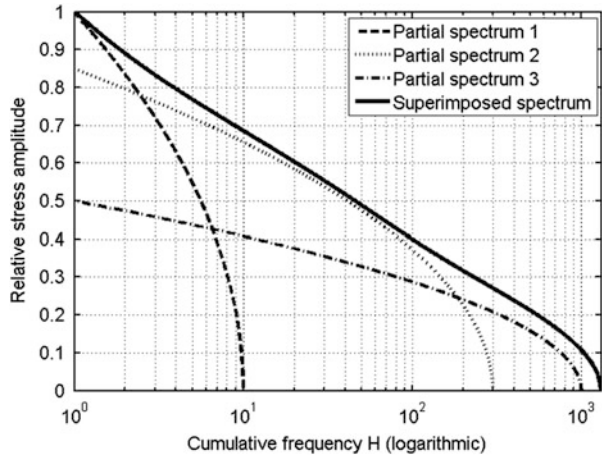
designations such as straight-line distribution are associated with specific representations. If it is assumed that the load cycles enter linearly into a damage accumulation analysis for estimating the fatigue life, the linear plot of the step frequency yields the more realistic representation of the respective damage content of a spectrum.

From all four plots, it is obvious that the normal distribution represents a considerably harder spectrum than the straight-line distribution. Consequently, this distribution is preferentially employed for fatigue-strength testing because of the associated saving in time required for testing. The two plots of the step frequency at the bottom can be interpreted as damage distributions of the load horizons, if the slope of the S-N curve is  $k = 0$ , and the Miner elementary modification is applied.

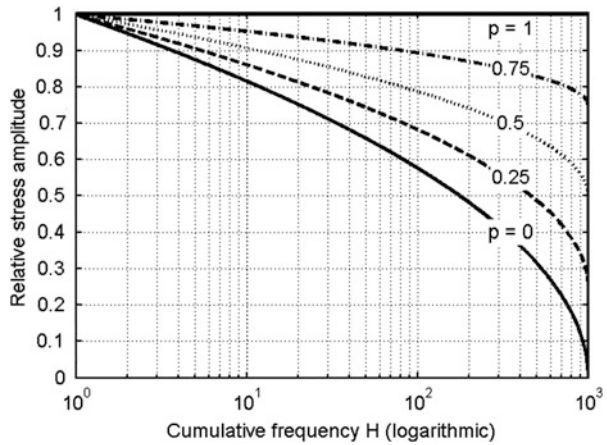
If lengthy measurements are performed for different stationary processes (rough seas, roadways), a straight-line distribution often results. This distribution is obtained as an overall spectrum by superposition of normally distributed partial spectra which differ with respect to the spectral maximum and the cumulative frequency  $H_0$ , Fig. 4.7.

Spectra which result from measurements on motor vehicles, aeroplanes, ships, machine plants, etc. under operational conditions differ drastically. From the wide variety of the spectra thus obtained, typical shapes are determined for these spectra and then idealised (standardized spectra). This approach has in fact been applied for

**Fig. 4.7** Superposition of three normally distributed partial spectra to yield a straight-line distribution



**Fig. 4.8** p-value spectra



a long time. Spectral shapes similar to those plotted in Fig. 4.5 have already been reported in [Gass64]. In this work, the so-called p-value spectra, which are important for crane construction, were also proposed. The weight of the empty trolley in the middle of the crane causes the occurrence of stress amplitudes during operation, and these frequently exceed a certain threshold value. In a relative plot, Fig. 4.8, the so called p-value denotes the ratio of this threshold value for  $H = 1000$  to the maximum for the spectrum for a variety of possible spectrum shapes [Rada07].

Besides the shape parameter  $\nu$ , a different parameter  $\nu$  is also available for describing the shape of the spectrum [Häne99], and is defined as follows:

$$v = \sqrt[k]{\frac{\sum_{i=1}^n h_i \cdot \left(\frac{S_{ai}}{S_a}\right)^k}{H_0}} \quad (4.7)$$

The range of values for  $v$  lies between zero and one. On this scale, constant-amplitude stress corresponds to the maximal degree of fullness,  $v = 1$ . The value also depends on the slope  $k$  of the S-N curve, in addition to the shape of the spectrum. In [Wirt87] and [Euli08], a value of  $k = 6$  is proposed; as specified in the FKM Guideline [Häne99, Häne03],  $k = 5$  applies in the case of bending, and  $k = 8$  applies to torsion.

Three parameters are thus available for describing the shape of the spectrum: the shape parameter  $\nu$ , the  $p$ -value, and the  $v$  parameter. Terms such as spectrum hardness or spectrum fullness are often employed for providing a qualitative description. The hardness or fullness of a spectrum increases progressively as the shape of the spectrum approaches a rectangular shape, which corresponds to a one-step stress.

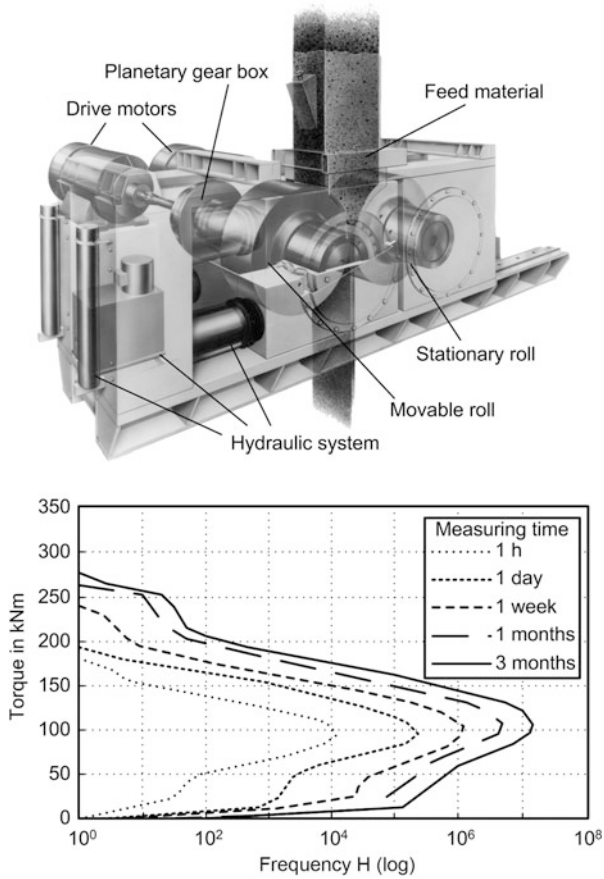
## 4.2 Extrapolation

For measurements on machine plants, the duration of measurement necessary for obtaining a representative spectrum is always a question of vital importance. Of course, the stress spectrum which results from measurements over a limited period will never be sufficient for accurately predicting the stresses over a long design life. However, a representative spectrum can be expected to become established after a certain period of measurement, which depends on the operating conditions. Consequently, this question reduces to a problem of extrapolation. The spectra measured over a day, over a week, and over 3 months on a high-pressure grinding-roll mill and on a shredder are plotted in the following, Figs. 4.9 and 4.10 [Gehl92]. In the case of the high-pressure grinding-roll mill, hardly any changes in the operational state are observed, and only slight alterations in the ground stock (cement clinker) occur in the course of time. As indicated in Fig. 4.9, a useful spectrum is available after a measuring period of 1 month. After a shift to the right by a factor of 3, the result would nearly coincide with the spectrum obtained after 3 months [Gehl92].

In the case of a shredder, the rotor torque is highly dependent on the type of scrap to be processed and also on the type of feed system. The shape of the spectrum obtained after 111 days cannot be inferred from the shapes of the spectra obtained after shorter periods of operation, Fig. 4.10 [Gehl92]. In this case, an attempt must be made to estimate a representative spectrum by means of a statistical analysis of the expected operational states (type of scrap), as shown in Sect. 10.2.

From a measurement under operational conditions during a limited period of time, a measurement of the maximal operational load value cannot be expected.

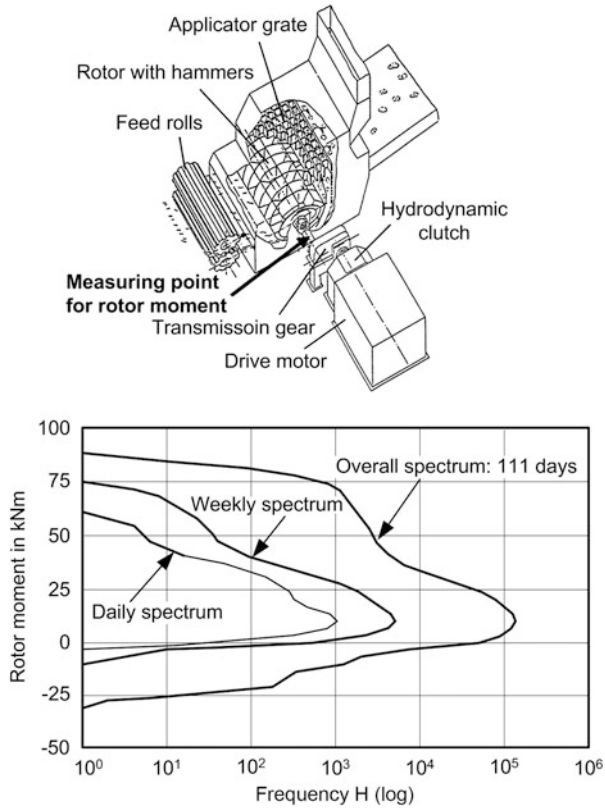
**Fig. 4.9** Torque spectra for a high-pressure grinding-roll mill after different measuring periods, see also basic sketch, Fig. 2.3 [Gehl92]



Instead, it is highly probable that the observed maximum for the spectrum during the entire design life will be decidedly higher. The ratio between the maximum expected for the spectrum during the design life and the maximum which actually occurs during the measurement depends on the duration of the measurement, as referred to the design life and the amount of load scatter, among other factors.

One possibility for estimating the maximum for the spectrum is the multiplication of the measured amplitudes by a safety factor. This factor is indicated as a function of the risk and consequences of damage in series of technical rules; see also Chap. 10. However, the aforementioned decisive parameters, that is, the duration of measurement and the load scatter, are usually not considered.

**Fig. 4.10** Torque spectra for a shredder, measuring point s, above, after different measuring periods [Geh192]

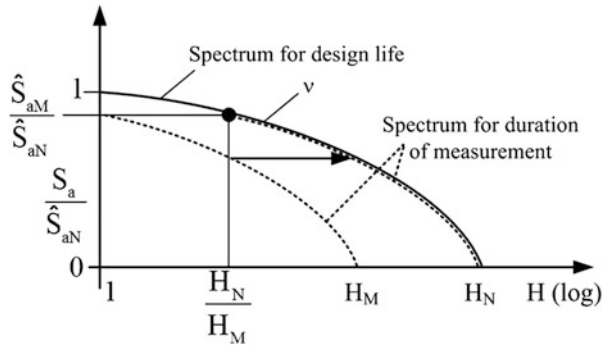


### 4.2.1 Analytical Extrapolation with the Shape Parameter

The problem of deciding how to perform an extrapolation is illustrated with the use of the following typical example: Results are available from a representative measurement under operational conditions during a limited period of time. From these results, the spectrum must be determined for the entire design life, see Fig. 4.11.

The stress spectrum for the operational measurement with the cumulative frequency  $H_M$  can be shifted toward the right, and the cumulative frequency  $H_N$  for the design life can thus be attained. In the special case of a load limitation, an increase in the maximum for the spectrum of measured values  $\hat{S}_{aM}$  would not be possible. That is, the spectrum for the entire design life would be limited from above by a horizontal line at the level of  $\hat{S}_{aM}$  (truncation). In the more general case, however, amplitudes  $S_{aN}$  which exceed  $\hat{S}_{aM}$  will occur during the design life. In many cases, therefore, it can be expected that the shape parameter  $\nu$  will not change. In the past, a graphical extrapolation was performed with the use of a curve template. As indicated in [Hink11], an analytical extrapolation can also be performed with the

**Fig. 4.11** Normalised spectra for the duration of measurement and for the design life



use of a constant shape parameter. It should be pointed out that the measured spectrum can also be a mixture, that is, a combination of partial spectra for different operational states. Hence, a prerequisite for the application of the method described in the following is the assumption that the fractional composition of the mixed spectrum does not change during the design life.

The starting point for the following consideration is Eq. (4.7) or a normalised spectrum such as that plotted in Fig. 4.5. In Fig. 4.11, a normalised spectrum has been plotted for the duration of measurement and for the useful life with the use of a shape parameter  $\nu$ .

For this purpose, an extrapolation factor

$$e = \frac{H_N}{H_M} \tag{4.8}$$

and an incremental factor

$$E = \frac{\hat{S}_{aN}}{\hat{S}_{aM}} \tag{4.9}$$

can be defined. For Eq. (4.6), this yields

$$H = H_0 \left( 1 - \left( \frac{S_a}{\hat{S}_{aN}} \right)^\nu \right) \tag{4.10}$$

In correspondence with  $\log H = \log H_N - \log H_M$ , or

$$H = \frac{H_N}{H_M} \tag{4.11}$$

and  $H_0 = H_N$ , the following is obtained



$$\frac{H_N}{H_M} = H_N^{1 - \left(\frac{\widehat{S}_{aM}}{\widehat{S}_{aN}}\right)^\nu} \quad (4.12)$$

In a logarithmic representation, this yields

$$\log H_N - \log H_M = \log H_N - \log H_N \left(\frac{\widehat{S}_{aM}}{\widehat{S}_{aN}}\right)^\nu$$

or

$$\frac{\log H_M}{\log H_N} = \left(\frac{\widehat{S}_{aM}}{\widehat{S}_{aN}}\right)^\nu \quad (4.13)$$

After a transformation, the following is obtained:

$$\frac{\widehat{S}_{aN}}{\widehat{S}_{aM}} = \left(\frac{\log H_N}{\log H_M}\right)^{\frac{1}{\nu}} \quad (4.14)$$

After insertion of  $e$  and  $E$ , this yields

$$E = \left(\frac{\log(e \cdot H_M)}{\log H_M}\right)^{\frac{1}{\nu}} \quad (4.15)$$

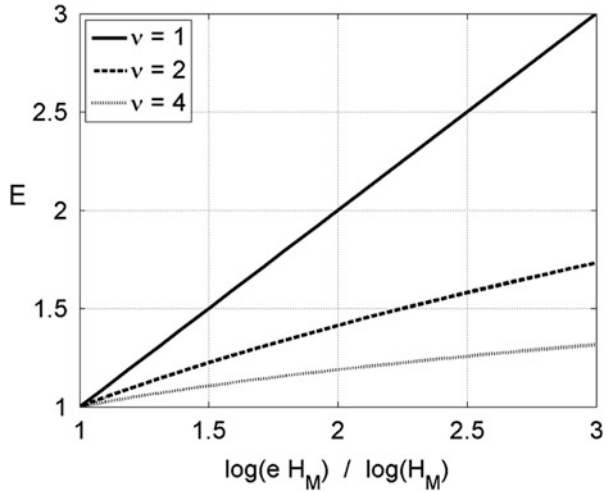
That is, the incremental factor for the maximum of the spectrum depends on the extrapolation factor, on the shape of the spectrum, and on the cumulative frequency of the measured spectrum. The latter can be interpreted as the duration of the measurement.

In Fig. 4.12, the incremental factor is plotted for three spectra (straight-line distribution, standard distribution, and  $\nu = 4$ ). A linear relationship is obtained for the straight-line distribution.

## 4.2.2 Statistical Extrapolation for Stationary Processes

Methods from the field of extreme-value statistics are also applied for extrapolation in the case of stationary processes. For instance, methods of this kind are also applied for performing estimates in order to deal with a “wave of the century” or extremely severe earthquakes. For extreme-value statistics as described by Gumbel [Gumb58], see also [Buxb92], it is assumed that the maxima for each series of measurements correspond to a log-normal distribution for  $n$  series of measurements (random samples) during a stationary random process. Hence, the measurement on an operational state (for instance, travel of a motor vehicle over a distance of 1000 km on a roadway) can be subdivided into  $n = 20$  equal intervals each

**Fig. 4.12** Enhancement of the spectral maximum with analytical extrapolation



**Table 4.1** Subdivision and order of the values

Ordinal number <i>i</i>	Maximum $x_{max}$	P (%)
1	510	3.3
2	490	8.2
3	:	:
:	:	:
:	:	:
10	:	47.5
:	:	:
:	:	:
:	:	:
<i>n</i> = 20	370	96.7

Where  $P = \frac{3i - 1}{3n + 1}$

50 km in length, and the maximum can then be determined for each interval. The maxima for the individual intervals *i* are sorted in decreasing order of their values, and a probability of exceeding a given value is assigned to each event (in accordance with Rossow) [Ross64], as shown in Table 4.1.

The individual pairs of values (maximum, level-crossing probability) can be plotted on a Gaussian probability grid and evaluated to determine the mean value and standard deviation.

From the total distance travelled during the design life and the length of a measured interval, the theoretical number of load intervals  $n_{tot}$  is then determined for the entire design life. The level-crossing probability for the greatest load event to be expected during the design life (*i* = 1) is thus:

$$P = 2 / (3n_{tot} + 1) \tag{4.16}$$

The maximum for the spectrum can thus be determined from the probability grid, see Fig. 4.13. The essential characteristic values from this evaluation are the

mean value and the standard deviation  $s$  of the maxima for the individual intervals.

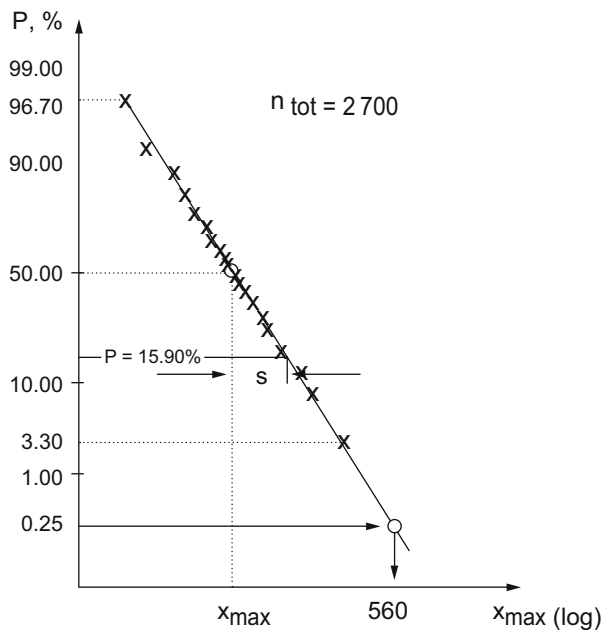
$$\log \bar{x}_{\max} = \frac{1}{n} \sum_{i=1}^n \log x_{\max,i} \tag{4.17}$$

$$s = \left[ \frac{1}{n-1} \sum_{i=1}^n (\log x_{\max,i} - \log \bar{x}_{\max})^2 \right]^{\frac{1}{2}} \tag{4.18}$$

Level-crossing probabilities have been determined for the extreme values, but the published results differ somewhat [Buxb92, Gude95]. Among other factors, these results are based on extended safety and reliability considerations. For ensuring the consistency of the equations employed here, the method of estimation due to Rossow, which is preferred for normally distributed values, is applied for the probability of exceeding a given value.

Stationary processes are a prerequisite for this method. Consequently, extrapolation is permissible only for partial spectra; that is, the method is not applicable to mixed spectra. A graphical plot of the maxima on a probability grid is well suited for determining whether a stationary process or a mixed distribution is involved. If the values are not well ordered along a straight compensation line in a logarithmic plot of the maximum, the method is not applicable. Two possible reasons can be

**Fig. 4.13** Distribution of the extreme values on probability grid



given for this restriction: The maxima may be associated with loads due to different physical causes, or load limitations may be present.

An example of loads due to different physical causes is the longitudinal force on the wheel of a motor vehicle. The longitudinal force can be caused by drive/braking processes, by rolling over bumpy roadways, or by inertial forces during longitudinal wheel oscillations.

Load limitations include, for instance, slipping clutches or predetermined breaking points, safety valves or bursting discs on pressure vessels, spring-excursion limiters, or the traction limit (on the wheels) of motor vehicles. Load limitations are often not attained during the measurement of operational loads. Consequently, the results obtained by extrapolation methods must always be checked for plausibility.

### 4.3 Derivation of Spectra from Matrices

The purpose of the following chapter is to show how the results of one-parameter counting methods can be derived from the results of two-parameter counting methods. As a rule, the objective is first to determine the matrices for the two-parameter counting methods and then to extract the results of one-parameter counting for the sake of easier visualisation. This approach ensures that the information content is maximised and that the advantages of the one-parameter counting methods can be utilised, see also [Haib06] and [Krüg88].

#### 4.3.1 Transition Matrix

The following explanation is based on the transition matrix known from Sect. 3.4.2, Fig. 4.14. The positive slopes of the stress-time function have been entered above the diagonal if the associated target class is larger than the start class. The negative slopes of the stress-time function have been entered below the diagonal if the

**Fig. 4.14** Transition matrix for the stress-time function Fig. 3.15 taken as example

		Target class							
		1	2	3	4	5	6	7	8
Start class	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								

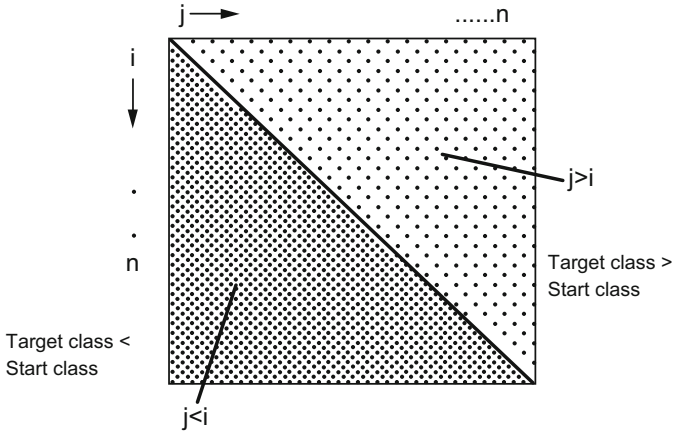


Fig. 4.15 Transition matrix with ranges for positive and negative slopes

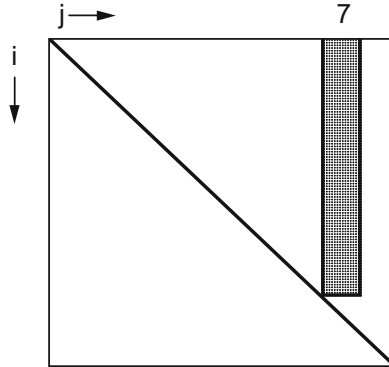


Fig. 4.16 Transition matrix with a range for the number of peaks in class 7

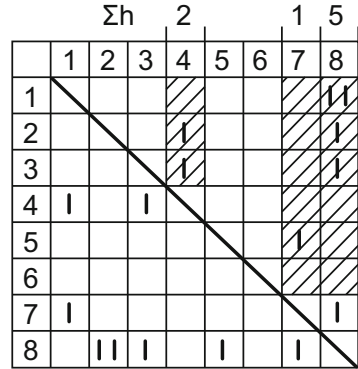
associated target class is smaller than the start class, Fig. 4.15. The peak values per class are shown in Fig. 4.16.

The cross-hatched area in Fig. 4.17 contains the step frequency  $h$  of the maxima which occur in class 7. As a result, the same values are indicated as those which resulted from one-parameter peak counting, Fig. 4.17. A corresponding result applies to minima, Fig. 4.18. The cross-hatched area contains the total number  $n_1$  of evaluated maxima for the stress-time function, Fig. 4.19.

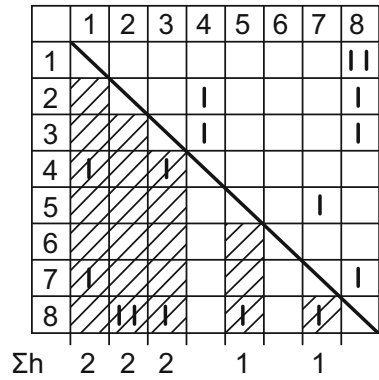
### Determination of the Irregularity Factor I

For the application of counting methods, the irregularity factor  $I$  of a stress-time function is often regarded as an important criterion. It is defined as the ratio of the number of mean-value crossings in one and the same direction  $n_0$  to the number of

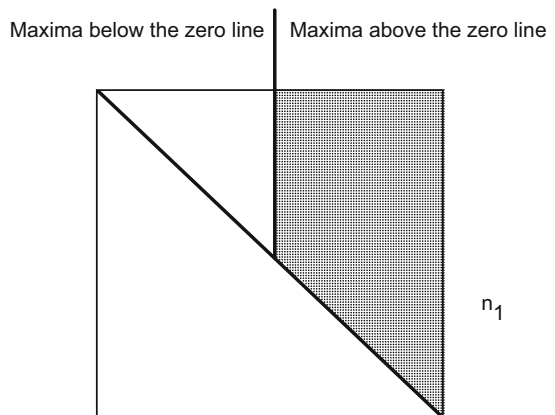
**Fig. 4.17** Transition matrix with ranges for positive slopes



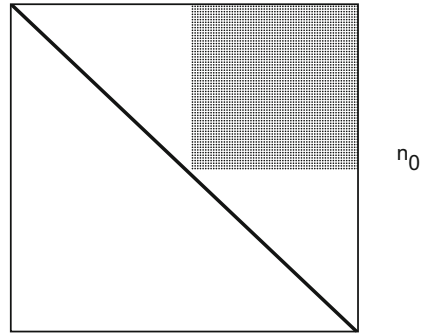
**Fig. 4.18** Transition matrix with ranges for negative slopes



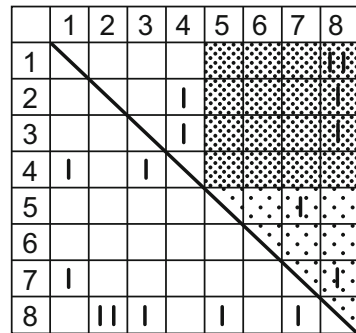
**Fig. 4.19** Transition matrix with a range for maxima (above and below the zero line)



**Fig. 4.20** Transition matrix with mean-value crossings in the positive direction



**Fig. 4.21** Transition matrix with irregularity factor I for the stress-time function taken as example



maxima  $n_1$  [ISO13]. From the number of mean-value crossings in the positive direction  $n_0$  (cross-hatched area), the irregularity factor  $I = n_0/n_1$  can be determined, Fig. 4.20.

In this context, it is important to note that an irregularity factor  $I = 1$  designates a regular load sequence, that is, without mean-load fluctuations. In a strict sense, therefore, the irregularity factor  $I$  really designates the regularity of a load sequence.

For the stress-time function taken as example, the resulting irregularity factor  $I$  is equal to  $4/6 = 0.67$ , Fig. 4.21. The procedure for the minima is analogous.

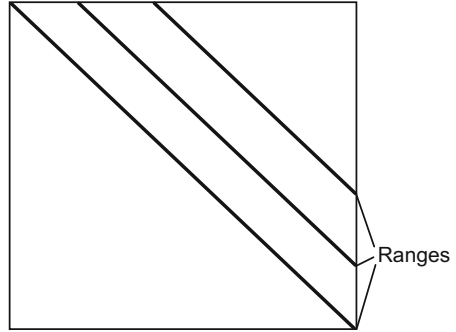
Equal stress ranges are located on diagonals which are parallel to the main diagonal. The result of range counting can be obtained with this procedure, Fig. 4.22.

The results obtained for the stress-time function taken as example corresponds to those of one-parameter range counting, Fig. 4.23, see Sect. 3.3.3.

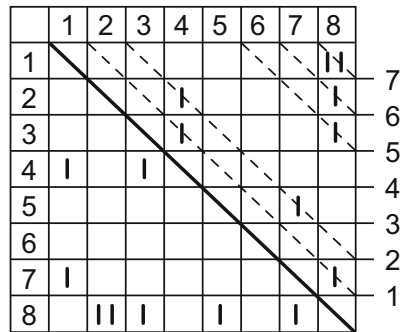
The same procedure is also applicable in the case of range-mean counting, Figs. 4.24 and 4.25. The lines of equal mean value are perpendicular to the main diagonal. The result is the matrix already known from Sect. 3.4.1, Fig. 4.26.

In the negative range, a similar result can be indicated underneath the main diagonal. For the present matrix, the area indicated in Fig. 4.27 includes the crossings of level  $a$ . For the stress-time function taken as example, the resulting

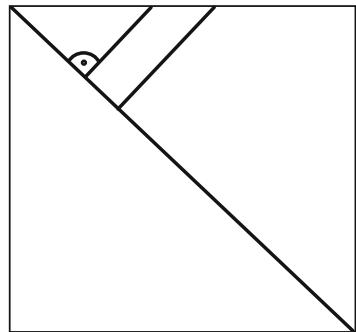
**Fig. 4.22** Transition matrix with ranges of equal size



**Fig. 4.23** Transition matrix with ranges of equal size



**Fig. 4.24** Transition matrix with main diagonal and lines of equal mean values



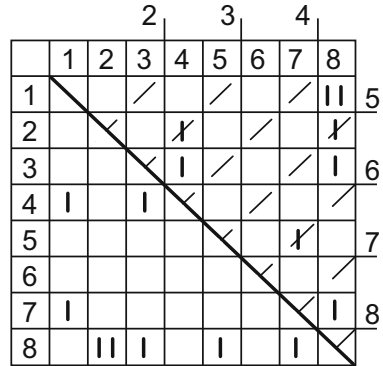
cumulative frequencies for the level crossings are indicated in Fig. 4.28, see also Fig. 3.8.

This result can also be compared to that of level-crossing counting, see Sect. 3.3.2, Fig. 3.8.

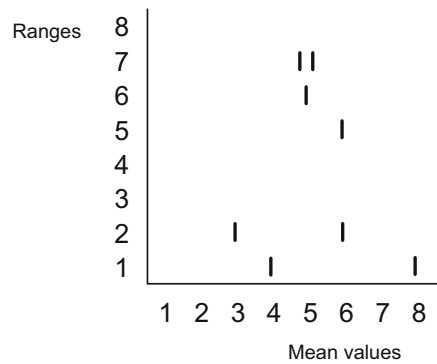
**Comment** Besides the irregularity factor, range counting, range-mean counting, and level-crossing counting can be derived from the transition matrix, but not range-pair counting.



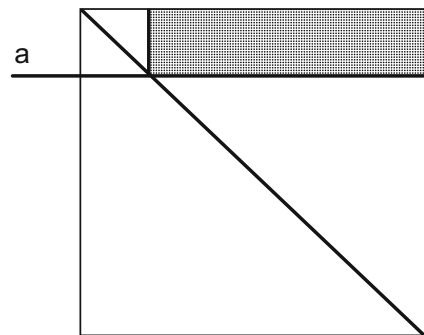
**Fig. 4.25** Transition matrix with lines of equal mean values



**Fig. 4.26** Result of range-mean-counting



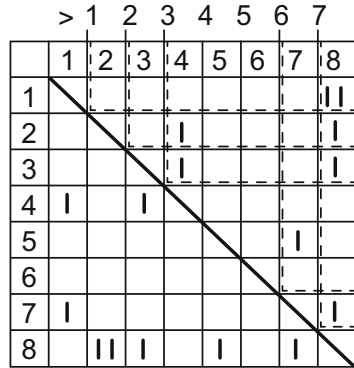
**Fig. 4.27** Transition matrix with a level



**4.3.2 Rainflow Matrix**

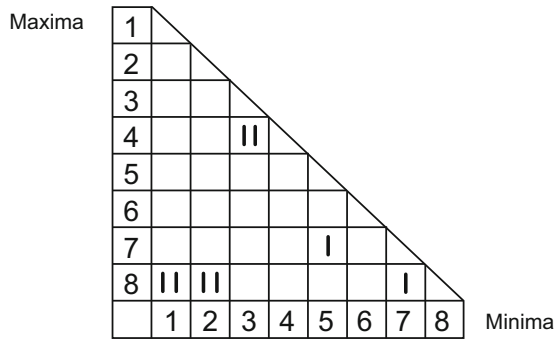
The following considerations are referred to rainflow half-matrices for the sake of clarity; similar considerations apply to full matrices. The result matrix from Sect. 3.4.4 is employed as an example, Fig. 4.29. In each case, the number of complete cycles is entered between the minima and maxima.

**Fig. 4.28** Transition matrix with the result of level-crossing counting



Level	Crossings
1	
2	
3	
4	
5	
6	
7	

**Fig. 4.29** Rainflow matrix for the stress-time function as example



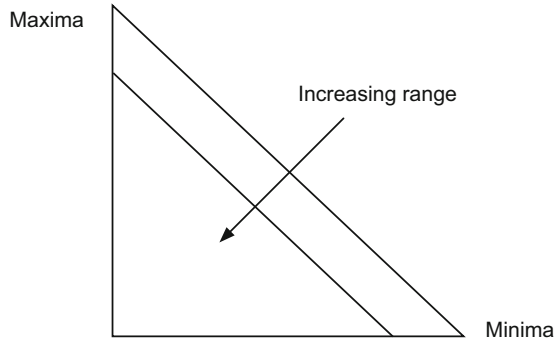
Lines for cycles of equal range are oriented in parallel with the diagonal. The stress range increases in the direction indicated by the arrow, Fig. 4.30.

Lines of equal mean value are perpendicular to the main diagonal; the mean values themselves increase in the direction of the arrow, Fig. 4.31.

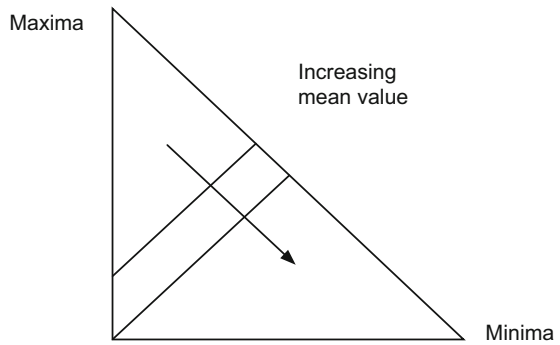
The following one-parameter counting methods can be derived from the rainflow matrix: range-pair counting, range-pair-mean counting, level-crossing counting, range counting, and peak counting.

The results of range-pair counting are obtained by summation of the cycles along the diagonal which is parallel to the main diagonal, Fig. 4.32.

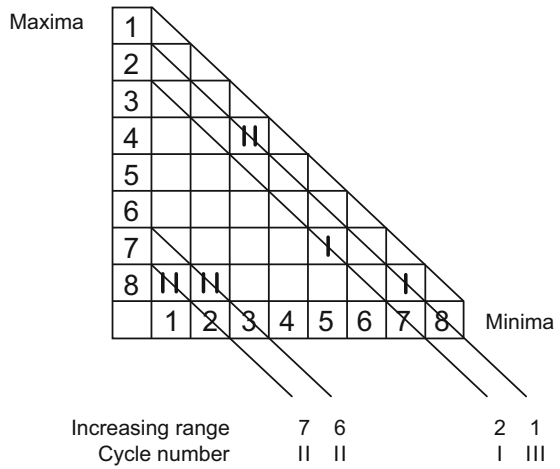
**Fig. 4.30** Rainflow matrix with a diagrammatic representation of the increasing range of the cycles



**Fig. 4.31** Rainflow matrix with a diagrammatic representation of the increasing mean values of the cycles



**Fig. 4.32** Rainflow matrix with range pairs of equal range



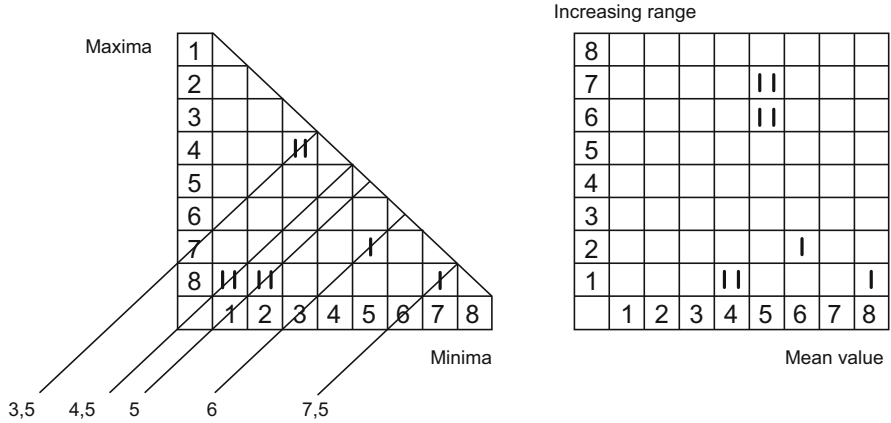


Fig. 4.33 Rainflow matrix and the included mean values of the range pairs

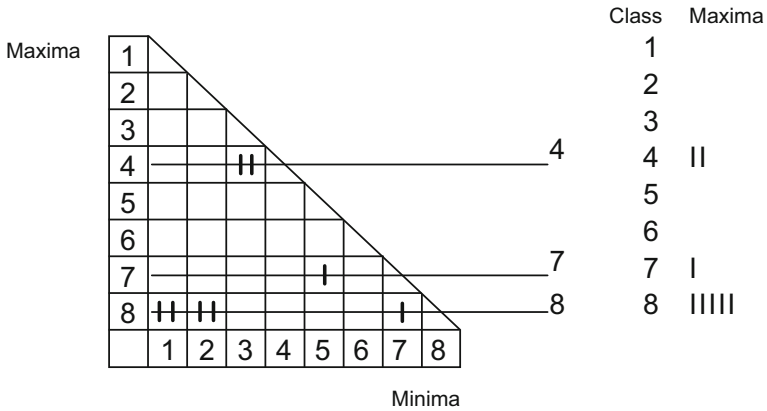


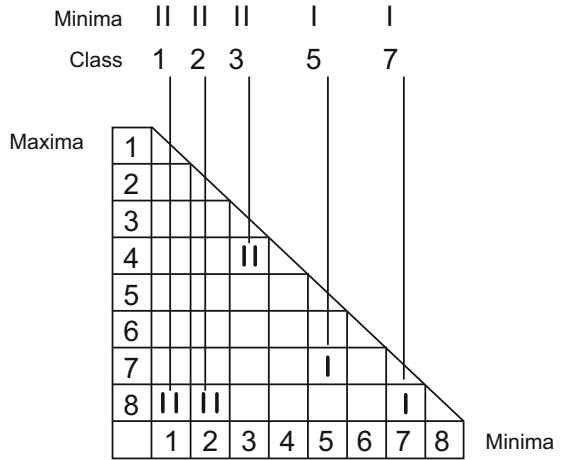
Fig. 4.34 Rainflow matrix with maxima of equal size

The rainflow matrix, Fig. 4.29, can be transformed to the plot for range-pair-mean counting, if the associated mean load is also indicated, in addition to the stress range, for the respective cycles, Fig. 4.33. In this process, the values 3.5, 4.5, and 7.5 have been rounded upward.

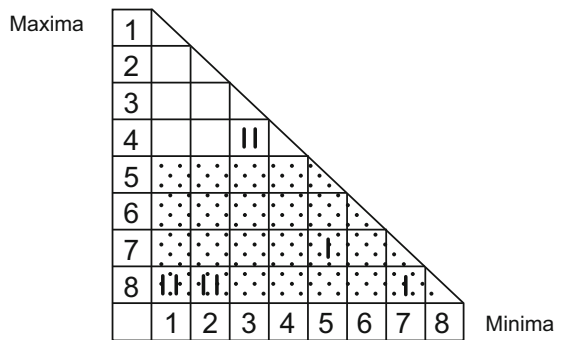
The results of peak counting can be calculated from the maxima and minima, Figs. 4.34 and 4.35.

The number of maxima per class is obtained by summation of the horizontal rows; the number of minima is obtained by summation of the elements in the vertical columns.

**Fig. 4.35** Rainflow matrix with minima of equal size



**Fig. 4.36** Rainflow matrix with maxima above the zero line



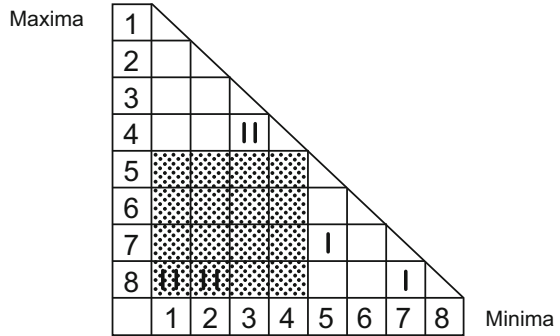
**Determination of the Irregularity Factor I** In Fig. 4.36, the rainflow matrix is shown with the maxima above the mean load, for instance, > class 4. The sum of these values corresponds to  $n_1 = 6$ ; compare Fig. 4.19.

The number of zero crossings, or of the crossings of a mean load which deviates from zero,  $n_0$ , is plotted in Fig. 4.37. As a rule, the mean value crossings are referred to the average operational load which occurs during the measurement.

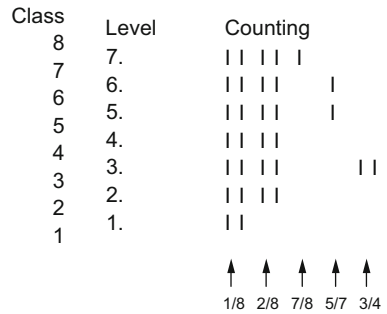
The number of zero crossings is  $n_0 = 4$ ; compare Figs. 4.37 and 4.20. For the stress-time function taken as example, the result is  $I = n_0/n_1 = 0.67$ .

Results of level-crossing counting can also be derived from the rainflow matrix. For instance, cycles which are entered into field 8/1 are crossings of levels 1–7 with the plotting procedure for level-crossing counting in which only positive slopes are evaluated. If the level crossings are summed in this manner for all fields of the matrix, the result shown in Fig. 4.38 is obtained with this counting method. This result is identical with the spectrum, Sect. 3.3.2, Fig. 3.8.

**Fig. 4.37** Rainflow matrix with zero crossings



**Fig. 4.38** Level-crossing counting from the rainflow matrix



**Comment** Besides the irregularity factor, range-pair counting, range-pair-mean counting, and level-crossing counting can be derived from the rainflow matrix. Moreover, other counting methods, such as range counting and peak counting, can be derived from rainflow counting, although this is not described in more detail in the present chapter.

### 4.4 Standardised Load Sequences and Spectra

The purpose of standardised load sequences is the specification of typical load sequences and spectra for particular structural components and applications. The original idea was to define a representative load sequence for improving analytical fatigue life predictions or for performing comparative tests on structural components. However, the load sequence was not intended for providing a suitable load assumption for designing and dimensioning. The first standardized load sequence was the eight-step block program test defined by E. Gassner in 1939. This standard fatigue-strength test was applied for decades in Germany. With the widespread application of the servohydraulic testing technique, the restriction of applying variable amplitudes as a blocked load sequence was eliminated. As a result, load

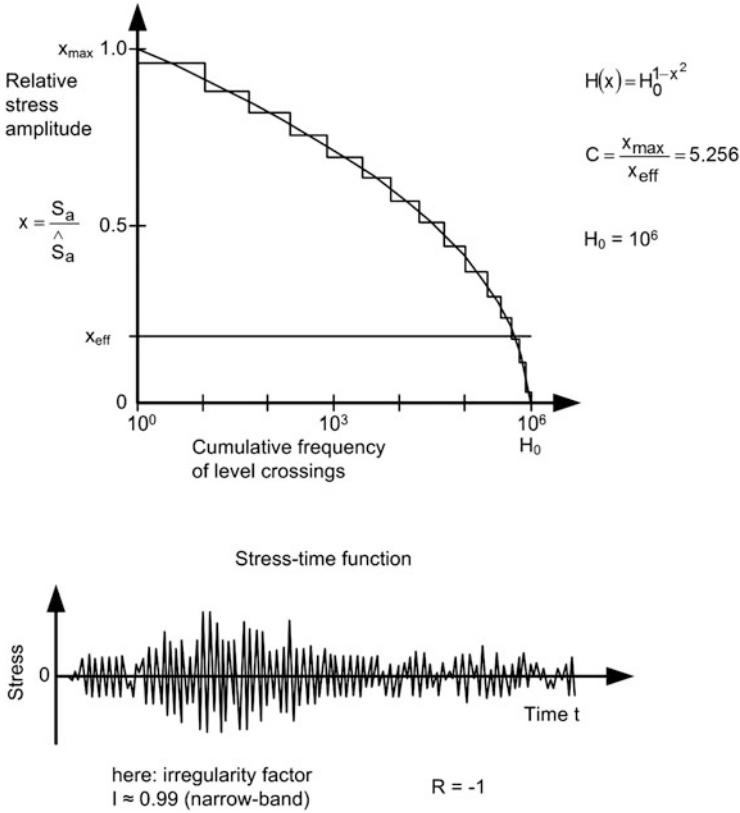


Fig. 4.39 Standardised stress-time function taken as example

standards first became established in the aircraft and jet-engine industry in the 1970s, and later also in the machine-plant and automotive industries. The Gaussian standard was especially important for numerous applications because it allowed the performance of random tests with acceptable test duration. In the example shown in Fig. 4.39, a section of the Gaussian standard is illustrated with a diagrammatic representation of the range-pair spectrum. The crest factor is defined as  $C = 5.256$  for this standard, see Sect. 2.3 [Fisc74, Have89, Heul98, Köbl76, Schü89, Schü94, Schü99]. During the first random tests, the set point setting was imposed with noise generators. Since the crest factor was not adjustable, the results of tests performed on different test stands were not mutually comparable.

The first standards were intended for defining one-channel load sequences, which could be supplemented by temperature variations if necessary. However, more recent standards, especially the various car-loading standards (CARLOS), represent multi-channel load sequences, which are correlated in time.

By way of departure from the original intention, standardised load sequences are employed as spectra for designing and dimensioning specific structural components

**Table 4.2** Standardized load sequences in chronological order

Name	Purpose	Channel	Country/ region	Year
8-step block programme	Fatigue-strength test under variable stress amplitude (blocked) [Buxb92]	1	D	1939
TWIST	Wings of transport aircraft Transport wing standard	1	EU	1973
GAUß	Fatigue-strength test under variable stress amplitude (random) [Haib76, Hück76]	1	EU	1974
FALSTAFF	Wing of fighter aircraft loading standard for fatigue [Aich76]	1	EU	1975
TRANSMISSION	Traction engine drive train (tractor)	1	US	1977
SUSPENSION	Motor vehicle chassis (bending)	1	US	1977
BRACKET	Motor vehicle aggregate bearing	1	US	1977
MiniTWIST	Shortened TWIST [Lowa79]	1	EU	1979
Mini FALSTAFF	Shortened FALSTAFF	1	EU	1980
HELIX, FELIX	Helicopter rotors (swivelling, fixed)	1	EU	1984
Eurocycle	Motor vehicle wheels and wheel bearings	2	EU	1981
Eurocycle	Motor vehicle wheels [Grub83]	2	EU	1983
HELIX/32, FELIX/28	Shortened HELIX/FELIX	1	EU	1984
ENSTAFF	FALSTAFF with temperature for CFK environmental FALSTAFF	2	EU	1987
Cold TURBISTAN	Cold discs of fighter aircraft engines	1	EU	1985
WISPER	Wind turbine rotor blades	1	EU	1988
WASH	Offshore structures [Kam92, Schü90a]	1	EU	1989
Hot TURBISTAN	Hot discs of fighter aircraft engines	2	EU	1989
AGRICULTR TRACTOR	Axle transmission gear	2	US	1989
LOG SKIDDER	Axle transmission gear, typical load cases	2	US	1989
WAWESTA	Rolling stand drive trains [Brun90]	1	D	1990
CARLOS (Part 1)	PKW Fahrwerksantriebe Car Loading Standard, [Schü90b]	1	EU	1990
CARLOS multi	Motor vehicle front-wheel suspension [Schü94]	3	EU	1994
CARLOS PTM	Motor vehicle transmission, manually shifted	3	EU	1997
CARLOS PTA	Motor vehicle transmission, automatic shift	3	EU	2002
CARLOS TC	Motor vehicle trailer equipment	3	EU	2003
CARLOS TC LT <sup>a</sup>	Light truck trailer equipment	3	EU	
CARLOS TC BC <sup>a</sup>	Motor vehicle trailer equipment, bike carrier	3	EU	
Eurocycle Respectively Stamas	Wheels and components with wheel control function [Heim06]	Multi- axial testing	EU	2006

<sup>a</sup>Not public

Overviews: [Have89, Heul98, Köbl76, Schü89]



nowadays. A prerequisite for this application is the development of appropriate methods for specifying the load level for the respective application in conjunction with the standardisation, in addition to the use of a representative load sequence as basis. These load standards can thus be applied instead of load assumptions and operational load measurements. A comprehensive set of standardised load sequences is indicated in chronological order in Table 4.2.

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# Chapter 5

## Comparison of the Counting Methods for Exemplary Load Time Functions

For the comparison of various counting methods, four different load time functions are evaluated by means of the following counting methods: level-crossing counting, range-pair counting, transition matrix (transition counting), and rainflow counting.

### 5.1 Load-Time Functions Selected for the Investigation

The selected load time functions include two different random functions with different irregularity factors. Furthermore, a step function with a superimposed damped load-time function is selected; this combination represents an operating cycle of a reduction pass on a rolling stand. Finally a superimposed sinusoidal-sinusoidal function, which is especially well suited for differentiating among the counting methods, is shown in Fig. 5.1.

### 5.2 Random Load with an Irregularity Factor $I = 0.99$

For the narrow-band random load-time function,  $I = 0.99$ , the four counting methods, level-crossing counting, range-pair counting, transition matrix (transition counting) and rainflow counting, yield very similar results. In the matrices, the diagonals are occupied from lower left to upper right, respectively, compare Fig. 5.2.

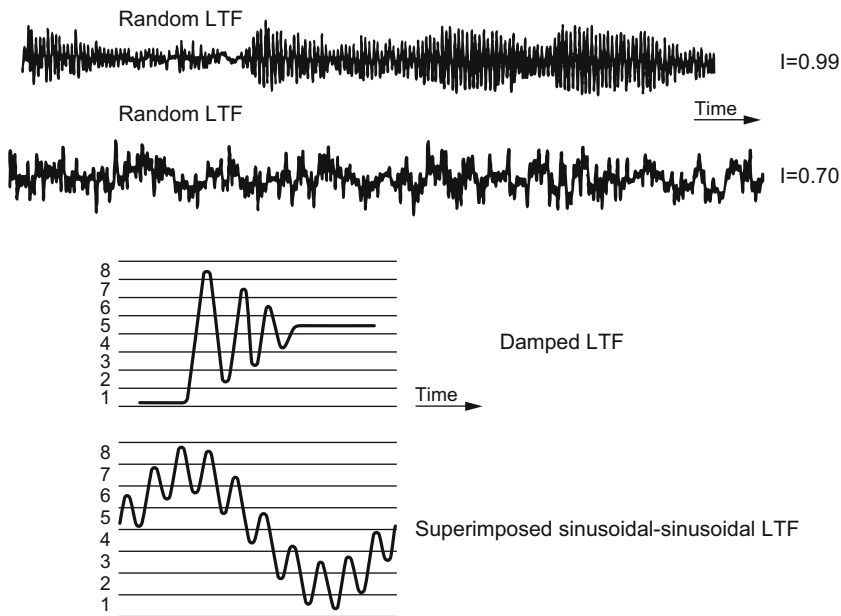


Fig. 5.1 Load-time functions taken as examples

### 5.3 Random Load with an Irregularity Factor $I = 0.7$

For a random load with  $I = 0.7$ , level-crossing and range-pair counting yield very different results for the evaluation of the amplitudes, Fig. 5.3. Level-crossing counting yields values of the amplitude which are larger than those actually present. The matrices are occupied over a larger range than for  $I = 0.99$ . The result is presented in the form of the transition and rainflow matrix in Fig. 5.3.

### 5.4 Step Function with Superimposed Damped Load-Time Function

For a damped load-time function, for instance, the torque in the rolling mill drive at the start of the milling process (see Fig. 2.11) no closed hysteresis loops result. Consequently, no events are counted during range-pair and rainflow counting. The half-cycles are entered as a residue. For the analytical fatigue-life prediction with shorter stress-time functions, attention must be paid to the fact that the half-cycles entered as a residue can contribute to the fatigue damage. For the fatigue damage accumulation, therefore, the assignment of one-half of the full-cycle fatigue damage to each of these half-cycles has been proposed [Clor85], see also Sect. 3.4.4.

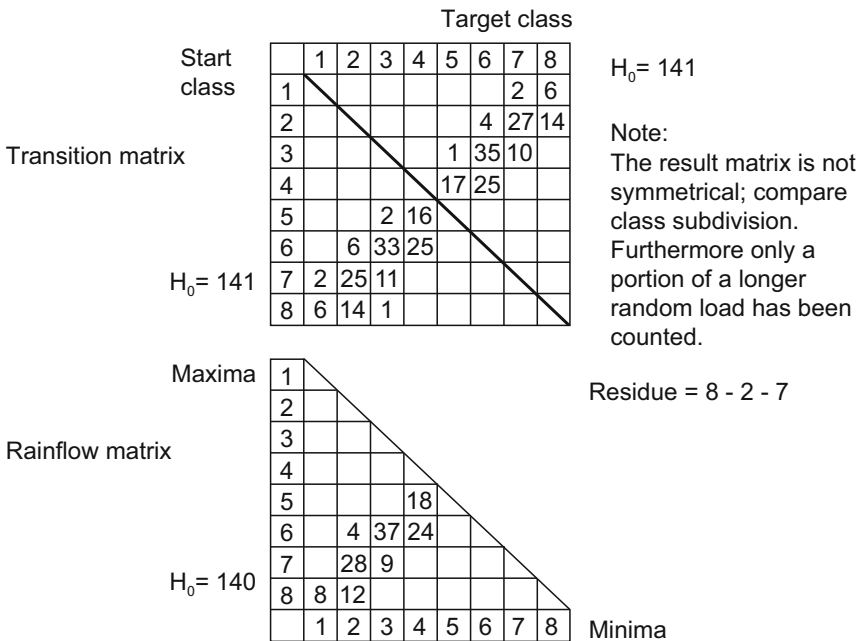
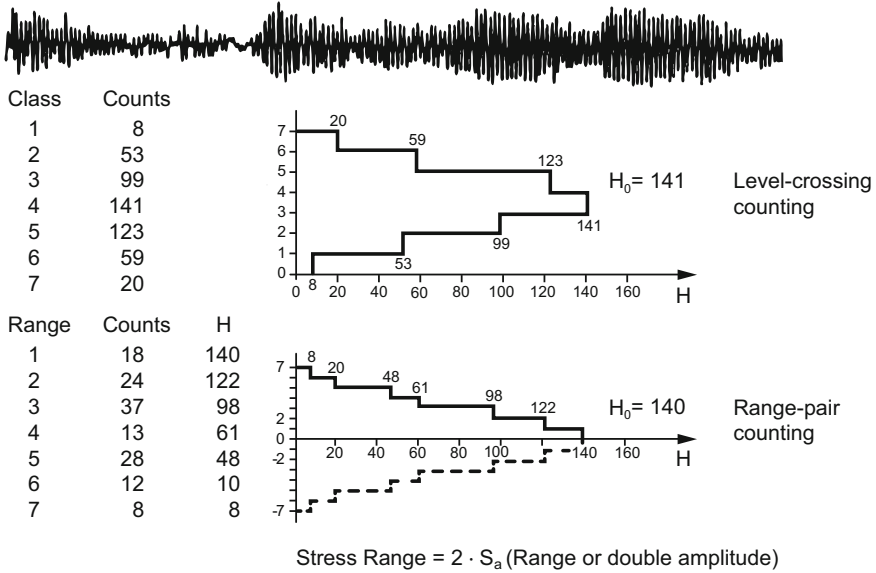


Fig. 5.2 Results of counting for random load,  $I = 0.99$

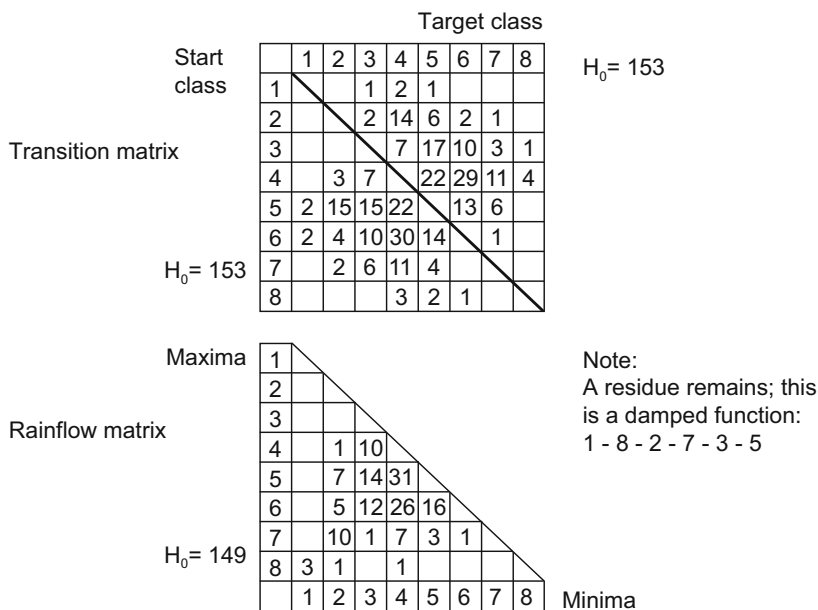
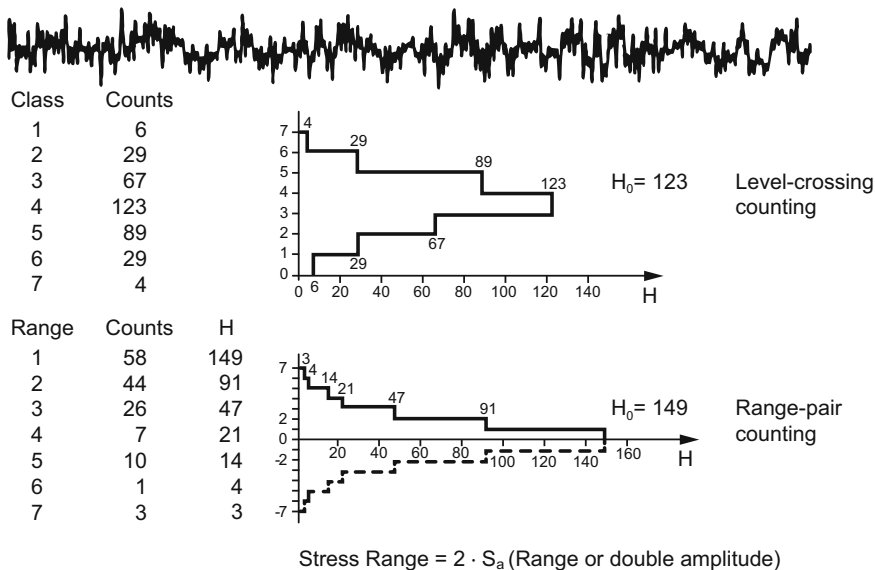


Fig. 5.3 Results of counting for random load, I = 0.7

The results of level-crossing counting, range-pair counting, transition matrix (transition counting) and rainflow counting for reduction-pass stress are shown in Fig. 5.4.

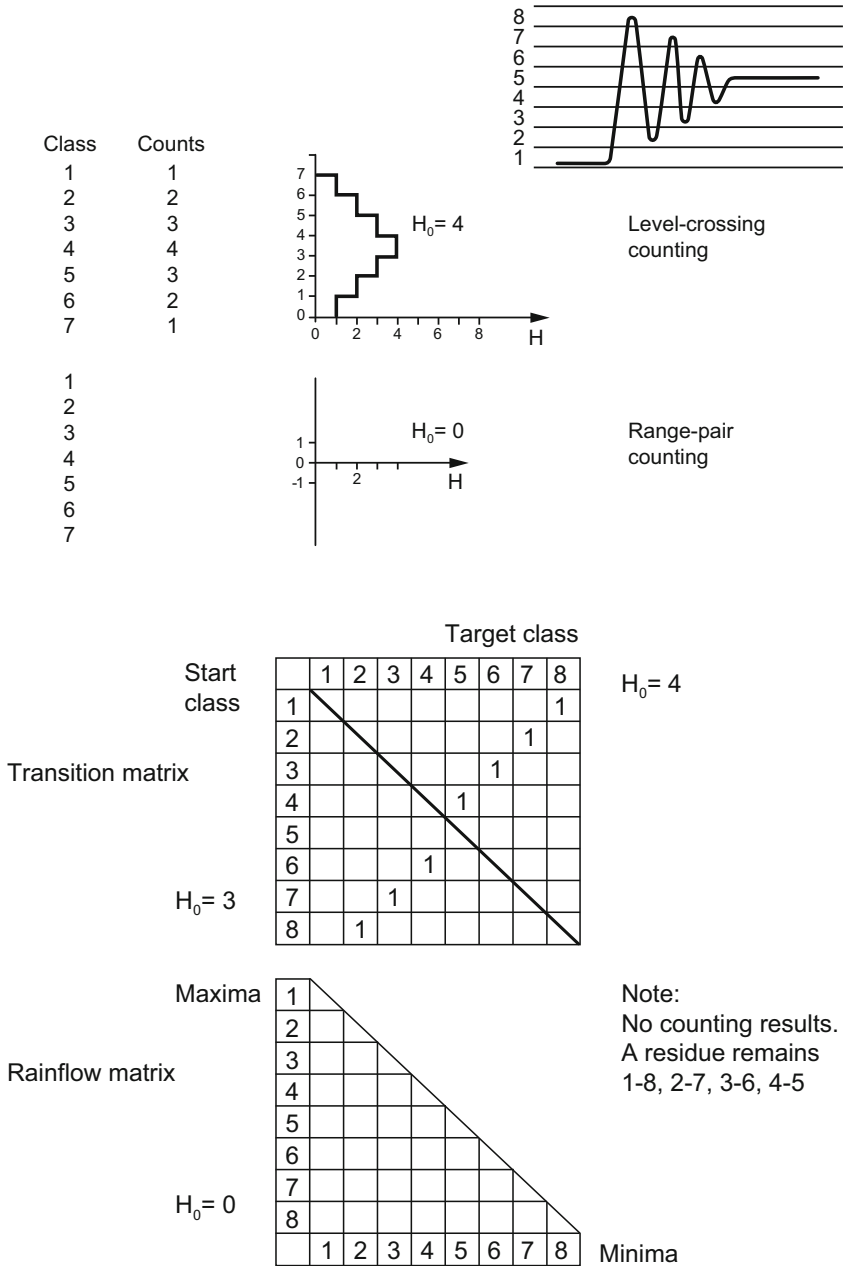


Fig. 5.4 Results of counting for damped load-time function

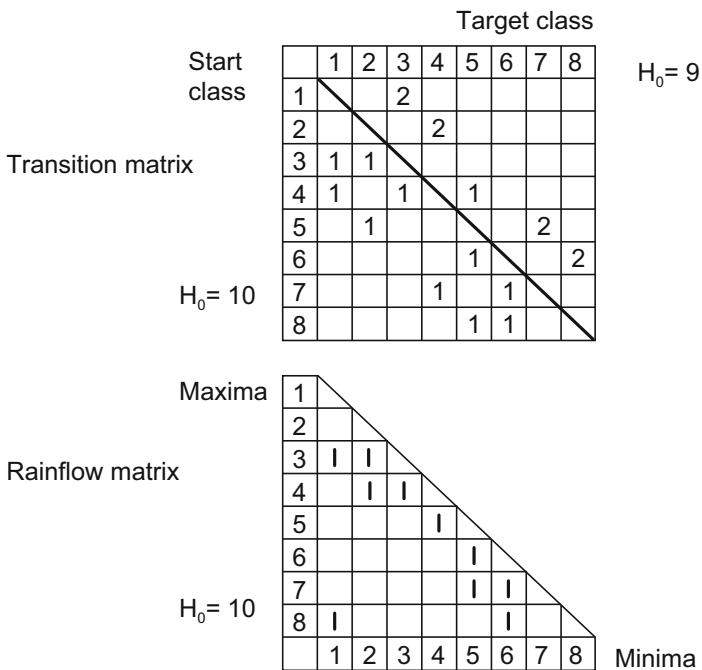
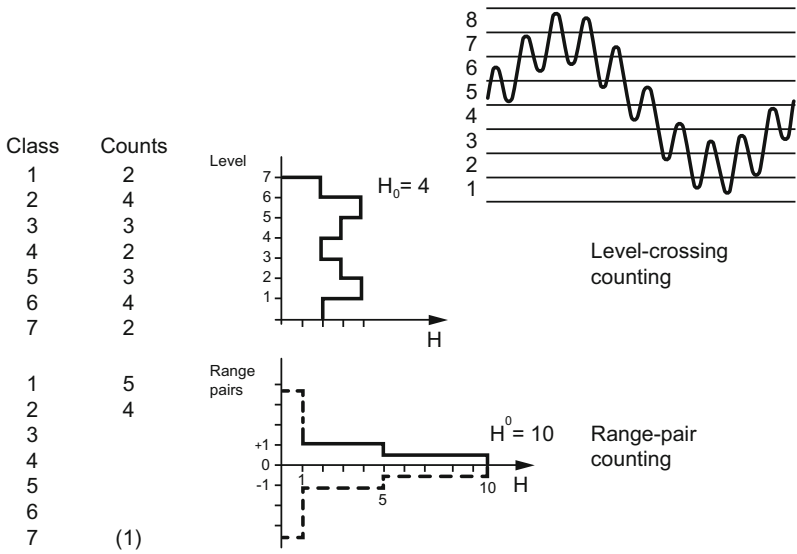


Fig. 5.5 Results of counting for superimposed sinusoidal-sinusoidal load functions



## 5.5 Superimposed Sinusoidal-Sinusoidal Load-Time Function

For superimposed sinusoidal-sinusoidal oscillation, the differences between the results of level-crossing and range-pair counting for the evaluation of the amplitudes and their cumulative frequency are extremely large, compare Fig. 5.5. In the case of range-pair counting, the largest cycle is not recorded during a load period; instead, it is recorded after two periods [Lipp67]. Consequently, it is indicated in dashed form in the figure.

The matrices are occupied in the diagonals from upper left to lower right. In this case, however, in contrast to the transition matrix for rainflow counting a large load cycle (8-1) is recorded, compare Fig. 5.5.

As shown by these results, the largest differences between the counting methods occur in cases of large mean-load fluctuations or small irregularity factors  $I$ .

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# Chapter 6

## Multiaxial Loads and Stresses

Many design tasks are concerned with structural components or systems subject to multicomponent loads. In this case, several forces and moments, which are partially time-independent, can act on the structural component to be evaluated. The local stress state and its variation with time at relevant locations result from the superposition of the local stresses. These stresses are caused by external loads acting on the component.

For this reason, the performance and analysis of measurements on structural components subject to multicomponent loads require a considerable degree of experience and understanding of the systems concerned. The appropriate choice of analytical methods depends especially on such questions as what is to be evaluated and where the measurements can be performed.

### 6.1 Definition of Terms

A definition of terms is first necessary in correspondence with the measurement of multiaxial load cases under operational conditions and their evaluation.

The external forces and moments which act on a structural component are designated as loads. If at least two load components occur, one speaks of a combined load.

The internal forces and moments, which are caused by external loads in an arbitrary cross-section of a structural component are designated as stresses. The stress at an arbitrary infinitesimal point of the component is described by the stress state or stress tensor.

In general, a distinction must be made between the variation of the forces (loads) acting on the component with time and the local stress state. As dictated by the structural component and the load situation, combined loads can cause local uniaxial stresses, for instance, the stress at the edge of a hole. On the other hand,

even only one constant load can cause oscillating multiaxial stresses, for instance, the stresses at the root of a gear tooth at constant driving moment.

A further distinction concerns the variation of loads or stresses under consideration with time. If the ratio of these quantities remains constant with time, one speaks of proportional load or stress. In this case, the direction of the principal stresses also remains constant with time. In the case of synchronous stress, the frequency is equal for all stress components, and the phase shift is zero. However, the ratio of the quantities can vary with time. Further cases are phase-shifted stresses and stresses of different frequencies. A planar stress state is present at the surface of a structural component (free surface). For the general case of a multiaxial, oscillating stress, the direction of the principal stresses varies with time.

The general multiaxial loading condition, for which no relationship exists between the load components, is designated as uncorrelated. Fatigue-strength tests have shown that the variation of the stress components with time (amplitude ratio, phase and frequency relationship, signal shape) is decisive for the fatigue life of a structural component [Zenn00].

## 6.2 Measuring and Recording Technology

As dictated by the particular application, the forces or moments, which act on the structural component can be measured directly or indirectly. A direct measurement can be performed, for instance, by placing load cells in the path of the load (such as wheel-hub load cells).

### 6.2.1 *Calibrated Strain-Gauge Measuring Points*

If the geometry of the structural component permits (rod, bar, plate, disc), the load can be determined from the forces or moments which act in the nominal cross-section. A prerequisite is the linear elastic deformation of the nominal cross-section. For this purpose, calibrated strain-gauge measuring points, that is, structural components with applied strain gauges, are employed. Before the measurement, these devices are subjected to defined loads in a special test (for instance, with the use of a tensile-testing machine), in order to unambiguously determine the relationship between the external load and the local strain.

### 6.2.2 *x-y Strain Gauge or Multiple Strain Gauge*

A direct measurement of the forces acting on the structural component is often not possible. In this case, the reactions of the component—local stresses or strains—are

measured with the use of x-y strain gauges or multiple strain gauges. These strain gauges consist of two or three grids with mutually crossed measuring grid axes arranged on a support. Strain gauge rosettes with three measuring grids are employed for the analysis of stress states with unknown principal stress direction. An unambiguous assignment of the external loads to the measured values is often not possible with combined loads in such cases.

### 6.3 Counting of Multiaxial Load-Time Functions

With each of the previously described counting methods, every individual force component is determined separately without consideration of the correlation. If the wheel forces (longitudinal, lateral, vertical) are measured on a motor vehicle, for instance, the superposition of the wheel-force components is decisive for assessing the stress on the wheel-guiding components. For the counting of individual force components, the frequency, the oscillation shape, or the order of occurrence can be neglected. However, this procedure is not permissible in the case of combined loads, apart from exceptional cases. For combined loads, a different approach has proved to be applicable. With this method, a multiplicity of new, virtual time functions are generated by superposition (weighted superposition) of the measured stress-time function and counted afterwards [Brem97]. The following considerations constitute the background for this approach: The local stress on the structural component results from the superposition of the forces acting on the component. The transfer factors for the forces at the sites which are critical for failure are defined by the geometrical conditions for the structural component. However, the transfer factors and the critical sites can be determined only by elaborate finite-element simulations. For this reason, it is assumed that all possible combinations of normalised transfer factors can be generated by weighting factors between  $-1$  and  $+1$  per force channel. The weighting factors determined for the external forces can also be interpreted as unit or directional vectors. If superimposed time signals are now generated and classified for all “directions”, the loading situation which results from the superposition of the external forces can be completely described [Brem97]. This method is not suited for performing an analytical fatigue-life prediction; however, it is well suited for the comparison of measurements, for instance, of the wheel forces on a motor vehicle travelling on different routes.

Attempts to extend the rainflow method to the cycle counting of multiaxial loads (multiaxial rainflow) or to the extrapolation of rainflow matrices are the subject of current research work [Jones77, Dres92, Dres96, Joha06].

## 6.4 Counting of Multiaxial Local Stress States

For the evaluation of multiaxial strain measurements, an individual evaluation of the stress/strain signals is permissible only in the exceptional case of proportional stress. For complex, uncorrelated stress-time curves or for superimposed mean stresses, even the definition of a cycle presents a problem. For this reason, equivalent stresses or invariants are often employed. However, the classification of principal-stress-versus-time curves can result in considerable incorrect interpretations. Thus, for an alternating tensile-compressive load, the range of the first principal normal stress is only one-half of the maximal range, by definition. For determining the actual range, the respective principal normal stress with the larger absolute value must be employed for each instant in time. In this case, the use of the maximal shear stress is preferable.

The von Mises equivalent stress is always positive by definition. Consequently, it can be employed for the counting of multiaxial stress states only if the magnitude of the equivalent stress has the same sign as the respective normal stress (principal stress) with the larger absolute value at every instant.

During the counting of equivalent stresses, the information on the variation of the principal stress direction with time is lost. However, investigations have shown that this variation affects the fatigue life of a structural component. Hence, the application of the von Mises equivalent stress is useful only in the case of proportional stress.

The difficulty of performing a correct count in the presence of multiaxial stress is due to various basic causes. Designing and dimensioning for withstanding operational stresses represent a comparison between stress and strength provided that the stress and the capacity to withstand loads can be clearly defined. This is not the case in the presence of multiaxial stress, since a strength hypothesis or a damage criterion is necessary for the comparison. However, a generally applicable criterion of this kind does not yet exist:

- Both the strength hypothesis and the type of failure depend on the material and on the condition of the material (brittle, ductile).
- Variable principal stress directions can cause pronounced material hardening (out-of-phase hardening). A prerequisite for equivalent stresses as strength hypotheses is the possibility of comparing a multiaxial stress state with a uniaxial stress state. This condition is not satisfied in this case.
- Materials are often anisotropic. That is, the stresses (strains) which are referred to the material strength depend on the direction. (With pronounced anisotropy, the problem of multiaxiality may be simplified.)
- The effect of superimposed mean stresses depends on the material. That is, the action of mean-stress components must be described as a function of the material for multiaxial stress. Very little information is hitherto available on the effect of the mean stresses in the case of multiaxial stress.

- The effect of multiaxiality during the phase of microcrack growth differs from that during the phase of macrocrack growth. The separation of these phases is not sharp.
- A number of procedures have been proposed for calculation. However, these approaches differ decidedly in the evaluation of the individual oscillating and static components:
  - on the basis of local stresses
  - on the basis of local strains
  - on the basis of energy transformation.

Furthermore, there are two fundamentally different means of considering the problem: the principle of the critical intersection plane and the integral consideration of a material volume, compare [Joha14, Soci00, Sons04, Sons11, Sons17, Zenn04].

Thus, no generally applicable counting methods are yet available for assessing the fatigue strength under variable stress amplitude for arbitrarily complex multiaxial stress states. The purpose of a counting method is data reduction. A prerequisite for this is reliable knowledge of the signal properties, which affect the fatigue life. The assessment of the fatigue life under multiaxial stress is still the object of current research work. Hence, one must decide on the appropriate counting method for every application and in each individual case.

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# Chapter 7

## Time-at-Level and Level-Distribution Counting

Besides cycle counting of amplitudes further statistical analysis are useful for the analysis of load-time-functions. The time-at-level counting methods are applied in case the duration of a process or operation condition shall be evaluated. Although they are not applicable for life-time prediction directly the time-at-level counting methods are very often used to describe the distribution of different working conditions. In the following, one- and two-parameter time-at-level counting methods are described.

### 7.1 One-Parameter Methods

Two different one-parameter counting methods are described and assessed in the following chapter.

#### 7.1.1 *Time-at-Level Counting*

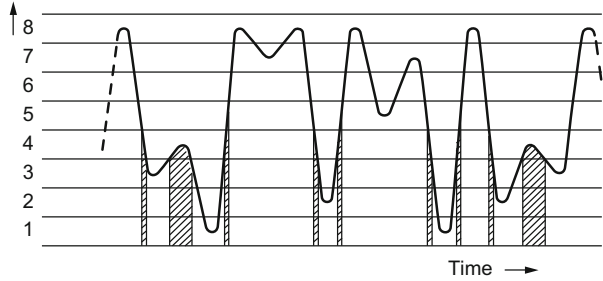
##### **Description of the Method**

During time-at-level counting, the sum is determined for the times during which the signal remains within the individual levels, Fig. 7.1. The duration of the time at level for all classes corresponds to the duration of measurement. The relative time at level is the time at level per class referred to the duration of measurement.

To determine the time at level, the signal value is determined by random sampling at equal time intervals and counted in the respective class. The frequency of the counting per class is a measure of the time-at-level in the respective class.



**Fig. 7.1** Stress-time function, time-at-level counting



### Description of the Counting Algorithm

For each class, the times at level  $\Delta t_i$  of the signal in the respective class  $i$  are plotted, as illustrated for the example of class  $i = 4$  in Fig. 7.1. The result of the counting is shown in Fig. 7.2. For this purpose,  $t_i = \sum \Delta t_i$ , see also Fig. 7.2.

By using a sampling algorithm, the stress-time function is subdivided into equidistant time intervals, Fig. 7.3. At the predetermined intervals, the respective measured value is counted in the class in which it occurs. The result of counting is plotted in Fig. 7.4. Since the introduction of digital measuring techniques, the measured signals are usually already available as equidistantly sampled stress-time functions.

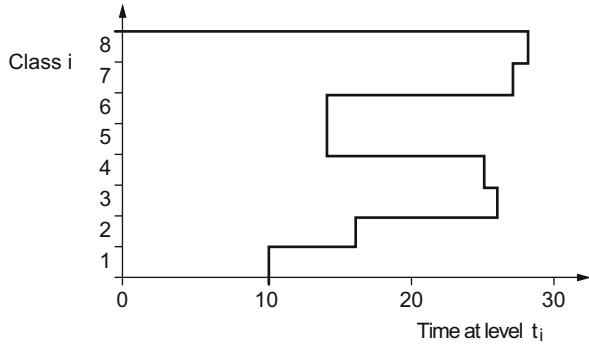
### Comment

The result of time-at-level counting does not contain any information on the frequency of the extrema and thus also contains no information on the magnitude and frequency of cycles. Hence, it cannot be applied for life-time prediction. Various applications are possible in the statistical analysis of time-related changes of process or operational parameters. The introduction of a range of restoration is of no use with this counting method.

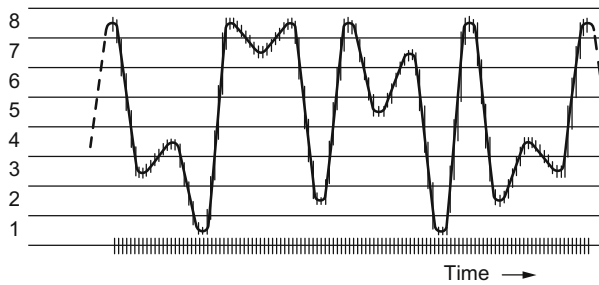
### Application

The level-distribution counting is applied for the statistical treatment of rotational speeds, pressures, velocities, and temperatures, for instance. A further application is the determination of the assessed value for the arithmetical and quadratic mean value of a signal in the time range. A special case of level-distribution counting is sampling of a signal as a function of time, rotational speed, or angle. Examples of such applications include the measurement of stresses on individual gear teeth and the determination of individual tooth stresses from the torque-time function [Schö84]. This counting method is thus a standard procedure for analytical fatigue-life predictions for gears and bearings at present.

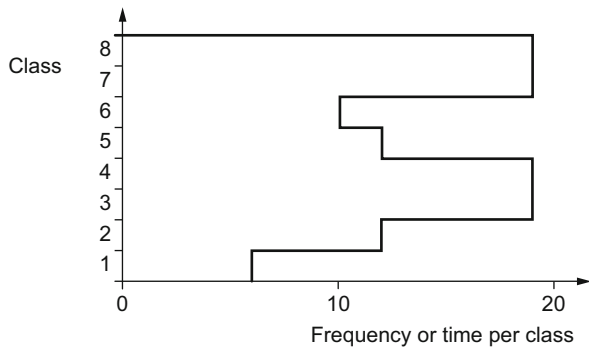
**Fig. 7.2** Result of time-at-level counting



**Fig. 7.3** Stress-time function, level-distribution counting



**Fig. 7.4** Result of level-distribution counting



**7.1.2 Relative Level-Distribution Counting**

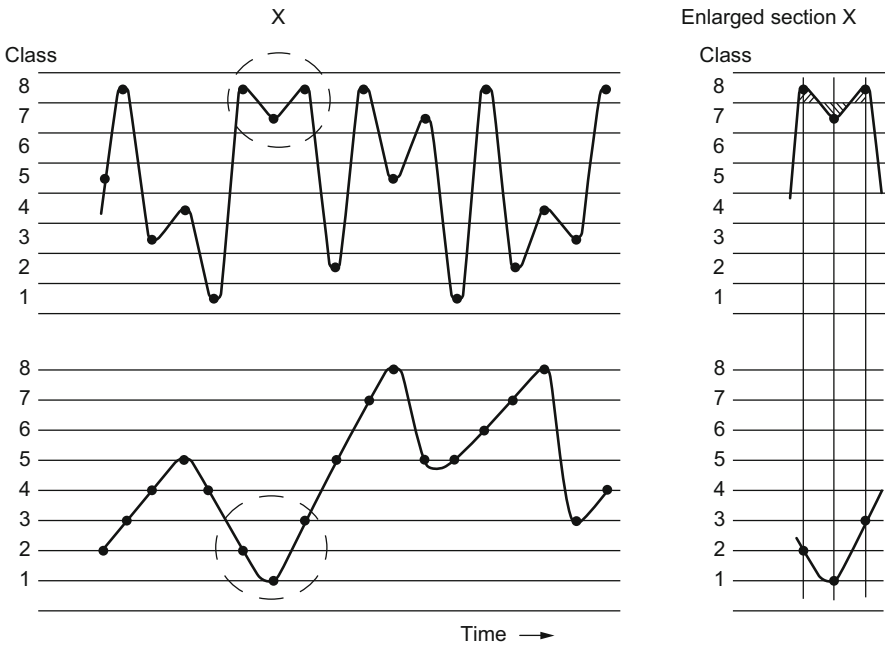
**Description of the Method**

To extend the time-at-level counting, a further reference signal which describes the process is required, in addition to the measured signal to be classified. As a practical example, a wheel-contact force is assumed as measured signal, and a wheel revolution is taken as reference signal. The rotational speed is usually also recorded

during measurements on rotating components. From the rotational speed signal (expressed in  $s^{-1}$ ), the number of wheel revolutions per measured value can be determined by division by the sampling rate (number of measured values per second). The sampling rate is usually considerably higher than the frequency of wheel rotation. Consequently, the number of wheel revolutions per measured value is considerably lower than one. In the following example, the number of wheel revolutions per measured value has been assumed to be  $\geq 1$  for the sake of clarity.

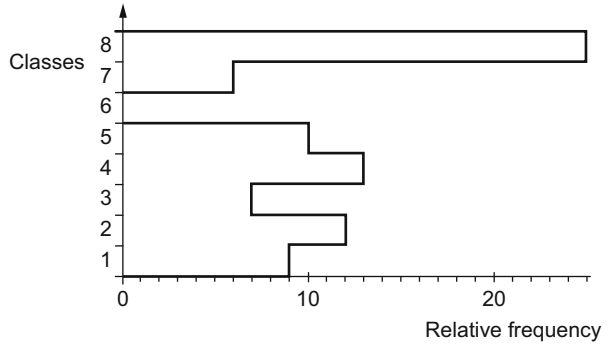
As described in Fig. 7.5, two measured values of the stress-time function are in class 1. At the first instant of sampling, the reference signal is situated at four revolutions; at the second instant of sampling it occurs at five revolutions. For the counting result, a frequency of nine is entered for class 1. For the example of the wheel-contact force, the wheel would have performed nine revolutions with a contact force in class 1 within the measuring sector. Two measured values of the stress-time function are again situated in class 2. The reference signal is situated at five revolutions during the first instant of measurement and at seven during the second instant of measurement. For the counting result this situation is represented by the entry of 12 revolutions in class 2.

As an example, the variation of the measured and reference signals with time are plotted in Fig. 7.5; the associated result of counting is presented in Fig. 7.6.



**Fig. 7.5** Time function, load (*top*) and revolution signal (*bottom*), relative level-distribution counting

**Fig. 7.6** Result of relative level-distribution counting



### Description of the Counting Algorithm

As in the case of level-distribution counting, the number of sampling processes for measuring a signal in a particular class is counted with the applied algorithm. However, this number is referred to the magnitude (class) of the reference signal at the instant of sampling (for instance, a revolution). By way of departure from level-distribution counting, however, the frequency of the respective measured value is not automatically equal to one. Instead, the magnitude of the reference signal is plotted as a frequency of the individual measured value.

### Comment

This method is especially well suited for the signal interpretation of rotating machine components with reference to distance or rotation. The cumulative frequency of the counting result is equal to the sum of all wheel revolutions. For a constant reference signal equal to one, the result would be the same as that of level-distribution counting.

## 7.2 Two-Parameter Methods

Two different methods of two-parameter counting are described and assessed in the following chapter.

### 7.2.1 Two-Parameter Time-at-Level Counting

#### Description of the Method

The time during which both parameters satisfy the respective class conditions is designated as the time at level in a two-dimensional class. The times at level

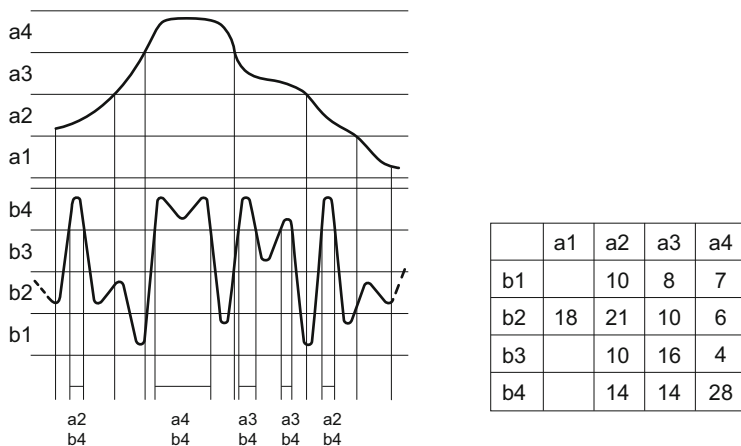


Fig. 7.7 Stress-time functions and result of two-parameter time-at-level counting

(in units of time  $t_i$ ) during which signal b is located in class number 4, and the associated class presence of signal a are indicated in Fig. 7.7. The result of counting can be plotted in two dimensions with contour lines (isohypses of the frequency hill). Bearing-load-rotational-speed spectra are an example of such applications. A three-dimensional perspective plot is also possible.

### Description of the Counting Algorithm

Both signals are subdivided into equidistant classes. The time during which the first signal is located in class  $a_i$  and the second signal is located in class  $b_j$  is counted as the time at level in a two-dimensional class  $a_i, b_j$ , Fig. 7.7.

### Comment

The two-parameter time-at-level counting is applied when statistical distributions of the duration of different operation conditions shall be evaluated for two different measured signals. It is therefore often used to describe the duration of two different forces or moments acting on a wheel of a vehicle or bearings.

As an example, the method can be applied to bearing-load-rotational-frequency plots [Grie73, Fisc80].

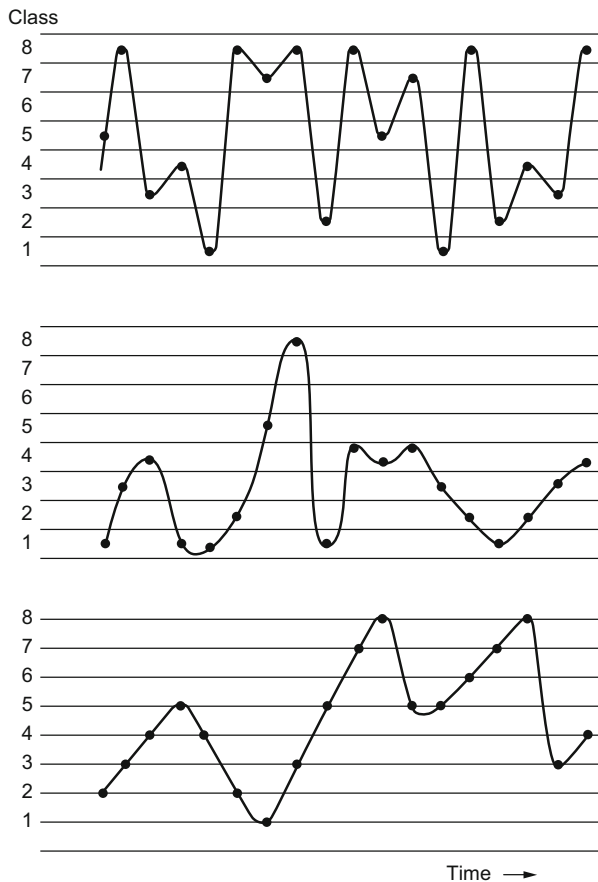
### 7.2.2 Relative Two-Parameter Level-Distribution Counting

#### Description of the Method

An extension of the two-parameter level-distribution counting method is relative two-parameter level-distribution counting [Jenn04]. As already discussed for the one-parameter version, a reference signal is employed for determining the frequency of the entries in the result matrix. As a practical example, two wheel forces (for instance, in the vertical and transverse directions) are taken as measured signals, and the wheel revolution is assumed to be the reference signal.

In the following example (Fig. 7.8), stress-time function 1 is situated in class 5, and stress-time function 2 is situated in class 1 at the first instant of sampling. The reference signal (revolutions) is located at 2. The resulting entry of two (revolutions) is located in cell 5/1 of the result matrix. At the second instant of sampling, stress-time function 1 is situated in class 8, and stress-time function 2 is situated in

**Fig. 7.8** Time functions 1 (*top*) and 2 (*centre*) as well as revolution signal (*bottom*), two-parameter relative level-distribution counting



**Fig. 7.9** Result of two-parameter relative level-distribution counting

		STF 1, Classes							
		1	2	3	4	5	6	7	8
STF 2, Classes	1	4	12	0	5	2	0	0	0
	2	0	0	0	8	0	0	0	8
	3	5	0	3	0	0	0	0	3
	4	0	0	4	0	8	0	5	11
	5	0	0	0	0	0	0	1	0
	6	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	3

class 3. The reference signal is located at three (revolutions). The resulting entry of three (revolutions) is located in cell 8/3 of the result matrix. The (revolution) frequencies at later instants of measurement with an identical combination of stress-time functions 1 and 2 are added (compare with one-parameter relative level-distribution counting). For the present example, the variations of the two measured signals and the reference signal (revolutions) with time are plotted in Fig. 7.8, and the associated result of counting is shown in Fig. 7.9.

The result of counting is a matrix in which the combinations of the measured quantities, stress-time functions 1 and 2, are plotted with the frequency of the reference signal, Fig. 7.9.

## Application

This method is especially well suited for the travel-route-related or revolution-related counting of rotating components on which several quantities, such as wheel forces, are measured simultaneously. The sum of all matrix entries for the result of counting is equal to the sum of all wheel revolutions. With a constant reference signal equal to one, the result would be the same as that of two-parameter level-distribution counting.

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# Chapter 8

## Application of the Counting Methods

In the preceding chapters, the various counting methods have been described and assessed. In the following, the criteria for selection are summarised. For this purpose, the graphical representation of the counting results is first considered. Subsequently, the suitability of these results for analytical fatigue-life predictions is discussed.

### 8.1 Criteria for the Selection of a Counting Method

As a rule, operational loads vary with time. The load cycles which occur in operations are almost always characterised by variable amplitude. The fatigue life of structural components under operational loads depends on the magnitude and frequency of the amplitudes which occur. Consequently, the task of transforming the load-time function to a frequency distribution of amplitudes has been the subject of intensive research for a long time. As early as 1932, load spectra have been measured on agricultural machines by W. Kloth and T. Stroppel [Klot32, Klot36, Klot61]. In 1941, E. Gassner completed his doctoral dissertation on the fatigue behaviour at variable amplitude [Gass41]. Numerous one-parameter counting methods have been developed for forming spectra; however, very few of these one-parameter methods are still in use today. Since the mean stress as well as the stress ratio  $R$  also affect the fatigue life, in addition to the amplitude, two-parameter counting methods were later introduced [Teic41, Mats68, Aich73, Günt73]. Non-sequential counting methods like range-pair and rainflow counting have been essential in the continuing development of counting methods for analytical fatigue-life prediction.

A common feature of all counting methods is data reduction with reference to the stress-time function. In the case of the counting methods applied for determining the fatigue strength under variable stress amplitude, the information on the frequency of the oscillation, the order of the occurrence of cycles, and on the shape



of the load time function (sinusoidal, triangular, trapezoidal, etc.) is lost. Consequently, the application of these methods is permissible only if the fatigue life of a structural component is not, or only slightly, dependent on these features.

A fundamental objection has been expressed in the following manner by E. Gassner in 1983: “All known counting methods apply to functional values (features) which are derived from the load-time function to be analysed, and whose importance for the fatigue-strength evaluation is assessed as though these values were independent individual results. However, these values are embedded in a continuous load-time function and exert their effect only in combination with the latter” [Gass83].

For the purpose of analytical fatigue-life prediction, the best-suited counting method in combination with an analytical damage accumulation hypothesis is that which best describes the fatigue damage caused by the stress-time function with variable amplitude. If the damage process at the critical point of a structural component is considered more closely, several phases can be distinguished: structural-mechanical changes in the material by cyclic plastic deformation, microcrack nucleation, microcrack growth, development of a macroscopic cracking, macroscopic crack propagation, and fracture [Mill93]. The fact that these processes are controlled in different ways by the normal and shear stresses which act in a volume element, as well as by the quotient of minimum and maximum stress, the stress ratio  $R$ , is evident. However, no damage model, which reliably describes this process chain is available to date. In particular, the transition from one phase to the next, and thus the duration of the phases, cannot be determined with sufficient accuracy.

Rainflow counting, 1968, and range-pair-mean counting, 1969, both of which yield the same results, are counting methods which have now become internationally established as standard methods under the name of rainflow counting. With reference to the volume element, closed stress-strain hystereses are counted with the rainflow method. The area enclosed by a hysteresis corresponds to the energy which is introduced into the material volume. This energy is necessary for cyclic plastic deformation, which in turn is the prerequisite for fatigue in metallic materials. Energy is also required for forming surfaces during crack growth. On the other hand, local heating which cannot be immediately associated with damage can also occur. With the counting of closed hystereses, rainflow counting is the only counting method with a physical background, that is, a background based on the mechanics of materials. Two facts speak in favour of the rainflow counting method:

- If the results of fatigue-life tests are compared with the calculated values, extensive scatter is observed. However, if the results obtained with the various counting methods are mutually compared, the least scatter results from rainflow counting [Euli94, Euli97], compare Table 9.1.
- From an existing rainflow matrix, a practically unlimited number of stress-time functions can be generated, as dictated by the algorithm for generating random numbers, and all belong to the same matrix. Test results have proved that the different stress-time functions yield practically the same fatigue life [Perr87].

Rainflow counting is in conformity with the local strain concept; that is, the total strain range of a closed hysteresis is employed as damage parameter for calculating the fatigue life.

## **8.2 Recommendations**

For the application of counting methods, attention must be paid to the limitations which result from the associated data reduction. Furthermore, the use of examples demonstrates that the local damage processes are not always determined by the load spectra alone.

### ***8.2.1 Graphical Representation***

For a graphical representation, for example, for comparing the loads along different routes or with different drivers, spectra provide a clearer illustration than matrices. Level-crossing and range-pair counting are recommended for the purpose.

With level-crossing counting, the maximum and minimum of the stress can be identified directly, but the amplitude cannot be determined. From the shape of the overall spectrum, partial spectra can be recognised in some cases. In the case of range-pair counting, the frequency distribution of the amplitudes is plotted. The height of the maximum and minimum cannot be determined. The mean stress is lost with both counting methods.

### ***8.2.2 Practical Experience***

The rainflow matrix should constitute the basis for the analytical fatigue-life prediction. In the case of short stress-time functions, attention must be paid to the half-cycles, which are entered as residue, since they can contribute to the damage.

As a matter of principle, the effect of the load frequency on the fatigue life, expressed in cycles, can be more pronounced in the presence of superimposed time-dependent processes, such as corrosion and creep. In this context, reference to special literature is recommended [Berg05, Rada07, Schi09, Soci00].

The order in which loads occur can affect the fatigue life; this fact has been proved, for instance, in the case of two-step high-low or low-high loads. Similar considerations apply to so-called consecutive loads due to torsion/bending or bending/torsion. The instant of operation at which a special event associated with overload occurs can also be decisive for the fatigue life of a structural component. However, extensive mixing of loads can usually be assumed for many types of

components, such as automobile parts. In such cases, the order of occurrence does not appreciably affect the results.

As shown in Table 9.1, the calculated damage sums and thus the calculated fatigue life depend on the counting method. For presenting the results of an analytical fatigue-life prediction, therefore, the counting method which has been applied must be indicated.

Further cases in which a critical consideration of the counting method is advisable for fatigue-life predictions in practice are described in the following:

If zero crossings of the torque with play (backlash, clatter) frequently occur during the operation of a transmission gear, the endurance limit of the tooth profile can decrease significantly as a result [Weck87]. This phenomenon has been associated with the sudden engagement of the retaining brake in wind-turbine units. As a result, the free oscillation of the rotor caused a faster development of seizing and abrasive effects on tooth profiles, in comparison with pitting due to fatigue. This effect would not have been recognisable from a measurement on the shaft as a result of counting alone [Fisc91, Zenn97].

A further example is wheel skidding on rail vehicles as a result of slippage of the driven wheels. The resultant bending and torsional stresses cause fatigue damage on the axles of the driving wheelsets [Stüh80]. In addition to loads of this kind, sliding, frictional, and slipping processes often occur simultaneously at other points of the drive system. These processes can cause specific local cracking and microstructural transformations (“butterfly” phenomenon) in large roller bearings [Wade93].

As can be seen from these examples, not only the load spectra are decisive for the fatigue life of a structural component. Local failure can also be caused by other ambient conditions, for example, overload due to special events, zero crossings, idle times, gradient of a signal, resonances, fretting corrosion.

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# Chapter 9

## Analytical Fatigue-Life Estimate

In the preceding chapters, the procedure for transforming measured stress-time functions to yield a stress spectrum and a frequency matrix has been described. This transformation is a prerequisite for the load assumption, which is necessary for designing and dimensioning of structural components with the required fatigue strength under variable stress amplitude. Dimensioning of components in this manner constitutes a comparison between the loading (stress) and the capacity to withstand loads (strength). For static and endurance-strength designing, characteristic parameters can be simply compared. For designing and dimensioning of components with a defined fatigue life under variable stress amplitude, however, characteristic functions must be employed. For the loading capacity, for instance, such a characteristic function is the S-N curve for the component, and for the stress, the spectrum or frequency matrix is such a function. These characteristic functions are the input parameters for damage-accumulation hypotheses by means of which an allowable number of load cycles to failure is calculated.

In the following, the basic principles of the analytical fatigue-life estimate are explained, but only to the extent necessary for the purposes of Chaps. 10 and 11. For a more comprehensive treatment of this topic, refer to the following books: [Buxb92, Seeg96, Gude99, Soci00, FKM03, Haib06, Rada07, Schi09].

For the analytical fatigue-life prediction, three concepts with different damage criteria are available:

Nominal stress concept	Macroscopic crack initiation or fracture
Local strain concept	Macroscopic crack initiation
Fracture-mechanical concept	Macroscopic crack propagation up to a specific crack length or to fracture

The following considerations are restricted to the nominal stress concept. For this purpose, forces or moments can also be employed instead of stresses. This limitation does not imply a valuation of any kind. The nominal stress concept has been selected because it is relatively simple and transparent and because it is

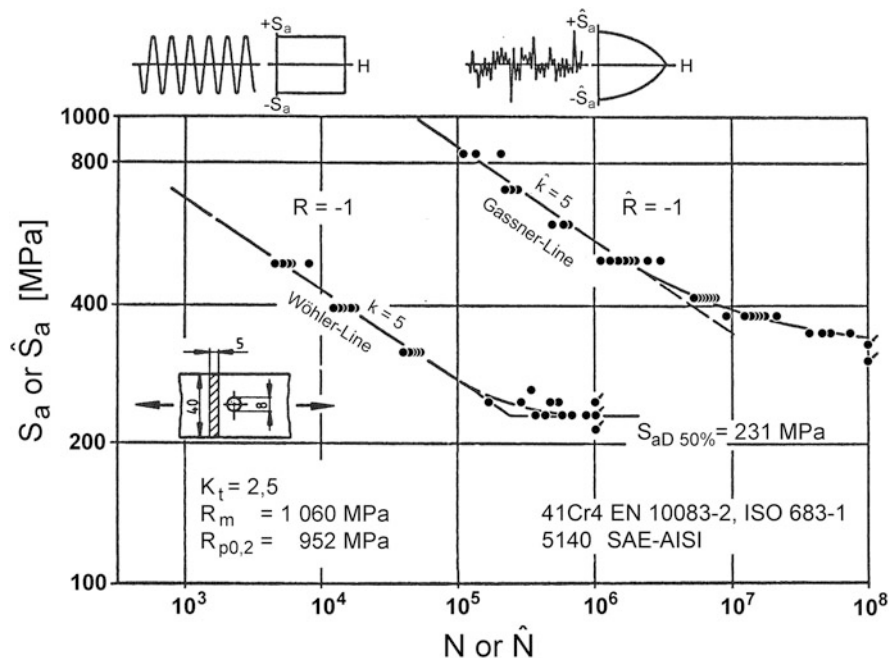
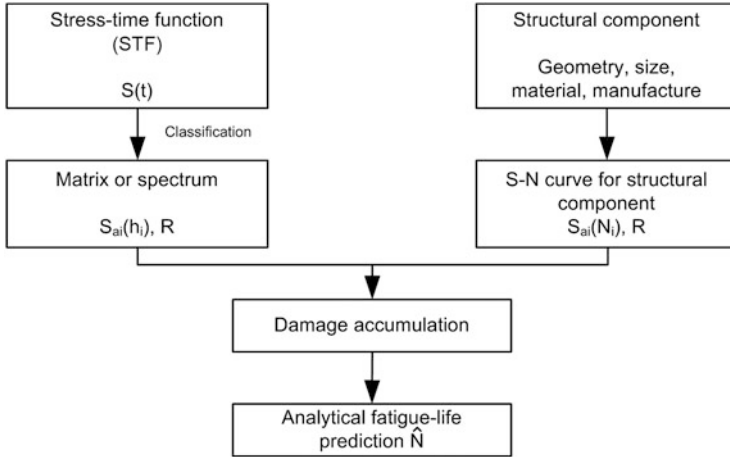


Fig. 9.1 S-N curves for constant and variable amplitudes, axial loading of notched specimens, QT-Steel (fatigue test by LBF Darmstadt)

sufficient for explaining the necessary procedure for the load assumption. This concept constitutes a phenomenological approach; that is, the progressing fatigue phenomena which occur in the structural component during the fatigue process are considered in a global manner. In practice, this concept has received widespread application, as indicated in the associated series of technical rules, for instance. It is well suited for evaluating service loads as described here, precisely because it does not depend on specific damage mechanisms. The question of whether or not this is permissible must be considered before application in each individual case.

Results of fatigue tests performed at constant as well as variable amplitudes are shown in Fig. 9.1. For this purpose, notched specimens of a QT steel have been investigated under axial loading. For constant amplitudes, the shape of the load spectrum is a rectangle. In the logarithmic plot of the nominal stress amplitude  $S_a$  versus the number of cycles  $N$  until fracture, the S-N curve (Wöhler-line) is a straight line with slope  $k$ . The value of the stress ratio  $R = -1$  indicates that the nominal mean stress is zero. In this case, the decrease in the curve is limited to a number of cycles higher than  $2 \times 10^5$ . An endurance limit  $S_{aD}$  is attained. With variable amplitudes, a load spectrum with a Gaussian distribution can be obtained, as in the present case.  $H$  is the cumulative frequency of cycles (log). The S-N curve (Gassner-line) results from the plot of the maximum nominal stress amplitude of the spectrum  $\hat{S}_a$  versus the number of cycles  $\hat{N}$ . This curve approximates a straight line



**Fig. 9.2** Procedural principle of the analytical fatigue-life prediction

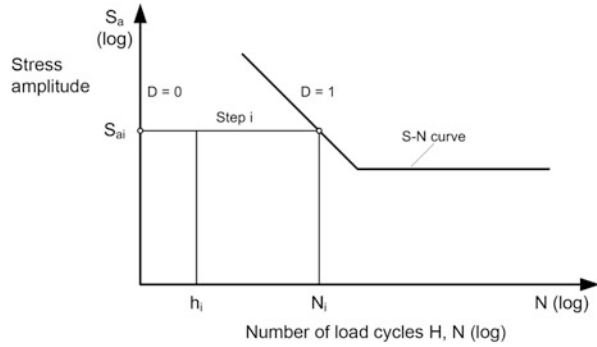
only in the upper range. A synthetic description of the Gassner-line is not usual. The theoretical course of these curves is derived from the damage accumulation hypothesis which is applied in the life-time calculation for a number of horizons.

The basic principle and procedure for calculating the fatigue life in accordance with the nominal stress concept are illustrated in Fig. 9.2. The input parameters for the calculation are the frequency matrix or the stress spectrum, which are derived from the measured stress-time functions  $S(t)$  or generated synthetically, and the S-N curve, which can be determined experimentally for the structural component or estimated, for instance, as a function of the component geometry, the material, and the manufacturing process. For the damage accumulation, an appropriate hypothesis and the assumption of a characteristic damage sum  $D$  (damage) for the failure are required. With the use of these data, the fatigue life can then be calculated as an allowable number  $\hat{N}$  of load cycles to failure.

## 9.1 Palmgren-Miner Rule

The term damage  $D$  is employed for designating the analytical contribution to the total damage by one load cycle or by a series of load cycles. The total damage has been attained if the limiting value for which failure is assumed has been attained by addition or accumulation of the damages. For this purpose, a damage criterion must be specified, for instance, macroscopic crack initiation or fracture. This limiting value corresponds theoretically to  $D = 1$ . The severity of the damage is a purely theoretical value which cannot in general be verified by testing of a structural component.

**Fig. 9.3** S-N curve  
Wöhler: schematic  
explanation of damage  
accumulation in accordance  
with Palmgren-Miner



By comparison of calculated values with test results, it will later be shown that the actual damage sum of failure can deviate extensively from one. This fact can be utilised for adapting the calculation if sufficient experience is available.

For the damage accumulation, a linear summation is usually performed in practice, as proposed by Palmgren 1924 [Palm24], by Langer 1937 [Lang37], and by Miner 1945 [Mine45]. A load horizon  $i$  of an S-N curve is considered, see Fig. 9.3:

For  $N = 0$  load cycles, the damage sum is  $D = 0$ .

For  $N = N_i$  load cycles, the damage sum is  $D = 1$ .

$N_i$  is the tolerable number of load cycles in the constant-amplitude fatigue test (S-N curve) for crack initiation or fracture. The damage per load cycle is equal to:

$$d_i = \frac{1}{N_i} \quad (9.1)$$

The damage sum for  $h_i$  load cycles on horizon  $i$  ( $h_i$ —step frequency) is then calculated as follows:

$$D_i = h_i \cdot d_i = \frac{h_i}{N_i} \quad (9.2)$$

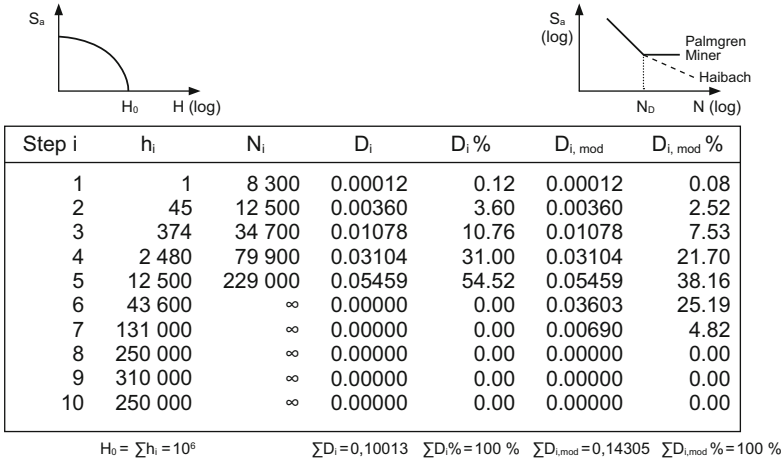
For  $k$  steps, this yields the total damage sum for a matrix or spectrum:

$$D = \sum_{i=1}^k D_i = \sum_{i=1}^k \frac{h_i}{N_i} \quad (9.3)$$

Equation (9.3) expresses the Palmgren-Miner rule. If a spectrum with a continuous distribution is present, discretisation of the stress is necessary for the calculation.

For calculating the number of load cycles until failure, see Fig. 9.4, the number  $z$  of allowable partial series is first determined. For this purpose,  $H_0$  is assumed for the example of  $10^6$  load cycles. For this partial spectrum with  $10^6$  load cycles,  $D_i = 0.10013$  is obtained as damage sum, see column 4.





**Fig. 9.4** Exemplary calculations for Palmgren-Miner D and a modification according to Haibach  $D_{mod}$

For the theoretical damage sum  $D = 1$  all the way to failure, the number of partial series is:

$$z = \frac{D}{\sum D_i} = \frac{1}{0.10013} = 9.987 \tag{9.4}$$

The calculated number of load cycles until failure is thus:

$$\hat{N} = z \cdot H_0 = 9.987 \cdot 10^6 \tag{9.5}$$

### 9.1.1 Consideration of the Mean Stress and Stress Ratio

As described in Sect. 3.4, the results of two-dimensional counting methods like rainflow counting are stored in a matrix. The cells of the matrix indicate the frequencies for the pairs of values (mean stress—amplitude). For calculating the resulting damage in accordance with the Palmgren-Miner rule, S-N curves for many mean stresses or R-values are necessary. In practice, two approaches are possible:

1. The mean stress is neglected for the pair of values. From the rainflow matrix, the range pairs with equal stress range parallel to the diagonal are extracted and added, as explained in Sect. 4.3. The result of range-pair counting is thus obtained. The damage accumulation is then calculated for this frequency distribution. This procedure is recommended only if the mean-stress differences for the pairs of values are small, or if the differences in the upward and downward directions are similar.

- The effect of the mean stress is considered for each pair of values by transformation of the amplitude. In correspondence with the effect of the mean stress on the fatigue life (number of load cycles to failure) as indicated in a Haigh plot, all amplitudes are transformed for a stress ratio  $R$  or a mean stress.  $R = -1$  or  $R = 0$  is usually recommended. This procedure is described in [Haib06], for example. After the amplitude transformation, the S-N curve for whose  $R$ -value or mean stress the transformation has been performed is then sufficient for the damage accumulation.

## 9.2 Modifications of the Palmgren-Miner Rule

The fact that large differences can occur between the fatigue life  $\hat{N}_{exp}$  determined by a fatigue-strength test and the calculated fatigue life  $\hat{N}_{calc}$  was recognised at an early stage; see for instance [Schü73]. For improving the agreement, a series of modifications have been proposed for the Palmgren-Miner rule. In designing for ensuring the necessary fatigue strength under variable stress amplitude, the amplitudes which exceed the fatigue limit cause damage at the critical points of a structural component. Consequently, even amplitudes below the original fatigue limit gradually begin to contribute to the damage accumulation, and this contribution continues to increase. This fact was regarded as an essential cause of the discrepancies between experiment and calculation. In general, the calculated lifetime according to Palmgren Miner rule is significantly longer than the lifetime determined in fatigue tests, see Sect. 9.3. In order to take this effect into account, modifications of the Palmgren-Miner rule have been developed. By means of these modifications, the contribution of small amplitudes to the damage accumulation is considered in different ways; see for instance the general surveys in [Euli94, Gude99, Haib06].

For the original Palmgren-Miner rule, the following applies, see Fig. 9.5

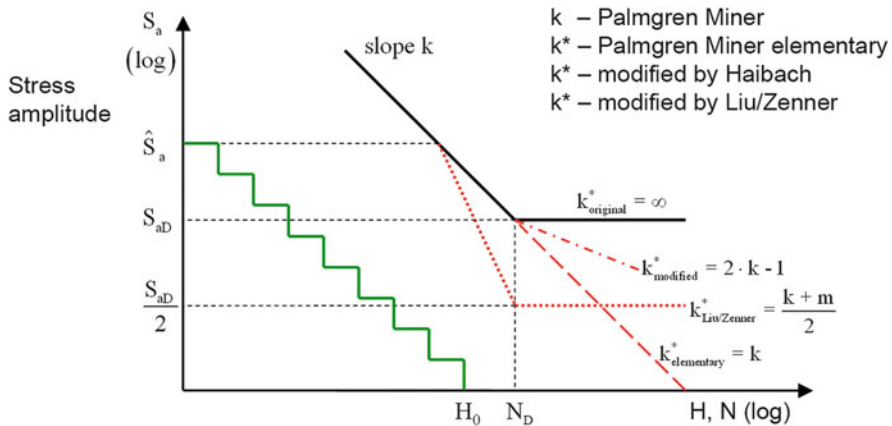


Fig. 9.5 Reference S-N curves Wöhler for original and modified Palmgren-Miner, after Haibach, modified by Liu and Zenner, and elementary

## A: Original Palmgren-Miner rule

$$S_a \geq S_{aD} : N = N_D \cdot \left( \frac{S_a}{S_{aD}} \right)^{-k} \quad (9.6)$$

$$S_a < S_{aD} : N \rightarrow \infty \quad (9.7)$$

With the modification given by Haibach [Haib70], the S-N curve is deflected at the level of the fatigue limit, that is, at the deflection point  $N_D$ :

## B: Modified Palmgren-Miner rule according to Haibach

$$S_a \geq S_{aD} : N = N_D \cdot \left( \frac{S_a}{S_{aD}} \right)^{-k} \quad (9.8)$$

$$S_a < S_{aD} : N = N_D \cdot \left( \frac{S_a}{S_{aD}} \right)^{-(2k-1)} \quad (9.9)$$

With the elementary Palmgren-Miner rule, the straight line for the finite-life fatigue strength is simply extended:

## C: Elementary Palmgren-Miner rule

$$S_a \geq S_{aD} : N = N_D \cdot \left( \frac{S_a}{S_{aD}} \right)^{-k} \quad (9.10)$$

In Fig. 9.4, an exemplary calculation is also presented for the modification due to Haibach, see column 6. Up to step 5, the calculated damage sums are the same as those obtained with the original Palmgren-Miner rule. With the modification, however, appreciable damage also occurs at steps 6 and 7. Thus, the damage sum as a whole becomes larger, and the calculated tolerable number of load cycles to failure is lower.

With the modification due to Liu and Zenner [Liu92], the straight line for the finite-life fatigue strength is rotated at the height of the maximum for the spectrum, Fig. 9.5. For this purpose, the slope  $k^*$  is assumed to be the mean value of the slope  $k$  from the finite-life fatigue-strength line and the slope  $m$  of the S-N curve for crack propagation. The slope of the S-N curve for crack propagation corresponds to the steepest possible S-N curve, for instance,  $m = 3.5$ . The endurable number of load cycles is decreased to 50%.

A further modification is the so-called Consequent Miner rule. In this case, the straight line for the finite-life fatigue strength is continuously shifted to the left as a function of the damage, and the fatigue limit is decreased. For this purpose, various approaches have been proposed [Haib71, Gnil81, Fran85, Repp86]. Refer to [Haib06] for the terminology. Because of the continuous variation of the

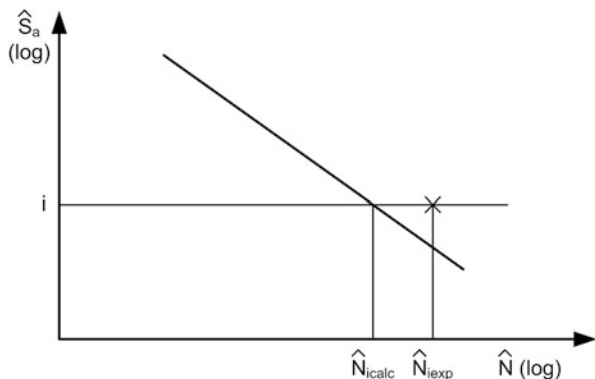


Fig. 9.6  $\hat{S}_a$ - $\hat{N}$  curve Gassner: individual sample on horizon  $i$

reference S-N curve according to the Concequent Mine rule, the damage accumulation is nonlinear.

### 9.3 Comparison Between Calculation and Experiment

If the calculated and experimental numbers of tolerable load cycles are available for a sample, the quotient can be interpreted as a damage sum, Fig. 9.6

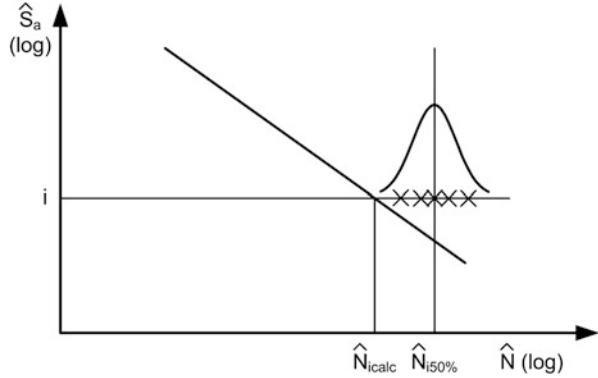
$$D_i = \frac{\hat{N}_{i,\text{exp}}}{\hat{N}_{i,\text{calc}}} \quad (9.11)$$

In this equation,  $i$  denotes the horizon of the maximum for the spectrum. Agreement between calculation and experiment implies  $D_i = 1$ . For  $D_i > 1$ , the computational estimate is conservative; that is, the experimentally determined fatigue life is greater than the calculated value. For  $D_i < 1$ , the calculated value is on the unsafe side.

In publications on analytical fatigue-life estimates, in the Series of Technical Rules, and in software programs, damage sums which have been obtained from a comparison between calculation and experiment are frequently indicated. This approach has been adopted in the present case, too, and is described in detail in the following. However, the differences between three types of damage sums are first explained for the sake of better understanding:

- If several samples are subjected to similar stresses on a stress horizon, a **mean damage sum**  $D_{i,50}$  can be determined. This average then represents an estimated

**Fig. 9.7**  $\hat{S}_a$ -N curve  
Gassner: several samples on  
horizon i



value for this horizon. These tests on a horizon are designated as a **test series** in the following. In practice, tests performed on different horizons for determining a Gassner curve are usually designated as a test series. In the present case, the distinction with respect to calculation groups merely results from the use of different formulae for calculation.

- If samples are tested on several horizons, and also if test series are performed with different spectra, or if different materials are included, for instance, the term **calculation group** is employed in the following. For a calculation group of this kind, a **characteristic mean damage sum**  $D_{50\ CG}$  can be determined.
- In publications, the term **effective damage sum**  $D_{eff}$  is often employed. The use of an effective damage sum is also proposed for calculation in the Series of Technical Rules, for instance. The effective damage sum can be regarded as having been derived from the mean damage sums for several calculation groups. In addition, a partial safety factor can also be considered in this case. The range of application for which an effective damage sum is representative must be determined.

In this context, it should be pointed out that deviations between calculation and experiment do not result only from the calculation.

If several samples  $j$  with the same spectrum are tested on a stress horizon  $i$ , the tolerable load cycles will be scattered, Fig. 9.7. In the same manner, the damage sum also exhibits scatter. This scatter results from the test scatter alone. The mean deviation between calculation and experiment is associated with the applied damage accumulation hypothesis. Thus, the mean value, the standard deviation, and the range of scatter of the damage sum for this horizon are given by the following equation:

$$\log \hat{N}_{i,50,exp} = \frac{1}{k} \sum_{j=1}^k \log \hat{N}_{i,j,exp} \tag{9.12}$$

where

$$\log \widehat{N}_{i,50,\text{exp}} = \frac{1}{k} \sum_{j=1}^k \log \widehat{N}_{i,j,\text{exp}} \quad (9.13)$$

with

j—ordinal number of the samples,  
k—extent of random sampling on horizon i

The standard deviation  $s$  and the range of scatter  $T_D$  of the damage sum for a test series are given by

$$s = \sqrt{\frac{1}{n-1} \sum_{j=1}^k (\log D_i - \log D_{i,50})^2} \quad (9.14)$$

The standard deviation of the damage sum corresponds to that for the number of load cycles on this horizon.

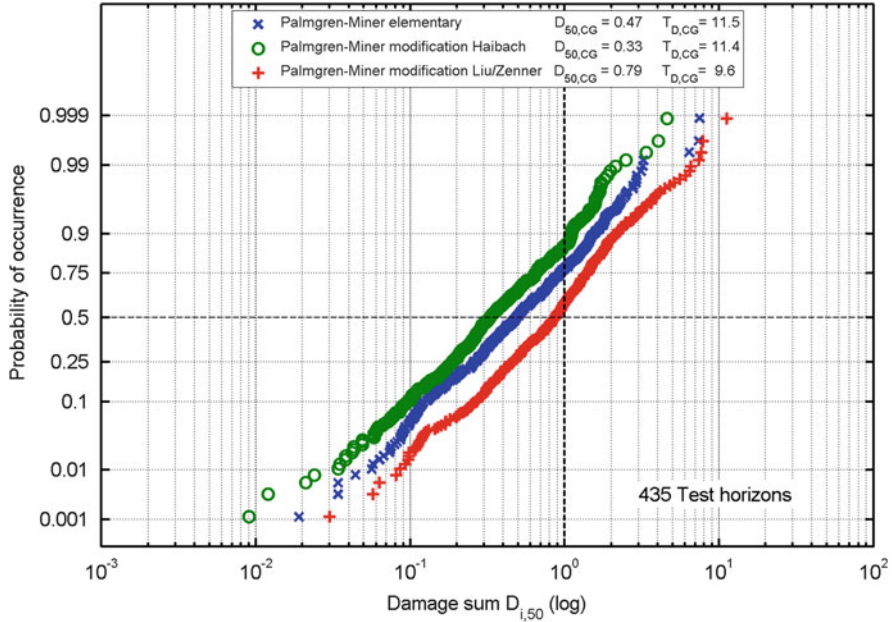
$$T_D = \frac{D_{90}}{D_{10}} \quad (9.15)$$

The range of scatter expressed as a quotient of the feature values for a probability of occurrence of 90% and 10% is often employed as a more easily visualised equivalent of the standard deviation. If a log-normal distribution is assumed for the damage sum, the following applies:

$$\log T_D = 2.56 \cdot s \quad (9.16)$$

Thus, the mean value and the scatter of the damage sum are known for a horizon with identical stress (maximum for the spectrum, shape of the spectrum, extent of the spectrum).

If extensive scatter of the mean values from test series is assumed, as indicated in Fig. 9.8, for instance, it may be possible to define calculation groups in such a way that the scatter of the mean values is decreased. This possibility should be examined in any case. A calculation group consists of several test series, which are selected in correspondence with the question under investigation. Parameters include, for instance, the material, the stress horizon, the type of stress, the stress ratio, the shape of the spectrum, the stress-concentration factor for the notch, the size of the structural component, the manufacturing



**Fig. 9.8** Distribution of the damage sums  $D_{i,50}$  for three Palmgren-Miner-modifications, rod samples of steel, tension-compression and bending

process, the counting method, and the damage accumulation hypothesis. For this purpose, it may prove useful to specify certain parameters and to leave others unspecified. Thus, a calculation group CG can be generated for steels, bending stress, unwelded components, rainflow counting, and elementary Palmgren-Miner rule, for instance. Other parameters (possibly delimited), such as the stress horizon, stress ratio, shape of the spectrum, stress-concentration factor, and size of the structural component, can remain variable. Further pertinent examples will be provided.

The mean value for a calculation group is calculated as follows:

$$\log D_{50,CG} = \frac{1}{n} \sum_{x=1}^n \log D_{i50,x} \tag{9.17}$$

$x$  – Ordinal number of the test series

$n$  – Number of test series

The standard deviation for the calculation group is given by

$$s_{CG} = \sqrt{\frac{1}{n-1} \sum_{x=1}^n (\log D_{i,x,50} - \log D_{CG,50})^2} \quad (9.18)$$

and the range of scatter is

$$T_{D,CG} = \frac{D_{CG,90}}{D_{CG,10}} \quad (9.19)$$

If a log-normal distribution is assumed for the damage sum, the following is obtained:

$$\log T_{D,CG} = 2.56 \cdot s_{CG} \quad (9.20)$$

For specific cases of application, the mean value and the scatter of the damage sum can thus be determined for calculation groups. This procedure is illustrated for 435 test series and for three Palmgren-Miner modifications in Fig. 9.8. In this case, the test samples are predominantly rod specimens. The stress-concentration factors range between 1 and 7.6. The amplitude spectra employed for the calculation of the fatigue life were determined by rainflow counting and amplitude transformation [Marq04]. The three distributions plotted in the figure indicate that a log-normal distribution can be assumed for the damage sum  $D_{i,50,x}$  with the three modifications. The mean values differ decidedly with the modifications. The ranges of scatter  $D_{90}/D_{10}$  extend over one decade. That is, the agreement between calculation and experiment is not satisfactory. Possible causes of this disagreement may be insufficient accuracy in determining the damage with the damage-accumulation hypotheses, characteristic functions which are not always appropriate for the calculation (estimated value), as well as inaccuracies in performing the tests. For determining the mean values and scatter for calculation groups, the individual values can also be weighted in correspondence with the extent of the random sampling; however, this procedure is not illustrated here.

In the following, a comprehensive collection of data and results of evaluations is presented as examples for illustrating the extent to which different parameters affect the mean value and the scatter of the damage sum for a calculation group [Euli94]. For this purpose, 964 test horizons for fatigue-strength tests are considered. The materials under investigation include steel, aluminium alloys, titanium alloys, cast iron, and cast aluminium. Most of the tests considered were performed on rod samples. In addition, a few results of tests performed on structural components are presented. Typical parameters include the type of stress, the stress-concentration factor, the stress ratio, the shape of the spectrum, the irregularity factor, the counting method, and the type of Palmgren-Miner modification.

First of all, the effects of the counting methods and of the Palmgren-Miner modification are considered, see also [Schü73, Zenn74, Euli94, Euli97], for instance. In Table 9.1, the mean damage sum and the range of scatter are indicated for the 964 test horizons.



**Table 9.1** Mean value and range of scatter of the damage sums for various Palmgren-Miner modifications and counting methods; calculation group: Fe-, Al-, Ti alloy [Euli94]

n = 964 test horizons	D <sub>50,CG</sub>	T <sub>D, CG</sub>
Palmgren-Miner elementary		
Rainflow	0.39	12.3
Range-pair	0.37	14.7
Level-crossing	0.81	24.2
Palmgren-Miner modification (Haibach)		
Rainflow	0.28	12.6
Range-pair	0.26	15.1
Level-crossing	0.61	26.0
Palmgren-Miner consequent		
Rainflow	0.29	12.7
Range-pair	0.27	15.3
Level-crossing	0.63	25.9
Palmgren-Miner modification Liu/Zenner		
Rainflow	0.75	9.2
Range-pair	0.70	11.8
Level-crossing	1.56	18.0

For calculation, four Palmgren-Miner modifications were applied, and three counting methods were evaluated. The mean damage sum can be employed as a criterion for determining whether the result of the calculation is on the safe or unsafe side. Damage sums < 1 indicate “unsafe”; that is, the calculated fatigue life is greater than the experimental value. Damage sums > 1 indicate “safe”; that is, the calculated fatigue life is lower than the experimental value. The range of scatter T<sub>D</sub> is a measure of the agreement between the calculated and experimental results. A larger range of scatter implies greater deviations.

### 9.3.1 Results

For the elementary Miner modification, all of the modified mean damage sums are consistently situated on the unsafe side. This result has been recognised in the FKM Guideline for Analytical Strength Assessment, where the calculation is based on damage sums  $D < 1$  for specific applications, [FKM03, FKM12]. In publications, it has often been claimed that the elementary modification is too conservative. This opinion is not confirmed by the values given in Table 9.1.

As expected, the mean damage sum is higher for level-crossing counting than for range-pair and rainflow counting. The range of scatter is very large for level-crossing counting. This is the reason why this counting method is not recommended for analytical fatigue-life predictions.

The mean damage sums for rainflow and range-pair counting differ only slightly, although the range of scatter is somewhat larger for range-pair counting, that is, without amplitude transformation.

With the Liu/Zenner modification, the mean damage sum is closest to the theoretical value 1, and the range of scatter is somewhat smaller than that for the other modifications [Gude99].

A proposal which was developed by Lipp and Svenson [Lipp67] in 1967 and which still appears in publications today [Kauf94, Sons07] should also be mentioned: If the difference between the frequency distributions for range-pair and level-crossing counting is larger than 1:3 (cumulative frequency), a geometric mean should be calculated from the two distributions. For this new frequency distribution, a damage sum which exceeds 0.5 is not permissible. By means of this procedure, the effect of additional damage by mean-stress variations should be taken into account, see also [Buxb92].

Since the level-crossing counting method is not recommended for estimating the fatigue life, Eulitz has examined the possibility of obtaining better results with the proposed geometrical averaging of the range-pair and level-crossing counting spectra for stress-time functions with mean-load variations [Euli99]. For this purpose, 79 test horizons have been evaluated. The associated stress-time functions were classified in the mean by rainflow counting (with amplitude transformation), by range-pair counting, by level-crossing counting, and by averaging for the latter two counting methods. The damage sum was calculated with the consequent Palmgren-Miner modification. The results are presented in Table 9.2.

Rainflow and range-pair counting yield similar mean values. The amplitude transformation results in a decrease of the scatter. As expected, level-crossing counting yields a high mean value with extreme scatter. The averaged results from range-pair and level-crossing counting exhibit significantly more scatter than rainflow and range-pair counting. Consequently, this approach is not recommended for an analytical fatigue-life prediction, [Euli99].

The effect of the material is illustrated for the examples of steel, aluminium, and cast iron for two Palmgren-Miner modifications in Table 9.3, [Euli94].

Larger differences in the mean value and range of scatter are caused by stresses of different types: tension-compression, flat bending, and torsion, Table 9.4. However, the extent of random sampling is small in the case of torsion. Consequently, this result is applicable only with reservations.

As indicated in [Hink10], the evaluation of comprehensive test data has shown that the damage sum depends on the shape of the spectrum. This effect can be described by means of the degree of fullness and the slope of the S-N curve. By regressive adaptation, the scatter of the fatigue-life estimate can thus be decreased.

**Table 9.2** Effect of the counting method on the mean value of damage sum and the associated range of scatter by calculation with the consequent Palmgren-Miner modification [Euli99]

Rainflow counting with transformation		Range-pair counting		Level-crossing counting		Mean for range-pair + level crossing	
D <sub>50,CG</sub>	T <sub>D, CG</sub>	D <sub>50,CG</sub>	T <sub>D, CG</sub>	D <sub>50,CG</sub>	T <sub>D, CG</sub>	D <sub>50,CG</sub>	T <sub>D, CG</sub>
0.53	29	0.50	38	1.91	211	1.33	82

**Table 9.3** Effect of the material on the mean value and range of scatter for the damage sum, calculation group for all test horizons [Euli94]

	Steel n = 525		Aluminium alloy n = 332		Cast iron n = 88	
	D <sub>50,CG</sub>	T <sub>D, CG</sub>	D <sub>50,CG</sub>	T <sub>D, CG</sub>	D <sub>50,CG</sub>	T <sub>D, CG</sub>
Palmgren-Miner consequent	0.24	10.3	0.34	15.2	0.38	13.8
Palmgren-Miner modified by Liu and Zenner	0.64	8.7	0.79	8.2	1.25	10.2

**Table 9.4** Effect of the stress type on the mean value and range of scatter [Euli94]

	Tension-compression n = 463		Flat bending n = 422		Torsion n = 21	
	D <sub>50,CG</sub>	T <sub>D, CG</sub>	D <sub>50,CG</sub>	T <sub>D, CG</sub>	D <sub>50,CG</sub>	T <sub>D, CG</sub>
Palmgren-Miner consequent	0.25	11.1	0.36	13.5	0.13	3.6
Palmgren-Miner modified by Liu and Zenner	0.65	8.4	0.90	9.8	0.48	7.2

**Table 9.5** Effective damage sum D<sub>eff</sub> in accordance with the FKM Guideline

	Unwelded structural components D <sub>eff</sub>	Welded structural components D <sub>eff</sub>
Steel, GS, Al	0.3	0.5
GGG, GT, GG	1.0	1.0

### 9.4 Relative Palmgren-Miner Rule

If the mean values of the damage sums differ for various parameters, as indicated in Tables 9.1, 9.2, 9.3, and 9.4, an appropriate adaptation of the calculated value to match the experimental value can be attempted. For this purpose, a representative value for an application profile can be employed, rather than the theoretical value of the damage sum 1. If suitable data are available, so-called effective damage sums D<sub>eff</sub> can be derived from the mean values of the damage sums for the calculation groups. An approach of this kind makes sense if similar stress-time functions and stress spectra as well as similar structural-component parameters are available, as already demonstrated in [Schü72].

In the series of technical rules, for instance, in the FKM Guideline, this approach is also adopted by introducing a “tolerable Miner sum” [FKM03, FKM12], which can be understood as an effective damage sum, as shown in Table 9.5.

The term “relative fatigue-life estimate” is also employed in some cases where an estimate of the absolute fatigue life is not necessary or intended. Instead, it may be necessary to determine the manner in which the fatigue life varies if individual structural-component parameters vary. For instance, the material strength or the geometry of the structural component may vary. Especially during the development process, relative information is important in the course of development loops for optimising the geometry of structural components or for a change in the selection of materials. For this purpose, it may be quite sufficient to correctly describe the tendency in the variation of the fatigue life with variations of the parameters. Decisive-parameter models which describe the S-N curve for a structural component as a function of various geometrical and technological effects are of paramount importance for assessing the changes in geometry or material, see for instance [Hück83, Berg99, FKM03, FKM12]. The results of investigations indicate that the effect on the fatigue life can be overestimated, especially in optimising the geometry of structural components subject to high stress [Pött03].

As a matter of principle, an application of the relative fatigue-life estimate yields correct results only if the parameters are similar to those for which the reference value has been determined. Such an appraisal can be difficult. In any case, a similar development of the progressing damage is a prerequisite.

As new test results continuously become available, a more comprehensive data base will accrue. Thus, the generation of calculation groups as well as the reliability of the effective damage sums can be improved.

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# Chapter 10

## Design and Dimensioning Load Spectra

For designing and dimensioning a structural component or a structure, a design and dimensioning spectrum which covers the design life is required. The data which are acquired from a measurement constitute only an initial basis. Since the measurement extends only over a short period of time in comparison with the design life, an extrapolation is necessary. In particular, however, a preview over the so-called operational profile during the design life is necessary. If it is assumed that different operational states can occur, an assumption is also required for determining the frequency with which these states occur, for example start and stop procedures, idle time, or different states of operation. For making an assumption of this kind, many years of operational experience are required; the results of a single measurement are often not sufficient. For this purpose, partial spectra can be analysed for individual operational states, and the associated damage content can be evaluated by means of a Palmgren-Miner calculation. The purpose of the fatigue-life estimate is to obtain a relative estimate of the damage contribution from each state of operation, rather than a qualitative lifetime estimate. This can be accomplished with the use of a fictitious S-N curve, for instance, with the slope  $k = 5$  for bending and  $k = 8$  for torsion (Miner elementary, steel component). In correspondence with the severity of the risk associated with a case of damage, the operational profile can be varied in such a way that the load assumption results in a reasonable partial safety factor. A second partial safety factor results from the specification of the tolerable loading capacity (the strength).

For machines and equipment which are manufactured in large series and which are employed in very different ways by the customer, the load assumption is often especially difficult. Bicycles, for instance, mountain bikes, are a prime example of such a problem complex. The loads acting on the bicycle fork, for example, depend on the weight of the cyclist, his manner of riding, on the type of terrain, and on details of construction such as the spring suspension. For which cyclist (recreation, not sport) is the bicycle to be designed and dimensioned? In this case, too, experience in the field is ultimately decisive for designing and dimensioning.

In the case of failure, material and production defects as well as faulty maintenance are often the first expected cause. Upon closer examination of the statistics on cases of failure, however, it becomes evident that many failures are the result of an incorrect load assumption and an erroneous determination of the local stresses. In plant designing, for example, it may be difficult to subsequently determine the load which actually occurred in a case of damage.

Furthermore, variations which occur during service of a machine or facility may not have been considered in designing and dimensioning. If a long-distance aircraft is inadvertently employed for short-haul traffic, the more rapid sequence of severe ground-air-ground load alternations, which occur during every flight, will contribute more frequently to the damage. If sheet metal is rolled at lower temperatures for technological reasons, the stresses at the drive train of the steel mill will be higher. In recent years it has become obvious that the load assumption for power plants is no longer in line with the changing operational requirements. Because of the increased share of renewable energy sources, conventional coal- and gas-fired power plants undergo more frequent start- and stop processes, which result in severe loads that had not been considered in the calculations.

These examples are intended to show that the load assumption, which is decisive for designing and dimensioning, can be difficult. Reliable designing and dimensioning for ensuring fatigue strength under variable stress amplitude is possible only with a reliable load assumption [Sons07, Brun09, Webe10].

## 10.1 Determination of Representative Load Spectra

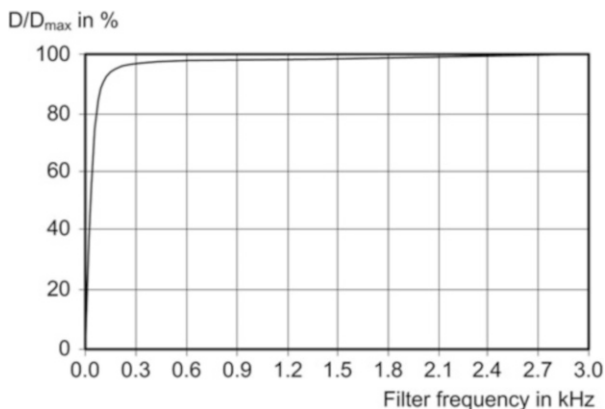
In general, stress spectra are derived from measurements which are performed under operational conditions. In most cases, the duration of operational measurements is only a fraction of the operating or design life. In many cases, it also is not possible to completely adapt the spectrum of individual operational states during the operational measurement to match the real operational conditions. In the following, the procedure for determining representative spectra despite the indicated limitations is described. In Chap. 8, rainflow counting has already been designated as the recognised counting method for determining the fatigue strength under variable stress amplitude. The information presented in the present chapter is referred essentially to rainflow counting or to range-pair counting derived from rainflow counting.

### 10.1.1 Definition of the Relevant Measuring Frequency

With the application of modern measuring systems with computer-aided digital data acquisition and recording, it is now possible to continuously sample a large number of measuring channels with a high signal frequency over an extended



**Fig. 10.1** Example illustrating the dependence of the damage sum  $D$  on the filter frequency



period of time. Because of limitations on memory capacity and also for simplifying further processing, however, the sampling rate and thus the volume of data must still be limited to a reasonable magnitude. A possible means of determining the necessary signal frequency has been proposed by S. Jenne [Jenn04]: Data are measured and recorded at a high sampling rate during a short interval at the beginning of a measurement under operational conditions. If at all possible, it should be ensured that all expected signal frequencies occur during this interval. In the course of the subsequent evaluation, the measured signals are fed to low-pass filters of decreasing frequency. From the differently filtered stress-time functions, rainflow matrices are determined, and the results of the range-pair counts are then extracted from these matrices. By means of a damage calculation, a damage sum  $D$  is determined for each range-pair count. The result is a curve which describes the increase in the damage sum with increasing filter frequency. An example of such a curve is plotted in Fig. 10.1. The variation described by this curve depends essentially on the frequency content of the signal to be measured.

In the present example, a filter frequency of 0.2 kHz was specified for the measurement. At this frequency, 96% of the maximal damage sum is attained. The sampling rate necessary for this filter frequency must be specified in accordance with the rules of the Nyquist sampling theorem [Nyqu28]. A prerequisite for the procedure described here is that no appreciably higher frequency components occur during the following measuring intervals.

### ***10.1.2 Recommendations for the Generation of Measuring Spectra***

In the vast majority of applications, it is not possible to perform a measurement under operational conditions in such a way that the distribution of the operational

states relevant to the stress correspond completely to the actual operating conditions. However, an overall spectrum can be generated from the totality of the measured data without weighting of the individual operational states only if this is the case. Under the given boundary conditions for a measurement under operational conditions, the required operational spectrum often cannot be simulated during the measurement. In this case, the measurement must be examined for relevant, decisive parameters, and partial spectra must be generated. After completion of the measurement under operational conditions, the resulting data are classified in correspondence with the loading condition, direction of travel, and roadway conditions, for example. Subsequently, a partial spectrum is determined for each operating or operational condition [Buxb79, Klät85].

Among other advantages, a separate evaluation for each partial spectrum offers the possibility of clearly illustrating the effect of certain operational parameters on the stress level with the use of the partial spectra.

For generating the complete spectrum, knowledge of the required design life or operating lifetime is necessary. In this context, the complete spectrum is a mixture of the partial spectra on the basis of the expected operational conditions [Zenn88, Zenn89, Heul98]. Thus, the task is to determine which operational state will occur during the design lifetime and how often it will occur. The decisive factor may be expressed in units such as kilometers (railway vehicles, motor vehicles), rotational speed in  $\text{min}^{-1}$  (rotating devices), time (power plants), number of flights (aircraft), or number of articles produced, etc. The necessary data can often be obtained from production statistics, observation on driver's cabs, performance specifications, or records from customer vehicles. In the following table, the manner in which the extension factor can be determined for various partial spectra is illustrated for the example of a railway vehicle, Table 10.1.

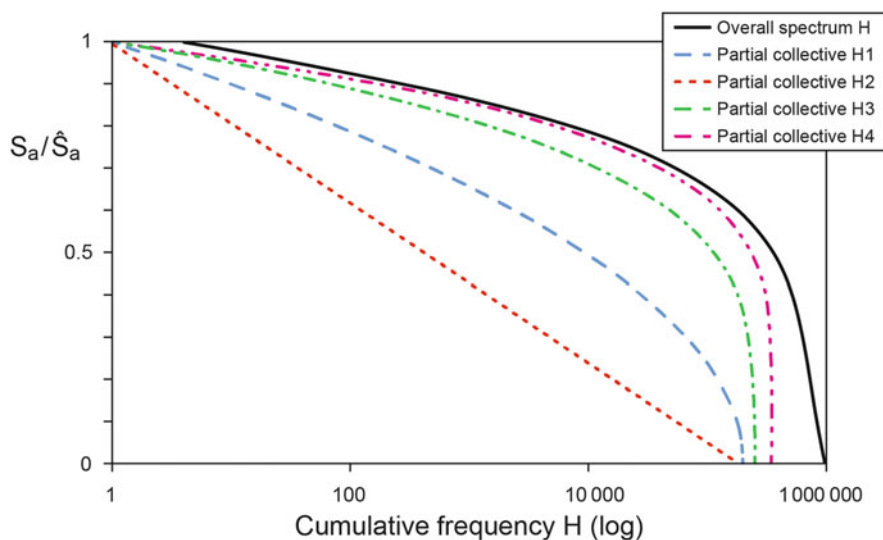
The frequencies of the counting events for the partial spectra are multiplied by the extension factors. The overall spectrum is calculated by adding the frequencies of the extended partial spectra, see Fig. 10.2.

Because of the logarithmic plot of the cumulative frequency, the overall spectrum corresponds approximately to an envelope. If the shares of the partial spectra in the overall spectrum are varied, the effects of a possible future change in the application can be estimated. Furthermore, the share of the most severe spectrum is often increased, in order to obtain a more severe overall spectrum for reasons of safety and reliability.

If an overall spectrum is employed for the load assumption, corresponding safety aspects must be considered in weighting the partial spectra (partial safety factor for the stress).

**Table 10.1** Composition of a spectrum with a design life of 1,000,000 km

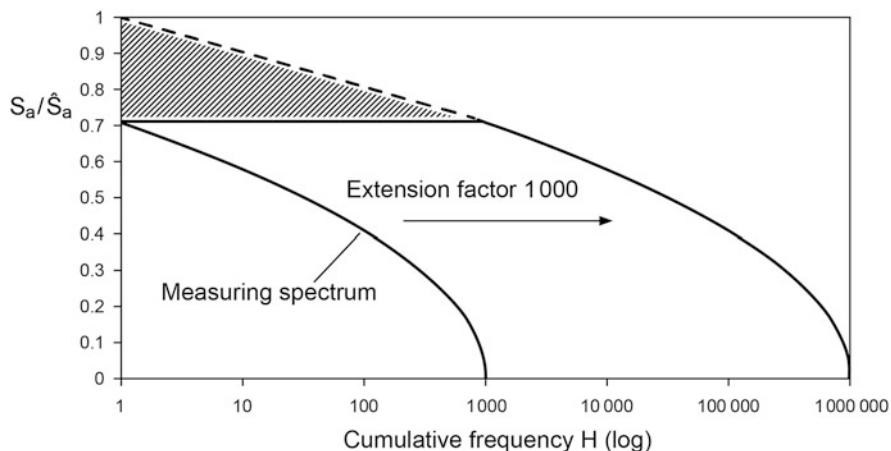
	Duration of measurement, km	Share in the overall spectrum, %	Share in the overall spectrum, km	Extension factor, $t_i$
Partial spectrum $A_1$	200	20	200,000	1000
Partial spectrum $A_2$	500	20	200,000	400
Partial spectrum $A_3$	10,000	25	250,000	25
Partial spectrum $A_4$	20,000	35	350,000	17.5



**Fig. 10.2** Generation of an overall spectrum from partial spectra

### 10.1.3 Determination of the Necessary Duration of Measurement

In the example considered, Table 10.1, an extension factor of 1000 is given for partial spectrum 1. The question may be raised as to how large the extension factor should be, as a maximum, and how comprehensive the measurement should be, at the least. A generalised answer to this question is not possible, since the maximal permissible extension factor depends essentially on the scatter of the stress level during the design life. In the case of large extension factors, the amplitudes of the



**Fig. 10.3** Extension of a range-pair count

stresses are often increased by a certain factor for ensuring a sufficiently high statistical confidence level for the frequency distributions. The question of the maximal permissible extension factor is considered on the basis of a fictitious range-pair count for the following example in Fig. 10.3.

The frequencies of the measuring spectrum were multiplied by the extension factor 1000. The result is an extended spectrum with the same maximum as that of the measuring spectrum, but with a parallel displacement to the right. The dashed line represents the possible extension of the real spectrum for the duration of a measurement which has been prolonged by the extension factor. The question of extrapolation has already been considered for spectral maxima in Sect. 4.2.

The effect of a short duration of a measurement or of a large extension factor on the overall spectrum can be estimated on the basis of a relative linear damage calculation. The slope of the S-N curve (not the position) affects the result of the calculation.

For a flat S-N curve (for instance, for torsion) the damage component of the cross-hatched area will be higher than that for a steep S-N curve (for instance, for bending), see Fig. 10.4 [Jung93]. In many cases, the damage sum for the extended measuring spectrum, including the cross-hatched area, will not differ appreciably from the damage sum for the measuring spectrum with a parallel displacement. This implies that the duration of the measurement was sufficient. This conclusion must be limited for the case in which the spectrum maximum which actually occurs is higher than the permissible static strength value.

During customer measurements on motor vehicles, it is assumed that a measurement yields statistically stable values after about 10,000 km. In this context, it must be remembered that this value depends decisively on the load parameter under consideration (longitudinal, transverse, or vertical force) and on the operational profile of the motor vehicle [Pött11]. In such cases, a measurement for a single, individual customer is not sufficient. In [Hors02] it is pointed out that some 2000

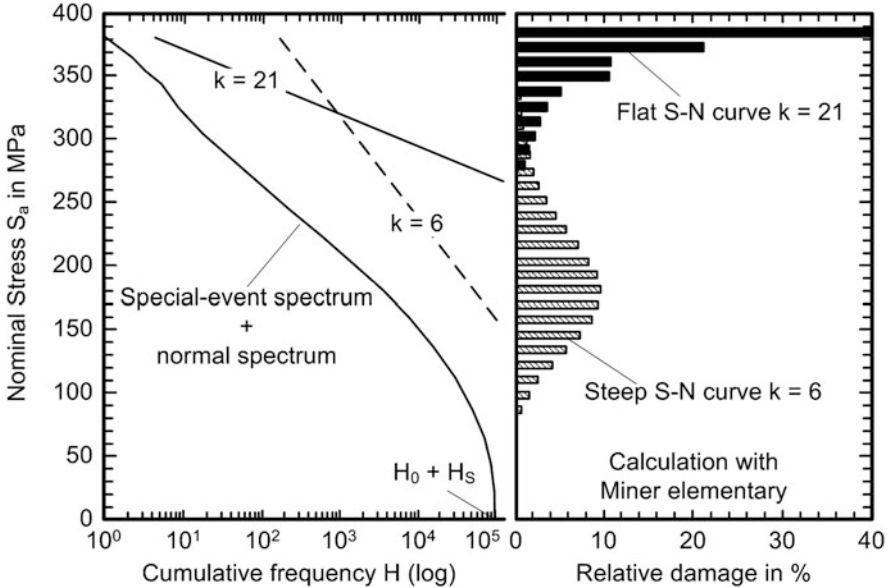


Fig. 10.4 Relative damage of the horizon as a function of the slope of the S-N curve

persons are interrogated for demoscopic opinion polls, in order to reach a conclusion with a sufficiently high statistical confidence level.

### 10.1.4 Recommendations for the Evaluation of Comprehensive Measurements Under Operational Conditions

The following recommendations are mostly the result of measurements performed on motor vehicles. Comprehensive measurements under operational conditions with more than 100 measuring channels over a period of several weeks are not unusual. For the evaluation of such comprehensive measurements, automated sequences are indispensable. The possibilities offered by the various computer programs which are available for evaluation differ considerably; hence, only basic recommendations for the procedure can be provided here:

- During the measurement, comprehensive documentation must be prepared for assigning the measured data intervals to the individual partial spectra.
- Prior to cycle counts, all measured data must be screened, and interference, such as spikes, noise, etc. must be eliminated from the measured signals.
- The maxima for the partial spectra should be determinable from the variation of the measured data with time and should be documented for plausibility control.

Moreover, an attempt should be made to identify the operational states for which the extreme stresses have occurred.

- The extraction of the results for one-parameter counting methods (level-crossing and range-pair counting) from the rainflow matrix should be performed only with the overall spectra. An extension of the one-parameter partial spectra and subsequent addition to the overall spectrum is of course possible, but results in a loss of information for the extraction.
- Equal level intervals should be selected for similar groups of load signals. They should not be specified individually for each load signal on the basis of maxima and minima during the measurement. If the measuring ranges for similar measuring points are very different, it is often advisable to substantially increase the number of classes, rather than to specify the parameter of the counting method individually. The parameters of a load signal must be equal for every partial spectrum, if they are to be combined to form an overall spectrum.

## 10.2 Example for the Generation of Design and Dimensioning Spectra

If different operational states occur, and if the partial spectra for these states are available, the problem of synthetically constructing an overall spectrum (mixed spectrum) must be solved, if this spectrum is to constitute the basis for designing and dimensioning. The individual steps are indicated in the following:

1. Specification of the total design life  $t_N$   
For instance, the design life may be 20 years for a machine in a processing plant 300,000 km for a motor vehicle, or 70,000 flights for a commercial aircraft.
2. Definition of the operational states or operating processes  $A_i$   
These must include all possible procedures during the design life.
3. Generation of the partial spectra  $H(B)_i$  (frequency of the stress) for the operational states or operating processes by load measuring, multibody simulation or on the basis of experience with similar types of machines
4. Weighting of the operational states or operating processes  $A_i$  by a weighting factor  $a_i$   
The weighting factor is determined from the share in the design life.
5. Extrapolation of the partial spectra for the duration of the measurement  $t_M$ ,  $H(B, t_M)$  to the respective share  $a_i$  in the total design life  $t_N$   
 $H_i(B, t_i)$  with  $t_i = a_i t_N$
6. Specification of special events (magnitude of the stress and frequency)
7. Superposition of the partial spectra to form the overall spectrum as described in Sect. 10.1.2

$$H(B, t_N) = \sum_{i=1}^k H_i(B, t_i) \quad (10.1)$$

### 10.3 Different Procedures for Load Assumptions

The load assumption, designing and dimensioning of structural components and systems, as well as the verification of reliability, depend on the specific circumstances which prevail for the respective operation or application. In general, safety factors are defined as functions of the hazard potential and of a risk-consequence estimate. Moreover, a number of differences exist. A few examples of such differences are indicated in the following.

- Are the load assumption and, if appropriate, the process of designing and dimensioning, based on series of technical rules, standards, and laws?

This is the case for naval construction, rail-vehicle construction, or aircraft construction, for instance, but not for the automotive or motorcycle industry.

- Is the development evolutionary or revolutionary? In other words, is experience or the result of measurement or of field observation available?

In the case of an evolutionary development, experience is available as a basis for the load assumption and for the specification of load spectra. In the case of a revolutionary development, the occurrence of load cases not previously considered in operations must be assumed.

- Can or must numerical methods be applied for the load assumption?

Despite rapid propagation, numerous load situations cannot be analysed with sufficient accuracy with the application of numerical methods, such as the multibody simulation. On the other hand, these methods offer numerous possibilities for the parameter study and for optimising.

- Is the operational profile of the product known for the entire design life? Does the operation proceed under unambiguously defined conditions? Can changes in application which can result in more severe loads during later operation be excluded with certainty?

In general, the extent to which possible load variations (magnitude and frequency  $h_i$  of the amplitudes) affect the design and dimensioning of structural components can be roughly estimated by means of an analytical fatigue-life estimate.

- Are maintenance and inspection considered within the scope of the design, dimensioning, and safety-verification philosophy?

In the case of aircraft, maintenance and inspection at regular intervals are specified from the start for “fail-safe” or “damage-tolerance” designing. In the automotive industry, however, operational safety and reliability must be ensured

even without proper maintenance of the motor vehicle. Consequently, the term “safe-life” designing is employed in this case, see Sect. 11.6.

- Is the load assumption referred to a verification test with due consideration of the test procedure, the scatter for the structural component, etc., or is it referred to an analytical strength assessment on the basis of reliable strength characteristics for the material or structural component, or both?

For performing and evaluating verification tests, simplifications are often necessary, in comparison with the real operational states which occur during operation. These simplifications must be taken into account by increasing the test loads in correspondence with a supplemental safety factor. Furthermore, these tests are usually performed on one or only a few test samples of the structural component concerned. If the strength assessment based on the tests with these structural components is intended to be representative for later series production, the load assumption must take the expected scatter of the strength for these components into account. The process of designing, dimensioning, and ensuring the required safety and reliability is decisive for this purpose. Consequently, the load assumption must always be considered in relation to the strength properties taken as basis for the strength analysis.

- Does the overall product development comprise several development loops? Are prototypes constructed and tested? Are the results obtained with these prototypes employed in the next development loop, or is the first prototype the product?

For large capital goods, such as power plants, steel works, or cement plants, etc., the assumptions taken as basis for designing and dimensioning cannot be checked before the start of operation. Hence, the assumptions must be sufficiently conservative.

In Chap. 12, examples are presented for making load assumptions for various applications. The obvious differences in procedure are justified by the aforementioned differences in the circumstances given by the different cases of application.

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# Chapter 11

## Safety Aspects

In this chapter aspects which are vital for the safety of structural components are considered. The discussion begins with the failure criteria which must be satisfied by a design for ensuring the necessary fatigue strength under variable stress amplitude as well as the respective characteristic parameters and characteristic functions for the loading capacity. The conventional safety concept and the reliability concept are described. Since the scatter of the load acting on a component and its strength decisively affects the probability of failure, this scatter is explained and discussed with the use of examples. Finally, the importance of safe and reliable designing is considered in a more general context. For this purpose, cases of damage of a component and consequential failure are described in detail, and possible means of avoiding or at least limiting such damage are discussed. Furthermore, the social acceptance factor is considered as applied to the complex process of safe designing and dimensioning of structural components and systems for ensuring safe and reliable operation.

### 11.1 Designing and Dimensioning of Structural Components

Designing and dimensioning of structural components, structures, and systems are based on a comparison of the load and the strength. The strength of a structural component must be greater than the load which acts on the component. For certification of the fatigue strength under variable stress amplitude, both the static stress and the dynamic fatigue stress must be considered. In the case of dynamic fatigue stress, a distinction is made between endurance-strength designing and designing for ensuring fatigue strength under variable stress amplitude for a given design life. For this purpose, failure of the structural component can be determined on the basis of various criteria.

In the case of **static designing and dimensioning**, damage criteria include forced ductile or brittle fracture, buckling, severe permanent distortion (bending, torsion, buckling, and bulging), local plastic deformation, or crack initiation (for instance, in case-hardened structural components).

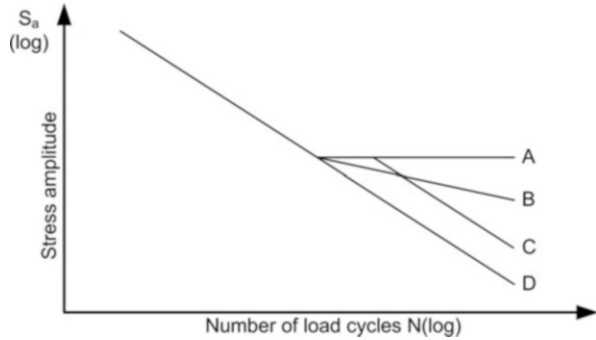
**Endurance-strength designing and dimensioning** is applicable to structural components which attain a large number of load cycles during their design life, for instance,  $N > 10^7$ , with a spectrum whose shape approaches a rectangle, which represents a vibrating load with constant amplitude. Examples include engine components such as connecting rods and crankshafts, wheel-set axles for rail vehicles, shafts for stationary turbines, as well as springs. The purpose of endurance-strength designing and dimensioning is to avoid a fatigue fracture. As a rule, a fatigue fracture occurs as a result of a process which comprises several phases: from dislocation motion to microcrack nucleation, from microcrack growth to macrocrack initiation, from macrocrack propagation to residual fracture. The fatigue limit can therefore be understood as the maximal amplitude for which no crack initiation process has yet been induced, or for which existing cracks do not propagate. For more information on the damaging processes from a more historical point of view, refer to the literature [[Hemp61](#), [Mans64](#), [Mill87](#), [Mura05](#)].

On the other hand, for metallic materials with a face-centred cubic crystal microstructure, such as austenitic steels and aluminium alloys, it has been known for a long time that the tolerable stress amplitude can decrease continuously with increasing number of load cycles. In other words, the fatigue strength does not attain a constant value. Furthermore, the crack-initiation process cannot be decisive for the existence of a fatigue-strength value, if crack-like defects already exist in a structural component in its original condition. As indicated by the results of recent investigations, the decrease in the tolerable amplitude in the high-cycle fatigue (HCF) range,  $N > 10^7$ , is not limited to materials with a face-centred cubic crystal lattice. It has also been observed in metallic materials with a body-centred cubic lattice and with a hexagonal lattice, for instance, ferritic/pearlitic steels and magnesium alloys [[Bath99](#), [Stan01](#), [Sons07](#), [Pytt11](#)]. This decrease occurs at temperatures far below the creep range and without direct corrosive action. In this context, a continuous decrease in the tolerable amplitude (curves B and D) must be distinguished from a stepwise decrease (curve C), that is, a further decrease after establishment of the conventional fatigue limit (curve A), as illustrated in Fig. 11.1, see [[Pytt11](#)]. This behaviour has been confirmed experimentally and depends on the material, the strength values, and on the condition of the material:

- A: for instance, steel, low-strength, Ck15
- B: for instance, aluminium alloy, EN AW 6082
- C: for instance, steel, high-strength, SUJ2
- D: for instance, steel, high-strength, 100Cr6

The stepwise decrease is explained by two different sites of crack initiation with different rates of crack propagation. At higher amplitudes, the crack usually starts at

**Fig. 11.1** Plot of S-N curves in the high-cycle-fatigue range, schematic



the surface and propagates from there. At low amplitudes, crack initiation can be caused by inhomogeneities within the volume of the material, see [Mugh06].

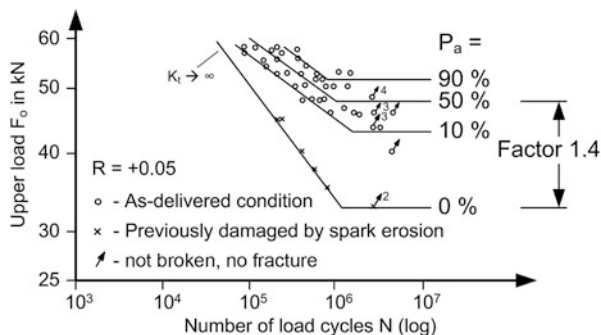
Numerous parameters affect the high-cycle fatigue characteristics of a structural component. These parameters include the microstructure of the material, the surface, notches, residual stresses, the type of stress, constant and variable amplitudes, stress ratio, and frequency, in combination with time-dependent processes (for instance, corrosion). At present, hardly any information is available on the manner in which these parameters act individually and synergistically. In particular, hardly any results of tests on structural components are yet available for the high-cycle fatigue range above  $10^8$  load cycles.

If structural components with cracks or crack-like defects are assumed, a fatigue limit in the macrocrack range is expected on the basis of fracture-mechanical considerations. This fatigue limit depends on the threshold value of the stress intensity (fracture-mechanical fatigue limit), see for instance [Rada07, FKM09]. The same conclusion applies to microcracks for whose propagation a threshold value can likewise be given. In particular, the existence of cracks which do not propagate (crack arrest) in combination with microstructural hindrance (barriers) or in the presence of notches must be mentioned, see for instance [Rada07, Schi09]. Finally, it should also be pointed out that a fatigue crack in a structure can come to a standstill if deformation-controlled conditions prevail, that is, if the incipient change in stiffness results in relief at the tip of the crack (statically overdeterminate system).

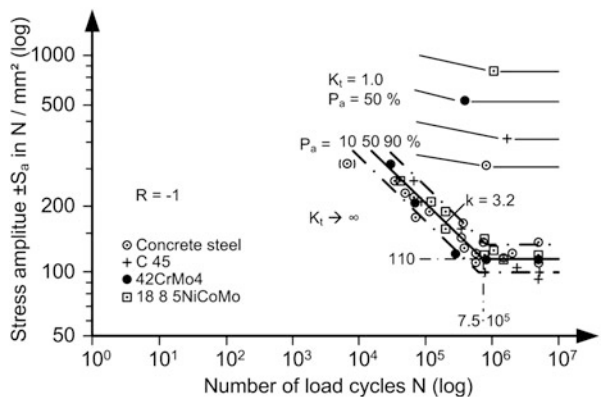
An interesting feature is the result of tests performed on samples and structural components which have been deliberately damaged with notches produced by spark erosion, [Hück84]. This procedure was assumed to generate a notch concentration factor,  $K_t \rightarrow \infty$ . The “worst” structural component should be characterised by these crack-like defects, which are associated with residual stresses. For connecting-rod screws, the results indicate that for a notch with  $K_t \rightarrow \infty$  the tolerable amplitude at the fatigue limit does not decrease to zero, Fig. 11.2.

Moreover, it should be noted that the decrease in strength for such a notch is independent of the material concerned (steels) and attains a common value, Fig. 11.3. A comparison with the mean values of the fatigue limit for the smooth

**Fig. 11.2** S-N curves for connecting-rod screws, principle of the “worst” structural component



**Fig. 11.3** Effect of the material for a smooth and an “infinitely” sharp notched sample



sample indicates that this decrease is greater with materials of higher strength. If this decrease is interpreted as the maximal scatter, this would imply that the scatter increases rapidly with increasing strength.

On the other hand, the scatter observed with a smooth sample differs considerably from that observed on a sample with crack-like defects, as shown in Fig. 11.3. If different causes are found for the variation of the S-N curve, as indicated in Fig. 11.2, more information on the respective scatter is necessary for an interpretation specifically related to applications.

On the one hand, a number of test results indicate that the tolerable amplitude decreases with increasing number of load cycles in the high-cycle fatigue range. On the other hand, however, a number of statistically well-founded S-N curves clearly indicate a fatigue limit all the way to the range with large numbers of load alternations ( $>10^7$ ). This observation implies that more extensive and stringent confidence testing is necessary for specific applications than in the past. In this context, the experimental strength analysis is of special importance. Furthermore, numerous machines have been and are still being safely operated in a range with very large numbers of load alternations, some of them over a period of decades.

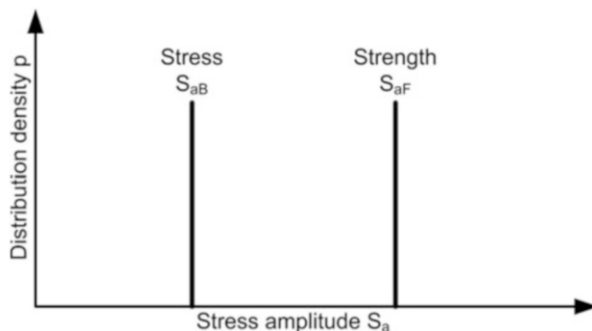
The detection of internal defects may present a fundamental problem if the process of fatigue is initiated by such defects, since the applicability of nondestructive materials-testing methods is limited in this case; see for instance [Rödl10]. Recommendations for lowering the S-N curve (specification of the number of load cycles at the deflection point and variation of slope per decade for the number of load cycles) in the high-cycle fatigue range are given in [Sons07] for steels, cast iron, wrought aluminium alloys, cast alloys, and magnesium alloys. A distinction is made between welded and non-welded materials.

**If designing is intended for ensuring the required fatigue life under variable stress amplitude**, amplitudes which exceed the fatigue limit are expressly allowed. This procedure, which was initiated by E. Gassner in 1938, has become established in all technical fields during the interim, first in the aircraft industry and then in the automotive industry. The decisive feature of this approach is the potential advantage for light-weight construction. Since loads which exceed the fatigue limit are permissible, the cross-sectional area of structural components can be considerably decreased. Further advantages include saving of resources and improved control capability for drive trains for instance. Structural components and constructions which have been designed for ensuring fatigue strength under variable stress amplitude do not necessarily attain the fatigue limit. As recognised by Gassner, however, it is not necessary to satisfy this condition if machine plants and means of transport quickly become obsolete because of continuing technical developments. Thus, the requirement for attaining an unlimited fatigue life is not in general tenable. (This view is justified from an economical, ecological, as well as safety-engineering standpoint.) Furthermore, some machines and devices, especially those subject to thermal-mechanical stresses, are subjected only to a relatively small number of load cycles during operation, for instance, the blades of steam turbines in power plants.

In designing for ensuring fatigue strength under variable stress amplitude, a simple comparison of characteristic parameters is not possible. Instead, so-called characteristic functions are employed for the loading capacity and for the stress. An S-N curve for the structural component, a strain S-N curve (Coffin-Manson) or a crack-propagation-rate curve (Paris straight line) can be employed for the loading capacity; see [Rada07], for instance. The load spectrum or frequency matrix can be employed for the stress. As dictated by the particular application, fracture due to fatigue, a loss of stiffness, a specific crack length, or macroscopic crack initiation can be employed as failure criterion. Designing and dimensioning are performed for a minimal fatigue life, which must be greater than the planned design life by a certain safety factor. Instead of the fatigue life in years or elapsed time of operation, reference values such as the number of load cycles, number of kilometres travelled, or number of flights can also be employed. In the series of technical rules, the intervals in the direction of load, that is, the loading capacity and the stress, rather than the tolerable and required fatigue life, are considered as the decisive criterion for designing and dimensioning, as shown in the subsequent safety consideration.

If an S-N curve for a structural component is taken as basis for dimensioning, a fatigue limit is considered only for the analytical fatigue-life estimate in accordance with the original Palmgren-Miner rule. That is, the assumption that amplitudes

**Fig. 11.4** Conventional safety concept of fatigue limit



below the fatigue limit do not contribute to the damage applies only in this case. As shown in Chap. 9, only modifications of the Palmgren-Miner rule for which the amplitudes below the fatigue limit also contribute to the damage are applied nowadays. If the tolerable amplitude can decrease in the high-cycle fatigue range, as illustrated in Fig. 11.1, a logical question concerns the manner in which this effect should be taken into account for fatigue-strength designing and dimensioning. With the modifications employed today, the decrease of the tolerable amplitudes is much greater than that indicated by the results of tests performed at constant amplitude. Hence, it may be stated that an effect of this kind is already taken into account by the modifications applied for fatigue-strength designing and dimensioning. In view of the fact that the modifications represent highly empirical procedures, this standpoint is certainly justified. These procedures are then appraised on the basis of their ability to decrease the failure and the scatter between calculation and experiment. For the reliability of the analytical fatigue-life prediction, the question of whether or not the mean value of the damage sum is equal to the theoretical value  $D = 1$  is only of secondary importance. The decisive criterion is the scatter of the damage sums. As also indicated in [Sons07], knowledge of the real S-N curve variation in the high-cycle-fatigue range is not necessary for fatigue-strength designing and dimensioning under variable stress amplitude.

## 11.2 Safety Concept

The essential challenge for reliable designing and dimensioning is the fact that both the loading capacity and the stress value are subject to scatter, regardless of whether characteristic values or characteristic functions are compared. In order to take this scatter and the associated uncertainty into account, a safety factor is therefore employed for designing and dimensioning.

In Fig. 11.4, the conventional safety concept for the fatigue limit is illustrated. The fatigue limit  $S_{aF} = S_{aD}$  is decidedly higher than the allowable stress amplitude

**Table 11.1** Safety factors for steel and wrought aluminium alloys

Safety factor		Consequences of damage		
		Severe	Moderate	Slight
Inspection at regular intervals	No	1.5	1.4	1.3
	Yes	1.35	1.25	1.2

of  $S_{aB}$ . ( $S_{aB}$  can be the amplitude of a constant-amplitude stress or the maximum of a cumulative load spectrum).

The safety factor is given by

$$S = \frac{S_{aF}}{S_{aB}} \quad (11.1)$$

Its value depends on the accuracy of the load assumption and the scatter of the loads, the scatter of the fatigue limit, and the risk upon occurrence of damage. The safety factors employed in practice are usually based on operational experience gained over the years and over decades. Representative data for the series of technical rules are the values indicated in the FKM Guideline, “Analytical Assessment of Fatigue Strength for Machine Components” [FKM12, Häne10]. These figures apply to unwelded structural components, Table 11.1.

The following explanation is given in the FKM Guideline [FKM12]: *The safety factors apply together with reliable load assumptions and a mean probability of survival  $P_{ii} = 97.5\%$  ( $P_{ii} = 1 - P_a$ ). The basic safety factor with reference to the fatigue limit under alternating stress is 1.5 for unwelded structural components. Under favourable conditions, the value can be decreased in correspondence with the inspection possibilities and the consequences of damage.*

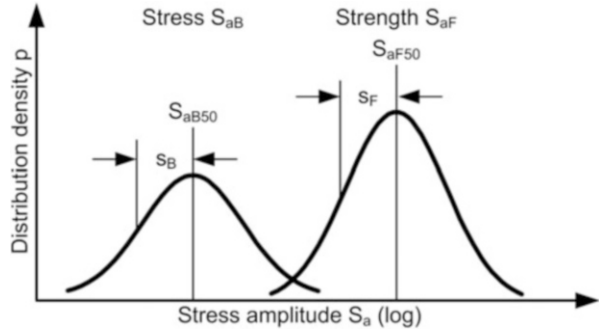
*Further safety factors are given for ductile cast iron materials (GS, GGG) in this FKM Guideline. For this purpose, a distinction is made between nondestructive and destructive testing. Information is also provided on safety factors for nonductile cast iron materials (GT, GG) and cast aluminium materials.*

## 11.3 Reliability Concept

If the strength and the stress are subject to scatter, the corresponding distribution-density functions may resemble those plotted in Fig. 11.5. This plot applies for the **fatigue strength**  $S_{aF}$  and the **service stress**  $S_{aB}$ . For this purpose, the value of the feature concerned is usually plotted on a logarithmic scale. The mean values  $S_{aF50} = S_{aD50}$  and  $S_{aB50}$  as well as the standard deviations  $s_F$  and  $s_B$  are indicated for the strength and stress. The mean value and standard deviation are independent of the distribution. That is, knowledge of the mean value and standard deviation from tests and measurements does not necessarily imply that the distribution function is known. For the following explanations of characteristic parameters



**Fig. 11.5** Reliability concept of fatigue limit



and reliability concepts as well as the associated considerations, a log-normal distribution is taken as basis for both the loading capacity and the stress.

Failure of a structural component will occur if  $S_{aB} > S_{aF}$ , that is, if a very high stress coincides with a “bad” structural component. For the two intersecting distributions, the probability of failure  $P_A$  (total probability of failure) can thus be calculated. For values of features with a log-normal distribution, a relative safety range  $u_0$  can first be calculated:

$$u_0 = \frac{\log S_{aF} - \log S_{aB}}{\sqrt{s_F^2 + s_B^2}} \tag{11.2}$$

This equation clearly indicates that the safety range increases if the difference between the mean values increases and if the standard deviations decrease. This result implies that the scatter of the loading capacity and the stress is described more transparently with the reliability concept than with the conventional safety concept. Moreover, the reliability concept yields a quantitative conclusion on failure. For this purpose, the mean values and the standard deviations must be known.

Failure occurs if the value of the difference  $z$  becomes negative:

$$z = \log S_{aF} - \log S_{aB} \tag{11.3}$$

The difference values likewise exhibit a normal distribution. The mean value of  $z$  is calculated from the spacing of the mean values:

$$m = \log S_{aF50} - \log S_{aB50} \tag{11.4}$$

The standard deviation of the difference values  $z$  is given by

$$s = \sqrt{s_F^2 + s_B^2} \tag{11.5}$$

After transformation to

$$u = \frac{z - m}{s} \tag{11.6}$$

the calculated probability of failure  $P_A$  is equal to the integral of the normal distribution

$$P_A = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{u_0} \exp\left(\frac{-u^2}{2}\right) du \tag{11.7}$$

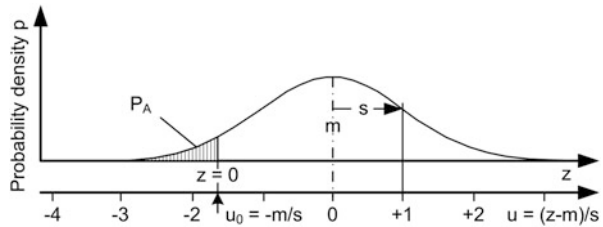
With the failure condition  $z = 0$ , the upper limit of integration is obtained

$$u_0 = \frac{-m}{S} \tag{11.8}$$

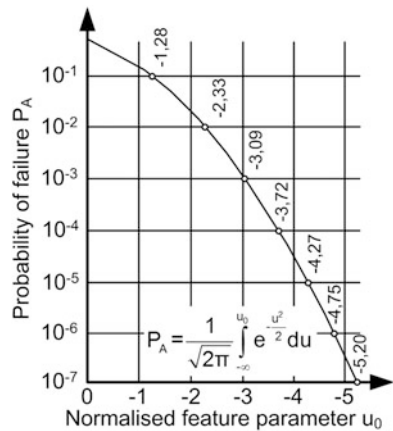
see Fig. 11.6 [Haib06].

For the relative value of the feature concerned  $u_0$ , the probability of failure  $P_A$  can be taken from Fig. 11.7. Appropriate tables are available in textbooks on statistics.

**Fig. 11.6** Calculated probability of failure for a log-normal distribution of fatigue strength and stress



**Fig. 11.7** Probability of failure  $P_A = f(u_0)$



A statistically justified safety factor  $j_S$  can thus be determined, in contrast to the safety factor  $S$  defined as

$$j_S = \frac{S_{aF50}}{S_{aB50}} \quad (11.9)$$

With  $\log j_S = m$  in accordance with Eq. (11.4) and with Eq. (11.8), the following is obtained:

$$\hat{j}_S = 10^{-u_0 \cdot s} \quad (11.10)$$

The determination of the failure probability  $P_A$  by a comparison of characteristic values, as demonstrated for the fatigue limit, is also applicable to the fatigue strength under variable stress amplitude. In this case, however, only spectra with the same spectrum shape and equal block length can be considered. With this limitation, and with a log-normal distribution for the loading capacity and for the stress, the same formulae can be applied as in the preceding discussion, provided that  $S_{aF}$  and  $S_{aB}$  are replaced by  $\hat{S}_F$  and  $\hat{S}_B$ , respectively. The stress-related safety factor is then given by

$$\hat{J}_S = \frac{\hat{S}_{aF50}}{\hat{S}_{aB50}} \quad (11.11)$$

If the required safety factor must be referred to the number of load cycles to failure, it can be obtained by a simple conversion with the use of the slope  $\hat{k}$ , provided that the Gassner curve is approximately a straight line in a double logarithmic grid.

$$\hat{J}_N = \hat{J}_S \hat{s}_k \quad (11.12)$$

As a matter of principle, safety factors can also be defined for a probability of failure and a probability of occurrence which are not equal to 50%, see [Seeg96, Rada07]. Critical comments concerning safety factors are given in Sect. 11.6.

For practical purposes, the effect of the scatter,  $s_F$  and  $s_B$ , can be assessed as follows. If the scatter ratio

$$v = \frac{s_B}{s_F} \quad (11.13)$$

is introduced, Eq. (11.3) can be transformed [Hück10]. The contribution of the scatter,  $s_B$  and  $s_F$ , to the total scatter  $s$  can thus be described as a function of  $v$ :

$$\left(\frac{s_B}{s}\right)^2 = \frac{v^2}{1+v^2} \quad (11.14)$$

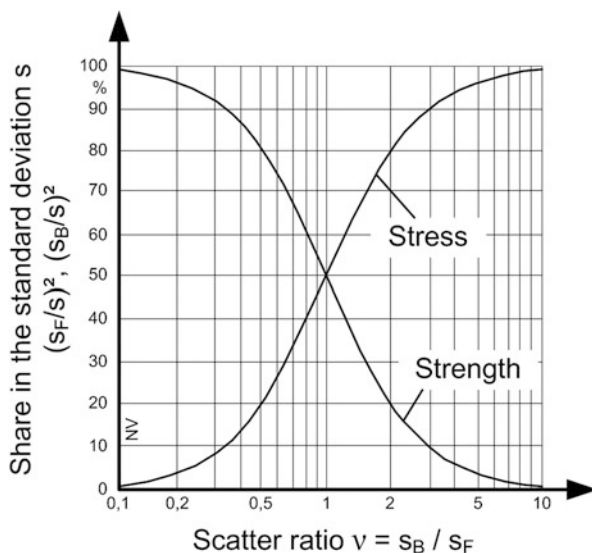
and

$$\left(\frac{s_F}{s}\right)^2 = \frac{1}{1 + \nu^2} \tag{11.15}$$

The variations of these curves are plotted in Fig. 11.8. If the scatter is the same for the strength and for the stress,  $\nu = 1$ , the corresponding contributions to the total scatter are equal. If the scatter of the stress increases, its contribution to the total scatter increases rapidly. On the other hand, the effect of the strength scatter decreases continuously. The case of increasing strength scatter is analogous. Thus, the total scatter which is decisive for the reliability concept can comprise various contributing components, and a higher value will result in a much larger contribution. For instance, the scatter of the stress caused by differences in driving habits, roadways, and payloads will be greater for chassis components than for engine components with a limitation on rotational speed of the engine. For a specific application, this result can thus be employed for determining whether or not the scatter of the strength can be appropriately decreased by specific measures.

The next question concerns the procedure to be adopted in the **general case of fatigue life under variable stress amplitude**. In this case, the scatter must be considered for the strength under fatigue stress, and the operational stresses of the spectrum also differ in shape and frequency, in addition to the maximum. For this purpose, the characteristic functions for the loading capacity and the stress can be transformed to a scalar value, that is, the damage sum, see also [Güth87] and [Zenn88]. For subsequently estimating a probability of failure  $P_A$ , the respective distribution functions of the damage sum  $D$  are required. As shown in Sects. 11.4 and 11.5, a log-normal distribution of the damage sum can be assumed for the evaluation of test data and measured data for the loading capacity as well as for the stress. Corresponding scatter values are given as standard deviations. In the

**Fig. 11.8** Effect of the scatter ratio on the share in the total scatter



following, this procedure is demonstrated for the nominal-stress concept, but the approach is also applicable to other concepts, such as the local-strain concept and the fracture-mechanical concept, as a matter of principle.

Let the loading capacity of a structural component be given by the S-N curve. With the use of a damage accumulation hypothesis, the fatigue life can be calculated for a given stress with variable amplitude. For this purpose, various Miner modifications are available, see Sect. 9.2. In the FKM Guideline “Analytical Stress Assessment for Machine Components” [FKM03, FKM12], for instance, the “elementary” and “consistent” modifications are recommended.

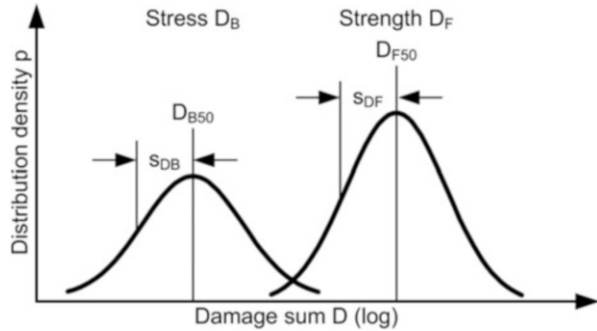
The fact that the calculated fatigue life deviates considerably from the experimentally determined fatigue life has been known for a long time. On the average, the ratio  $\hat{N}_{\text{exp}}/\hat{N}_{\text{calc}}$  and thus the damage sum  $D$  are lower than the theoretical value 1, see Sect. 9.3. Hence, in the FKM Guideline, a mean damage sum  $D = 0.3$  is proposed for the calculation in the case of unwelded steel structural components, for example. During the interim, experience with damage sums has been gained for numerous applications of various categories (material, type of stress, notch cases, manufacturing processes). On the basis of this experience, a mean value can be specified appropriately [Euli94, Euli97, Sons09, Hink10]. In the following, the mean value of such a representative damage sum is denoted by  $D_{F50\%}$ , and the standard deviation of this damage sum is denoted by  $s_{DF}$ .  $D_{F50\%}$  is determined from a large number of tests which belong to a calculation group, see Sect. 9.3. The mean value is calculated from the respective quotient,  $D_i = \hat{N}_{i,\text{exp}}/\hat{N}_{i,\text{calc}}$ , see Eq. (9.11), and depends on the respective Palmgren-Miner modification. The standard deviation is determined from the  $D_i$  values, Eq. (9.14). If the number of random samples employed for determining the standard deviation is small, only a coarse estimate will be obtained. Consequently, statistically more reliable values should be employed for the purpose, if at all possible; see Sect. 11.4.

If a series of stress spectra is available for designing and dimensioning of structural components, for instance, from measurements performed under operational conditions, a damage sum  $D_i$  can be calculated by damage accumulation for each spectrum. For this calculation, the same S-N curve and the same Palmgren-Miner modification must be employed as for the determination of  $D_{F50\%}$  and  $s_{DF}$ . From the logarithms of the damage sums, a mean value  $D_{B50}$  and the standard deviation  $s_{DB}$  can be determined, see Sect. 11.5.

In analogy with Fig. 11.5, the reliability concept can be indicated for the general case of fatigue strength under variable stress amplitude. For this purpose, the mean values and standard deviations for the fatigue strength and stress are replaced by the mean values and standard deviations of the damage sum for the fatigue strength and stress, see Fig. 11.9. In correspondence with Eq. (11.16), the calculation of the safety range  $u_0$  yields

$$u_0 = \frac{\log D_{F50} - \log D_{B50}}{\sqrt{s_{DF}^2 + s_{DB}^2}} \quad (11.16)$$

**Fig. 11.9** Reliability concept for the fatigue strength under variable stress amplitude in the general case



**Comments** This reliability concept for determining fatigue strength under variable stress amplitude involves an analytical fatigue-life estimate for the calculation of damage sums for the stress. As described in Chap. 9, this analytical fatigue-life estimate may be erroneous. Consequently, an experimental check on the damage sum to be employed for specific applications is recommended. For instance, fatigue strength tests can be performed on the corresponding structural component for different stress spectra.

For a normal distribution of density functions, a probability of failure  $P_A > 0$  always results, because the normal distribution is open on both sides. The question which logically arises, therefore, concerns the “very low” probability of failure to be associated with reliable designing and dimensioning, since a zero probability of failure does not exist [Mauc99]. In the past, probability distributions with a lower limit of  $P_a = 0\%$  (e.g. three-parameter Weibull-distribution) have repeatedly been proposed, for instance [Bert99]. However, this approach does not provide a solution to the problem, since the characteristic parameters for a probability  $P_a$ , or  $P_c = 0$ , or  $100\%$  are not known or cannot be assessed by experiment of other kind of observation. In the case of the loading capacity, for instance, tests usually comprise a number of specimens significantly less than 100. Therefore, intervals for  $P_a < 1\%$  are not occupied for the usual extent of random sampling. In an individual case, it may be possible to prove that the use of a specific distribution function yields a correct conclusion even for very low probabilities of failure. This can be achieved in a specific case, for example, by performing an extremely large number of tests. However, such a result cannot be generalised, since the scatter can be due to a very large number of highly different causes.

A similar situation applies to the probability of occurrence for very high stresses in measurement campaigns, since the number and duration of measurements under operational conditions are limited. Physical limits are an exception with respect to stresses. Examples include a stopping device which limits travel or path length, a slipping clutch for preventing overload, or a safety valve for preventing excess pressure. In such cases, stress maxima which are not exceeded can be assumed.

These comments are intended to emphasise the fact that limited distributions do not generally provide a satisfactory solution to the  $P_A > 0$  problem, that is, a total probability of failure equal to zero cannot be attained. On the other hand, it should be noted that the log-normal distribution taken as an example here does not satisfactorily describe the scatter either for the fatigue strength or for the stress in some cases. For more information on this point, refer to Sect. 11.6.

## 11.4 Scatter of the Strength

If fatigue tests are performed with  $n$  samples or structural components with the same load sequence (load horizon), for instance, as constant-amplitude tests in the finite-life fatigue-strength range  $i$  of the S-N curve or as fatigue-strength tests, the mean value

$$\log N_{50} = \frac{1}{n} \sum_{i=1}^n \log N_i \quad (11.17)$$

and the standard deviation

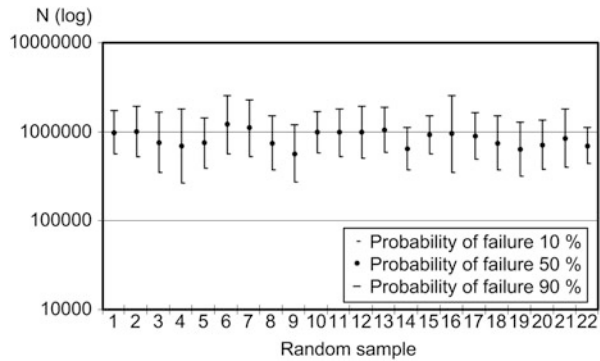
$$s = \left[ \frac{1}{n-1} \sum_{i=1}^n (\log N_i - \log N_{50})^2 \right]^{\frac{1}{2}} \quad (11.18)$$

can be calculated. These parameters are estimated values for the true mean value and the true standard deviation of the basic population. The accuracy in predicting the characteristic values of the basic population depends on the number  $n$  of random samples. With the number of random samples usually taken in the case of quality tests during mass production and also during research projects, the confidence interval for the value of the standard deviation is very large, apart from a few exceptions. That is, if the permissible range for the standard deviation must be estimated for a defined probability of a distribution function, the conclusion is relatively uncertain.

Furthermore, if a designed and dimensioned structural component is produced in large quantities and over a long period of time, the mean values and especially the scatter vary greatly from random sample to random sample, Fig. 11.10 [Mahn81], plot from [Aden01]. This behaviour has been confirmed by the results of tests on random samples in the course of quality control measures.

For determining the values, 18 test samples were taken from regular production each month, and S-N tests were performed on these samples,  $(-7 \pm 47 \text{ kN})$ . The

**Fig. 11.10** Variation of the fatigue-strength series scatter for a GTS-65 connecting rod



**Table 11.2** Scatter of results on test rods in the direction of the load cycles

Evaluation group	$n_s$	$s_{50,N}$	$s_{s,N}$
All sets of data (without Hück correction [Hück94])	1553	0.102	0.274
All sets of data	1553	0.106	0.273
All horizons	917	0.090	0.286
All chain-of-pearls methods	636	0.121	0.242
Crack initiation	67	0.105	0.268
Crack propagation	62	0.089	0.328
Fracture	788	0.096	0.284
S-N tests, $N \approx 2.4 \cdot 10^5$	409	0.090	0.276
Fatigue-strength test, $N \approx 5.4 \cdot 10^6$	508	0.102	0.292
Wrought aluminium alloys	255	0.091	0.290
Steel	642	0.097	0.284
Cast iron materials	60	0.092	0.272
Torsion, steel	41	0.187	0.176

mean value and the scatter range were determined for a total of 22 groups of random samples.

Hence, existing data from fatigue tests on specimens and structural components were collected and analysed, in order to derive appropriate reference values for designing and dimensioning. For this purpose, some 1500 sets of data from a total of about 17,000 individual tests were employed [Aden01]. In the following Tables 11.2, 11.3, 11.4, and 11.5, the results of this evaluation are compiled, and comments are provided where appropriate. The following values are indicated:

- $n_s$  Number of standard deviation values in the data pooling
- $s_{50,N}$  Mean value of the standard deviation in the data pooling in the N direction
- $s_{s,N}$  Standard deviation of the standard deviation values in the N direction
- $s_{50,S}$  Mean value of the standard deviation in the data pooling in the S direction



**Table 11.3** Scatter of results on test rods in the direction of the stress

Evaluation group	$n_s$	$s_{50,S}$	$s_{s,S}$
All sets of data with Hück correction, string-of-pearls methods, direction of stress [Hück94]	636	0.019	0.254
Crack initiation	103	0.018	0.233
Crack propagation	88	0.026	0.276
Fracture	446	0.018	0.247
S-N tests, $N \approx 2.4 \cdot 10^5$	284	0.019	0.247
Fatigue-strength tests, $N \approx 4.6 \cdot 10^6$	352	0.019	0.261
Cast iron materials	43	0.017	0.257
Steel	424	0.019	0.246
Wrought aluminium alloys	154	0.020	0.270
Torsion, steel	33	0.013	0.181

$s_{s,S}$  Standard deviation of the distribution of specific standard deviation values in the S direction

$s_{90,S}$   $P_{\bar{u}} = 90\%$  value of the standard deviation values in the S direction

The standard deviations are referred to the logarithm of the respective feature value.

The correction in accordance with M. Hück is the compensation for a systematic error during the calculation of the standard deviation  $s$  with the use of Eq. (11.18) for small numbers of random samples [Hück94]; see also [Mogw87].

The mean value of the standard deviation in the direction of the load cycles (corrected) is equal to  $s_{50,N} = 0.106$  for all sets of data, see Table 11.2. The scatter which results from a statistical evaluation by the string-of-pearls method is greater than that resulting from an evaluation of the individual stress horizons. The scatter of the fatigue life until macroscopic crack initiation is greater than that for crack propagation, see also [Schi09].

The scatter for steel, cast iron materials, and aluminium alloys differs only slightly. In the case of S-N tests on iron and aluminium alloys, the observed scatter increases with increasing number of load cycles. The ratio of the associated standard deviation values for  $10^4/10^5/10^6$  load cycles is 0.4/0.7/1.0. For fatigue-strength tests, no significant correlation is observed between the standard deviation and the number of load cycles ( $N < 10^7$ ). The slope of the S-N curve is less steep and correspondingly the scatter is decidedly higher in the case of torsion than for axial or bending stress.

For all data sets, the mean value of the standard deviation  $s_{50,S}$  is 0.19, see Table 11.3. The scatter in the direction of stress exhibits the same trends as in the direction of the load cycles, unless it is affected by the slope of the S-N or Gassner curve. Thus, the scatter is lower for torsion than for axial and bending stress in this case. The scatter of the fatigue life in the case of crack propagation is greater than that for the fatigue life until crack initiation, since the S-N curve for crack

**Table 11.4** Reference values for the scatter of the fatigue strength of structural components

Structural component	$\rightarrow s_{50,N}$	$s_{s,N}$	$\rightarrow s_{90,N}$	$s_{50,S}$	$\uparrow s_{s,S}$	$\uparrow s_{90,S}$	Comment
Steel, forged, machined by cutting processes	0.10	0.32	0.25	0.025	0.23	0.029	Edge layer of the component has been treated
Steel, forged, unmachined surface	0.13	0.17	0.22	0.023	0.19	0.040	
Cast iron, GG, GGG, GS, unmachined surface	0.19	0.26	0.42	0.029	0.33	0.078	
Wrought aluminium alloys, forged, unmachined surface	0.20	0.13	0.29	–	–	–	Confirmed by one component, 55 test points
Cast aluminium, unmachined surface	0.19	0.26	0.41	0.028	0.32	0.070	
Cast magnesium, unmachined surface	0.10	0.25	0.21	0.022	0.25	0.045	Confirmed by one component, 52 test points
<b>Welded joints</b>							
Line welds, sheet metal, steel	0.16	0.28	0.35	0.034	0.31	0.086	
Spot-welded joints, sheet metal, steel, fracture	0.17	0.25	0.35	0.033	0.29	0.077	
Line welds, sheet thickness >5 mm, steel	0.18	0.21	0.33	0.038	0.27	0.083	
Line welds, sheet thickness <2 mm, steel	0.19	0.26	0.39	–	–	–	
Friction-welded joints	0.15	0.27	0.34	0.025	0.29	0.059	Al-Al, St-Al, St-St
<b>Mechanical joints</b>							
Feather-key joints	0.11	0.18	0.19	–	–	–	
Screwed joints	0.23	0.16	0.36	0.026	0.21	0.049	
Al-/Ti riveted joints	0.14	0.26	0.29	0.031	0.23	0.060	
Aluminium clinched joints	0.27	0.15	0.42	0.047	0.21	0.088	

propagation has a steeper slope than that for crack initiation. In the case of wrought aluminium alloys, the standard deviation is higher by about 5% than for steel. The mean standard deviation is about 12% higher for steel than for cast iron materials.

On the basis of the comprehensive test data which are available, reference values of the scatter can be given for the finite-life fatigue-strength range of the S-N curve and of the Gassner curve  $\tilde{N} < 10^7$ , see Table 11.4. For structural components of steel and wrought aluminium alloys, a value of  $s_{50,S} = 0.025$  can be given for the standard deviation in the direction of stress. For cast iron and aluminium alloys, the

**Table 11.5** Reference values for the scatter of the fatigue strength of structural components

Structural components, edge-layer treatment	$\uparrow s_{50,S}$	$s_{s,S}$	$\uparrow s_{90,S}$	Comment
Gears	0.024	0.17	0.040	
Screws, strength class 8.8	0.039	0.21	0.074	Basis data from [Thom78]
Screws, strength class 12.9	0.050	0.14	0.077	Basis data from [Thom78]
Welded joints				
Line welds, sheet thickness >5 mm, steel	0.044	0.21	0.082	Basis data from DVS S-N curve catalog

value of  $s_{50,S}$  is  $=0.030$ . The standard deviation value of the evaluated data groups can be represented by a straight line plotted in the range from 5 to 95% on a Gaussian grid with a logarithmically subdivided feature axis [Aden01].

The aforementioned reference values are not applicable if the fatigue strength is affected by additional factors, such as corrosion, contact corrosion, elevated temperatures, etc.

For a few evaluation groups with low mean scatter  $s_{50,S}$ , the scatter of the scatter  $s_{s,S}$  is quite considerable. Consequently, a “safe” value of the standard deviation is more useful for the specification of reliable stress amplitudes than the mean value. In [Aden01] the 90% value  $s_{90,S}$  is regarded as a suitable value.

With regard to the fatigue limit for structural components, four evaluation groups have been formed, see Table 11.5. In the case of screws, an effect of the strength class on the scatter is evident.

For information on further reference values for the scatter, especially for welded joints, see the publications by [Oliv85, Ritt94, EC900, EC305, Hobb05, Haib06, Rada06].

## 11.5 Scatter of the Stress

Designing and dimensioning of components for series production almost always require the assumption that these components will be subjected to different stresses in operation. For chassis components, for instance, the roadway conditions, driving habits, and payloads will differ considerably. Consequently, the number of spectra which are generated during the design life is equal to the number of motor vehicles.

In principle, two approaches are possible for designing and dimensioning of structural components with the required fatigue strength. On the one hand, a maximal possible stress can be assumed to occur in the course of operation for the intended purpose. On the other hand, a load assumption can be derived on the basis of a mean value, a standard deviation, and a distribution function. As shown in Sect. 11.3, this applies to designing and dimensioning in the static and fatigue-limit cases, as well as the case of the fatigue life under variable stress. The first-mentioned procedure is usually proposed in the series of technical rules, for

instance, the FKM Guideline. For this purpose, it may be desirable to assign a probability of occurrence  $P_e$  to the case of maximal stress. The second procedure is based on the reliability concept, which is applicable especially in the case of large-scale series production, as in automotive construction. In particular, the reliability concept clearly indicates the importance of scatter, which is expressed in terms of the standard deviation  $s_B$  for the stress and  $s_F$  for the strength.

In the general case of a service load, the respective stress spectra are distinguished by the spectrum maximum, the spectrum shape, the frequency of the load cycles, and also by mean loads, for instance. As already discussed, the damage sum  $D_i$  is well suited for quantifying and characterising the respective operational stress in the general case. With the use of the damage sums, a conclusion concerning the scatter can be reached if appropriate data are available from measurements. For determining the damage sum by means of a damage accumulation calculation, a fictitious S-N curve can usually be employed, since the position of the S-N curve does not affect the result. If the slope of the S-N curve is not known exactly, a value of  $k = 5$  is recommended for structural components subject to bending stress, and  $k = 8$  for components subject to torsional stress. The calculation is usually performed without consideration of a fatigue limit, that is, with the use of the elementary Palmgren-Miner modification.

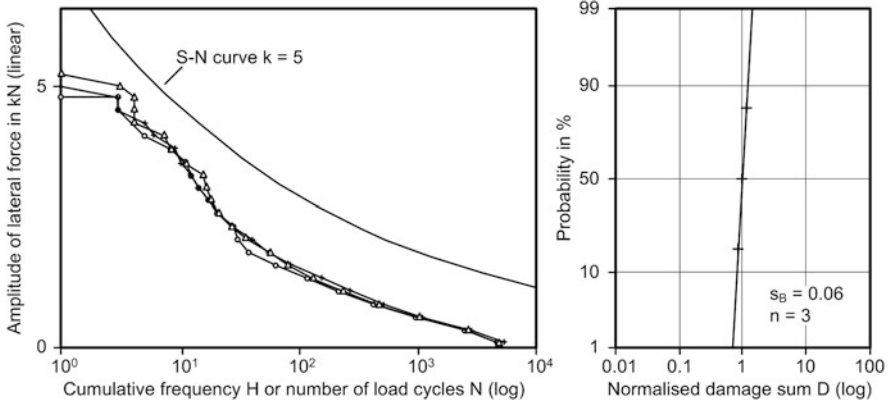
The result is illustrated with the use of three examples [Güth87]. In Fig. 11.11, three spectra are plotted for a driver who travels over a short, precisely defined route at the prescribed speed. The lateral forces at the front wheel are indicated. In the range of small amplitudes, which are caused by a slightly bumpy roadway, the spectra are practically identical. Even in the range of higher amplitudes which result from travel over curves, however, the observed differences are very small.

If ten drivers travel over a longer distance of about 100 km, including urban traffic, state highway, and motorway, the resulting scatter is greater at higher amplitudes. The longitudinal forces at the front wheel are indicated in this case, Fig. 11.12.

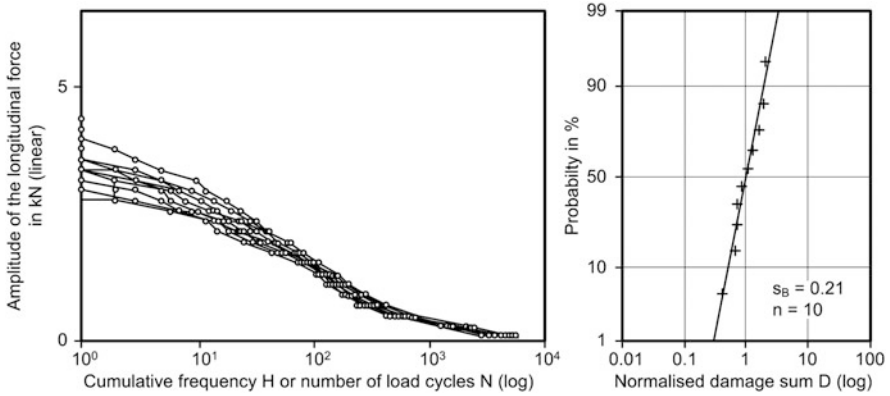
For travel over various routes, in this case 17 state highways, the scatter of the stress is even greater. The longitudinal forces at the front are indicated here, Fig. 11.13. Large differences are observed in the maximum and in the shape of the spectrum as well as in the frequency.

If the damage sums are calculated for all spectra, the standard deviation  $s_B$  can be given for each example. For plotting on a probability grid, the damage sum for  $P_e = 50\%$  was set to  $D = 1$ . The results can be described by a log-normal distribution. The standard deviation increases markedly from example 1 to example 3.

In an early publication on the scatter of the stress, braking pressure spectra were measured for motor vehicles over a distance of about 100 km. The route included urban traffic, state highway, and motorway, as well as roadway in bad condition [Wimm84]. Motor vehicles of three types were employed and were driven by 100 customers each. These customers also drove vehicles of the respective type privately and can be regarded as normal drivers. The vehicles were driven in all kinds of weather. The stress-time function of the braking pressure was classified by

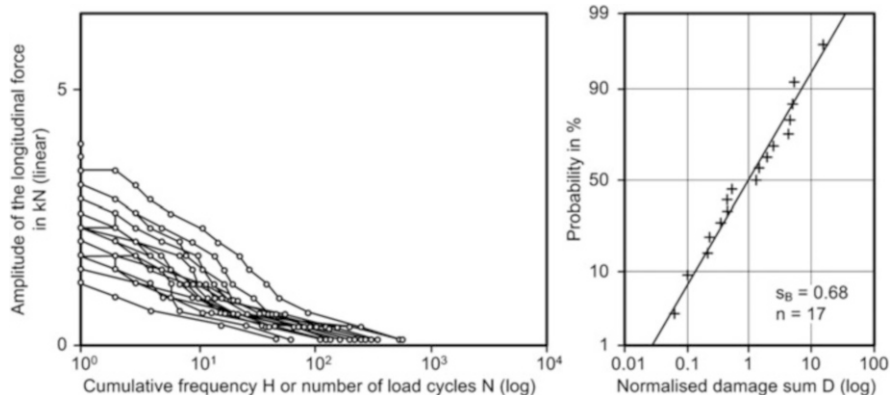


**Fig. 11.11** Three range-pair counting spectra for lateral forces, one driver, three times the same route, and probability distribution of the damage sums

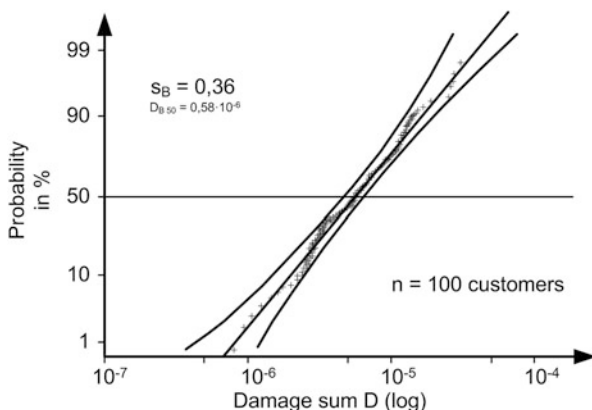


**Fig. 11.12** Ten range-pair counting spectra for longitudinal forces at the front, ten drivers, the same route, and probability distribution of the damage sums

level-crossing counting. For each measuring trip, the damage sum was calculated with the use of a fictitious S-N curve. As a good approximation for the distribution of the damage sum, a straight line thus resulted from a logarithmic plot of the feature on a probability grid. The standard deviation  $s_B$  is equal to 0.36, see Fig. 11.14. (The mean damage sum need not be discussed further, since the position of the fictitious S-N curve is not relevant for the structural component concerned.) A further result was obtained from these measurements: As expected, different spectra were obtained for the braking pressure with the three motor vehicles of different types employed for the measurements. However, if the spectra for the braking-pressure are transformed to spectra for deceleration, it can be shown that these spectra hardly differ at all for the individual motor-vehicle types. This result



**Fig. 11.13** Seventeen range-pair counting spectra for longitudinal forces at the front, 17 different highways, and probability distribution of the damage sums



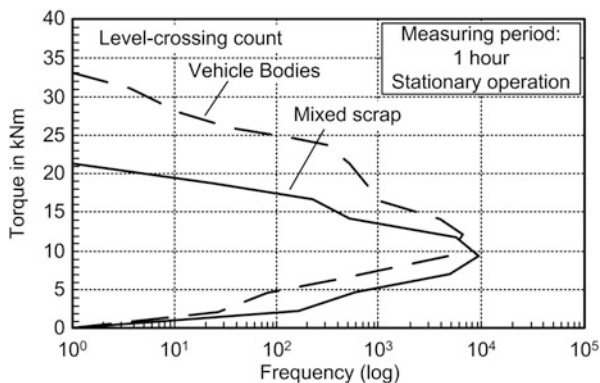
**Fig. 11.14** Distribution of the damage sums and confidence range for braking pressure spectra with 100 drivers and a mixed route of about 100 km

implies that the braking delay is determined exclusively by the behaviour of the driver. Thus, conclusions concerning the scatter can be reached independently of the types of motor vehicle.

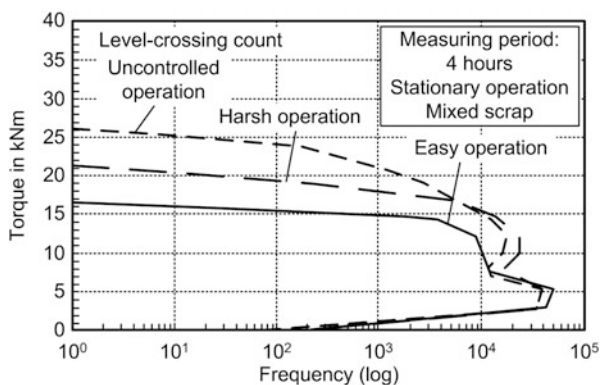
As repeatedly verified with the results of numerous measurements, the log-normal distribution provides a good approximation for describing the scatter of the stress, see Sect. 12.2.

The knowledge of the standard deviation  $s_B$  and of the distribution function is a prerequisite for the application of the reliability concept. If various fields of industry are considered, great differences are evident as far as the available knowledge is concerned. In the case of large-scale series production with global application, a high standard of knowledge has been achieved as a result of measurements over a period of decades. Examples include the construction of aircraft, automobiles, rail vehicles, wind turbines, and ships. Machine elements which can

**Fig. 11.15** Torque spectra of a shredder for different types of scrap



**Fig. 11.16** Torque spectra of a shredder for different modes of operation



be employed in highly different fields present a problem. Transmission gears, clutches, and screw connections are typical examples of such elements. Particularly for medium-sized industrial enterprises, it may be difficult to reliably estimate stresses for different applications. The drive train of a shredder is taken as an example for illustrating the extent to which the spectra encountered in operations can differ. As indicated in Figs. 11.15 and 11.16, the load which occurs during operation is highly dependent on the type of scrap and on the mode of operation [Gehl92]. The person operating the shredder can minimise the load by feeding the material as uniformly as possible. With this “careful” mode of operation, approximately the same material throughput is achieved as with the “severe” mode, which results in much higher amplitudes. Uncontrolled feeding results in extremely high torque all the way to blocking processes.

Thus, it can be concluded that the scatter of the stress depends on the type and application of a product and therefore varies from case to case. For a large production series with a wide range of applications, the problems involved differ decidedly from those which are typical of a small series with a very limited range of

applications. By means of measurements under operational conditions, more comprehensive information can be obtained at reasonable expense in many cases.

Unfortunately, it is not in general possible to compile reference values of the standard deviation for the stress in the same form as that for the loading capacity.

## 11.6 Safety and Engineering

In the preceding sections, the failure criteria under realistic operational stresses, the safety and reliability concepts, as well as the scatter of the loading capacity and of the stress have been considered. In Sect. 9.3, a comparison has been performed between calculated and experimental values for the analytical fatigue-life prediction. Thus, a fundamental question concerns the importance of testing in the context of the fatigue-strength analysis.

Technical safety and reliability are of such elementary personal and social interest that further standpoints must be considered, in addition to the technical aspects. As is evident from numerous reports in the public media, questions of technical safety are by no means viewed and perceived in a rational manner by the general public.

### 11.6.1 *Calculation or Test, or Both*

A reliable load assumption and a specification of the strength characteristics have already been indicated as prerequisites for a reliable fatigue-strength analysis. Especially for the certification of fatigue strength under variable stress amplitude, uncertainties are associated with the method applied for calculation, as already demonstrated. This conclusion has been unambiguously verified by a comparison between the calculated and experimental values. An essential cause of this uncertainty is the fact that the damage-accumulation hypotheses hitherto applied do not describe the real damage sequence with sufficient accuracy. The application of an effective damage sum can be viewed as a pragmatic approach for integrating the experience gained from experiment into the calculation.

The damage accumulation under fatigue stress with variable amplitude constitutes only one example. For other technical damage sequences, too, a sufficiently accurate description of the processes in progress is often not possible, either with the application of hypotheses or by means of model construction and simulation. Examples include processes under corrosive action and as a result of wear. Consequently, a logical approach is the application of an experimental simulation for eliminating this shortcoming. For determining the fatigue strength under variable stress amplitude, tests are performed on safety-relevant structural components under realistic service stresses (Gassner test). This procedure was first applied in the field of aircraft construction and later became established in automotive



construction. The development of servohydraulics during the 1970s constituted an important step in this direction, since realistic stress-time functions can thus be reproduced. In many cases, the ambient conditions are also simulated as realistically as possible.

This approach has become standard procedure for many applications, since it provides an additional safeguard which extends beyond the calculation itself, even though this applies only in the respective, specific case. This experimental verification is an extremely critical task, which includes the following special requirements:

- Correct selection of the operational loads to be simulated
- Ensuring the correct stresses at the critical point
- Control of complicated testing and measuring equipment
- Recognition of the simplifications (idealisations) which may possibly be involved and which may thus limit the validity of conclusions
- Short testing times and optimising of costs
- Clear-cut, transparent presentation of the results and conclusions

### ***11.6.2 Cases of Damage Analysis***

The history of technology is also a history of failure and damage cases. With the technical development of motor vehicles, rail vehicles, and aircraft over the past decades, for instance, safety engineering has also undergone numerous changes and improvements. These developments include the concepts, the possibilities for calculation and simulation, the load assumptions, experiments and tests under realistic loads and ambient conditions, constructive measures, control of manufacturing processes, quality assurance, and nondestructive testing, monitoring in operations, maintenance measures, and many other factors.

Cases of damage have often triggered continuing developments in the series of technical rules. In fracture mechanics, for instance, even the development of a specialised scientific field can be explained by the occurrence of specific cases of damage. After all, damage must be avoided. With this objective, comprehensive literature has been published on the subject of damage cases, for instance [Pohl56, Alli84, Lang87, Lang01, Schm05, Schü08]. This development can be illustrated with the example of the VDI Guideline, “Damage Analysis”. On sheet 1 of this guideline, the “fundamentals, concepts, definitions”, and the “procedural sequence of a damage analysis” are treated [VDI04]. On sheet 2 “damages caused by mechanical stresses” are considered, and the corresponding types of damage and features are explained for forced rupture and fatigue fracture. On sheets 3–5, the analysis of damage in a corrosive environment, during thermal exposure, and under tribological stress is described. The record to be prepared for the analysis of damage is explained on sheet 1 (translated from the German original document by the authors) as follows [VDI04]:

*The purpose of a record is to ensure the availability of background information on the case of damage. If this information is lacking, the analysis of the damage will be more difficult, the circumstances may be misleading, or the analysis may even prove to be impossible.*

*For determining the case history of failure, subjective information, such as statements made by the operating personnel, must be distinguished from objective information, such as operational data which are recorded by electronic or mechanical devices. The subjective statements may include inadvertent perceptive errors as well as deliberately erroneous or biased information associated with particular, individual interests.*

*For the record of the case history, the design and construction documents must be examined, the relevant production steps must be evaluated, and the available operational data must be included. Relevant standards and series of technical rules must also be taken into account.*

*The following information may be important:*

*Type of plant, operator, equipment manufacturer, business category, date of manufacture, factory location, date of commissioning, conditions of application, time of operation, date and time of auditing and third-party inspections, test certifications.*

*The damaged structural component or part constitutes the “data carrier” for its constructive design – including the dimensions and the condition of the material, the manufacturing process and production technique, as well as the stress to which the component was subjected during operation. The following information is important:*

- *Type, manufacture, further processing, quality testing of the material*
- *Constructive design, manufacture, quality testing of the structural component*
- *Function of the structural component, operational conditions during the operating period and shortly before occurrence of the damage*
- *Temporal sequence of the damage*

These are important details which must be clarified at the beginning of a damage analysis. For determining the fatigue strength under variable stress amplitude, however, two further aspects must be emphasised, see also [Buxb00]:

- *How reliable was the load assumption which was taken as basis for designing and dimensioning? Is it possible that the stresses predicted by the spectrum for designing and dimensioning have been exceeded during real operation?*
- *Has the fatigue-strength analysis been performed with sufficiently high quality and reliability?*
- *Do the applied methods correspond to the state of the art? Should the fatigue-strength analysis have been supplemented by an experimental fatigue-strength determination?*

These questions are considered again in the course of the present chapter, especially in the section on “safety-relevant measures”.

During an investigation of damage on aircraft, for instance, 119 fatigue failures were analysed and assessed on the basis of various criteria for failure: design and

construction, material, manufacture, operational failure, faulty maintenance, repairs, and unexplained causes [Lang87]. Among these criteria, only four failures are ascribed to material defects (microstructural defects, inclusions, internal cracks, shrink holes). In this case, too, however, criteria such as an erroneous load assumption or insufficient reliability of the fatigue-strength analysis have not been explicitly mentioned. For many cases of failure, a typical feature is the lack of reliable information concerning the pertinent operational stresses during their occurrence.

During recent decades, comprehensive stress measurements have been performed on motor vehicles, rail vehicles, aircraft, as well as machine plants and facilities during operation. These measurements were necessary for reliable designing and dimensioning. In the course of these measurements, unexpected results were obtained. These results have provided a more profound understanding of the relationships among the loads and deformations concerned, especially in the case of complex systems. By means of multibody simulation as well as structural and modal analyses, the relationships can often be explained and optimised nowadays. Well-proved units consisting of transmission gears and generators have been employed for decades in conventional power plants. However, if these units are employed for power generation with wind-turbines, they operate in an entirely different environment and are thus subjected to entirely different stresses, see for instance [Schl08].

The rupture of a wheel tyre on a German high-speed train (ICE) near the town of Eschede, Germany, in 1998 is one example which illustrates the differences in expertise on an extraordinarily serious case of damage [Essl04]. The wheel tyre of a rubber-cushioned wheel ruptured at a speed of about 200 km/h. (A hard-rubber cushion is situated between the wheel tyre and the wheel disc.) An extremely improbable chain of events, including the displacement of a switch point by entanglement of the ruptured wheel tyre, collision of a derailed train section with a bridge pier, and collapse of the bridge over a portion of the train, resulted in 101 deaths and in part severe injury of over 100 persons. In 2003, the responsible employees of the wheel manufacturer and of the operator had to appear in court. Over a period of several weeks, 13 experts provided their testimony concerning the background factors which led to the damage. After evaluation of the presentations, the court dropped the prosecution on the following grounds: *“To the extent that the accused could be proved guilty, if at all, their guilt would most probably be very slight, since neither wrongful intent nor negligence can be discerned from the evidence hitherto available”*.

This case is considered here as an example of a situation in which the experts agreed on many points, but definitely disagreed on a few points, too. This situation is typical of those observed with complex technical processes in other fields, too. Each expert evidently considers the problem especially from the standpoint of his own specialised field. An expert on nondestructive testing will first ask whether or not the possibilities of nondestructive testing have been considered adequately. A corrosion specialist will pay particular attention to the moisture present between the rubber cushion and wheel tyre and to the temperatures which prevail there. A specialist in the field of machine dynamics and vibration will check to determine

whether excitations may have occurred in the range of eigenfrequencies at certain speeds. The materials expert will pay special attention to the material, its condition, and the fracture surface. The fatigue-strength specialist will ask whether high overloads can occur during travel over obstacles, for example, and whether the complex interaction of several decisive parameters (surface, local stresses, residual stresses, etc.) may have contributed. Thus, a possible, logical conclusion is the following: A damage analysis must be an all-encompassing consideration. For complex cases of damage, this problem is an interdisciplinary task. For this purpose, it may be necessary to establish a forum for mutual discussion of the individual aspects before the submission of individual assessments.

### ***11.6.3 Comparison of the Concepts***

In Sects. 11.2 and 11.3, the safety concept and the reliability concept have been described. In the present section, the essential differences are considered further, and the difficulties encountered in practice are discussed in more detail.

The safety concept is the older concept from the standpoint of historical development. For the comparison of characteristic values for fatigue strength and stress in accordance with this concept, a safety factor must be employed. It has repeatedly been emphasised that the uncertainties associated with the comparison must be taken into account by the safety factor. On the one hand, these uncertainties include the scatter of the fatigue strength and the stress during operation. On the other hand, inadequacies or uncertainties may also be associated with the strength analysis which has been performed. The value of the safety factor to be chosen depends essentially on experience. It is not possible to indicate a probability of failure. Aspects such as the possibility of inspection and consequences of damage can be taken into account with the specification of a safety factor; see for instance [FKM12]. This procedure is also conceived as deterministic designing and dimensioning; for this purpose, allowable stresses are compared with limiting values of the operational stress caused by external forces [Buxb92]. For many fields of machine and plant construction, this is the only practicable approach. As a rule, this procedure results in very conservative designing and dimensioning; that is, the potential for light-weight construction cannot be fully utilised.

The introduction of the reliability concept was considered in publications as of 1950. With this stochastic designing and dimensioning method, the fatigue-strength characteristics and operational stresses are interpreted in a statistical manner. The safety factor is replaced by a probability of failure. The objective of this procedure is to “*improve the reliability of designing and dimensioning. That is, the phenomena of ‘operational loads’ and ‘fatigue behaviour’ can be described more realistically by probability functions than by the observance of global limiting values*”, as stated by O. Buxbaum. In 1958, A. Erke developed an approach for combining the scatter of the fatigue behaviour with that of the operational loads, in order to determine an overall probability of failure [Erke58]. E. Haibach developed numerical solutions

for logarithmic normal distributions of fatigue strength and stress [Haib67], see Sect. 11.3.

The reliability concept has been applied in various forms for a long time. This concept is especially useful wherever large quantities (major series) of structural components are produced and are subjected to different stresses during operation, for instance, in automotive construction. A prerequisite for the application of this concept is that the scatter of the fatigue strength as well as that of the stress have to be known. These scatter values can be described by means of the standard deviation. The status of present knowledge and the difficulties encountered in determining the scatter have been discussed in Sects. 11.4 and 11.5. A fundamental problem is the fact that a correct distribution function usually cannot be given. However, precisely this condition is necessary, since the ranges of very low fatigue strength and very high stress are especially decisive for the probability of failure. For the statistical description of experimentally determined strength characteristics, a number of distribution functions which agree well with the test results can be given. However, this applies only in the range which is covered by sampling to the usual extent, that is, small sampling size. For determining the values in the range of very low probability of failure, extrapolation from the experimentally determined range is necessary. In this process, various distribution functions, such as the log-normal distribution, the arc sin  $\sqrt{P}$ - distribution, or the Weibull distributions, yield very different results. Hardly any values are available from experience for indicating which distribution is correct. Furthermore, since the scatter and the statistical distribution are due to several causes, the existence of a generally applicable distribution cannot be expected. *“The operational loads as well as the fatigue behaviour usually encompass many decisive parameters. Each of these parameters can be a statistical random variable; the mutual dependence among such variables is often unknown, and their significance for the resulting fatigue behaviour can vary from case to case”* [Buxb92].

The safety factor is essentially a value obtained from experience, and is by no means transparent for many applications. In contrast, the probability of failure is a very clear-cut and informative parameter. A value of  $P_A = 10^{-5}$  for the probability of failure implies that one structural component among 100,000 will fail, as a statistical average. However, such a conclusion frequently causes difficulties. On the one hand, the accuracy of this conclusion must be examined more closely, since it is not self-evident. On the other hand, this conclusion implies that the probability of failure is greater than zero.

The question concerning the accuracy can be easily answered on the basis of the preceding discussion. If the standard deviation of the fatigue strength and stresses are expectation values, and if the distribution functions represent only approximations, the accuracy of the probability of failure is limited. On the other hand, the following standpoint is also justifiable: The calculated value represents an order of magnitude, which is certainly informative. Ultimately, of course, experience is decisive for determining the calculated probability of failure to be taken as basis for designing and dimensioning in this case, too.

The fact that a probability of failure which is greater than zero can and must be accepted is evident from many applications in practice. Examples include many products for everyday use, such as leisure articles and sporting equipment. On the other hand, one may ask whether or not the assumption of a failure probability equal to zero is a fictitious construct, which fails to take into account the indicated physical realities associated with the scatter. Safety can be increased, but one-hundred-per-cent safety is not attainable.

Among the probability distributions which are often proposed, the Gaussian normal distribution is unlimited whereas for example the three-parameter Weibull distribution and the arc sin  $\sqrt{P}$  distribution are limited. Examples include the three-parameter Weibull distribution and the arc sin  $\sqrt{P}$  distribution. For a more detailed description, see [Mauc99], for instance. If a lower limit can be given for the fatigue strength and an upper limit can be given for the stress, then a probability of failure equal to zero is imaginable. As already explained in Sect. 11.3, the problem is not really solved with the use of such models. Instead, the problem is merely shifted, because these limiting ranges cannot be sufficiently well confirmed experimentally, that is, by actual tests and measurements, as already mentioned in conjunction with the specification of distribution functions. Furthermore, no unambiguous physical relationship exists for determining which specific condition of the material would ensure a probability of failure equal to zero under a defined load.

A further point must be considered in connection with the reliability concept: As a matter of principle, the aforementioned safety index  $j_S$  must be distinguished from the safety factor employed with the safety concept. For the example of the fatigue limit, a safety index can be defined as the quotient of the fatigue strength values and the stress, as indicated in Sect. 11.3:

$$j_S = \frac{s_{aF50}}{s_{aB50}} \quad \text{See Eq. (11.9)}$$

This quantity is none other than the numerator of the normalised safety range  $u_0$  in antilogarithmic form:

$$u_0 = \frac{\log S_{aF50} - \log S_{aB50}}{\sqrt{s_F^2 + s_B^2}} = \frac{\log j_S}{\sqrt{s_F^2 + s_B^2}} \quad (11.19)$$

That is, an individual term is extracted from the basic equation of the reliability concept in order to reach a conclusion on the safety, see also [Rada07]. For  $S_{aF50} = S_{aB50}$ ,  $j_S = 1$  and thus  $u_0 = 0$ . The condition  $u_0 = 0$  corresponds to a probability of failure,  $P_A = 50\%$ , see Fig. 11.7.

The relationship between  $P_A$ ,  $s$  and  $j_S$  is plotted in Fig. 11.17, with

$$s = \sqrt{s_F^2 + s_B^2} \quad \text{see Eq. (11.5)}$$

as indicated in Sect. 11.3. For  $j_S = 1$ , the probability of failure  $P_A$  is constant, that is, independent of the standard deviation. For  $j_S > 1$ ,  $P_A$  decreases with decreasing scatter. This is understandable. At a defined ratio of the mean values,

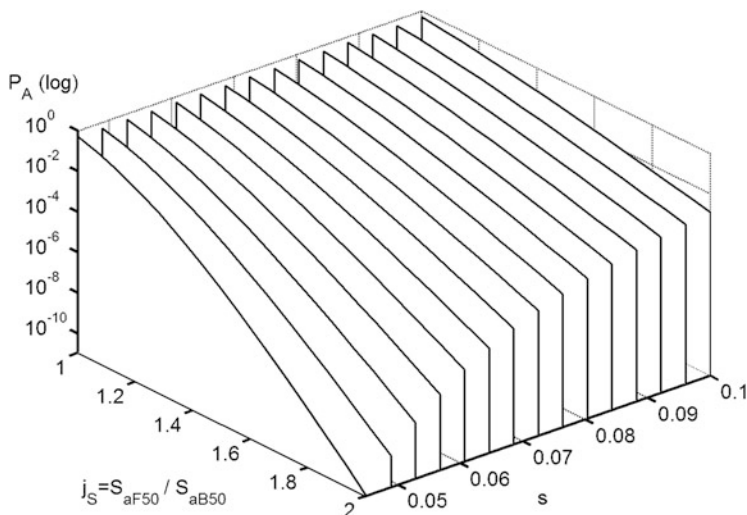


Fig. 11.17 Relationship between the probability of failure, the scatter, and the safety index

$S_{aF50}$  to  $S_{aB50}$ , the probability of failure decreases with decreasing scatter  $s$ . With relatively severe scatter, however, the observed probability of failure decreases only slightly with increasing safety index. If the probability of failure is taken as starting point for reliable designing and dimensioning, this result indicates that the safety index alone is not sufficient for reaching conclusions on the safety of a structural component.

In Fig. 11.18, the safety index is plotted as a function of the scatter for the probability of failure  $P_A = 10^{-5}$ ; for this purpose, the standard deviations  $s_F$  and  $s_B$  can assume arbitrary values. The safety index increases as the respective individual values of  $s_F$  and  $s_B$  increase. These examples are intended to emphasise the severe limitations on the validity of conclusions reached on the basis of a safety index.

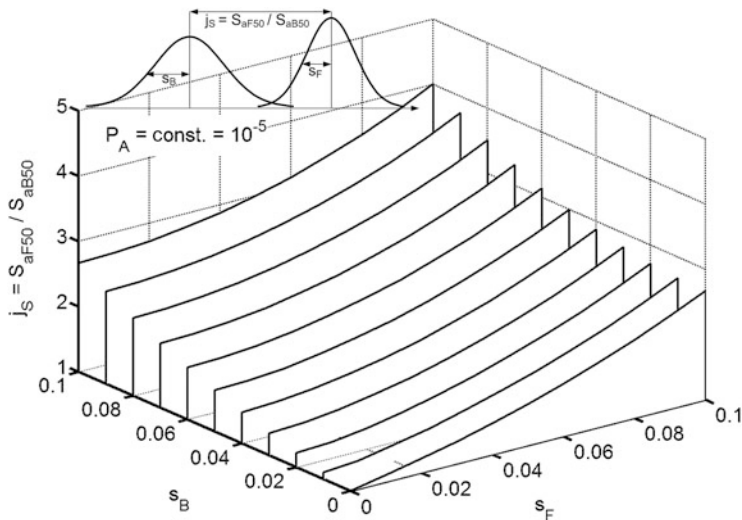
In many cases, the definition of a safety index as a quotient of quantiles of the distribution functions has been proposed, rather than as a quotient of mean values. In fact, a classical assumption which is frequently mentioned in publications is based on a fatigue strength value with a probability of one thousandth,  $P_a = 0.1\%$ , and a stress value with a probability of 1% (1-per-cent customer, see Sect. 12.2.4). With the definition employed here, the so-called 1-per-cent customer corresponds to a probability of occurrence  $P_e = 99\%$ .

Thus, a safety index can be defined as

$$j_{S,xy} = \frac{S_{aF0,1}}{S_{aB99}} \tag{11.20}$$

for example.

In Fig. 11.19, the safety index is plotted as a function of the standard deviations  $s_F$  and  $s_B$  for the overall probability  $P_A = 10^{-5}$ . As can be seen from this plot, the



**Fig. 11.18** Dependence of the safety index on the individual scatter levels for a constant probability of failure

safety index depends on the values of the scatter in a complicated manner for  $P_A = \text{constant}$ . In correspondence with the particular combination of the scatter values, the safety index can increase or decrease as one of the individual scatter values increases. This implies that no conclusion concerning the probability of failure  $P_A$  can be reached even with a safety index which is formed from quantiles of the distribution functions. In comparison with Fig. 11.18, the safety indices are considerably lower for the same probability of failure  $P_A = 10^{-5}$ .

With the application of the reliability concept, two conditions are especially important for the fatigue-strength analysis: The scatter of the actual characteristic values for the stress and the loading capacity must be known. The values employed are always expectation values for allowing a conclusion concerning the basic population. That is, assumptions involving the statistical characteristics must be made during the development of products. These characteristics need not necessarily coincide with those which apply after manufacture. Uncertainties in the specification of the statistical characteristics can have various causes. The characteristic parameters and associated effects which are important for this purpose are indicated in Table 11.6.

The specification of the characteristic parameters for the stress and the loading capacity, as well as the results of the fatigue-strength analysis are essentially predictions which may be subject to error. Possible causes of such errors are indicated in Table 11.7.

Consequently, the manner in which these uncertainties can be taken into account in the fatigue-strength analysis must be considered. For example, a correction factor can be introduced if inaccuracies of measurement are involved, and an “effective” damage sum can be employed for the damage accumulation, see Chap. 9.



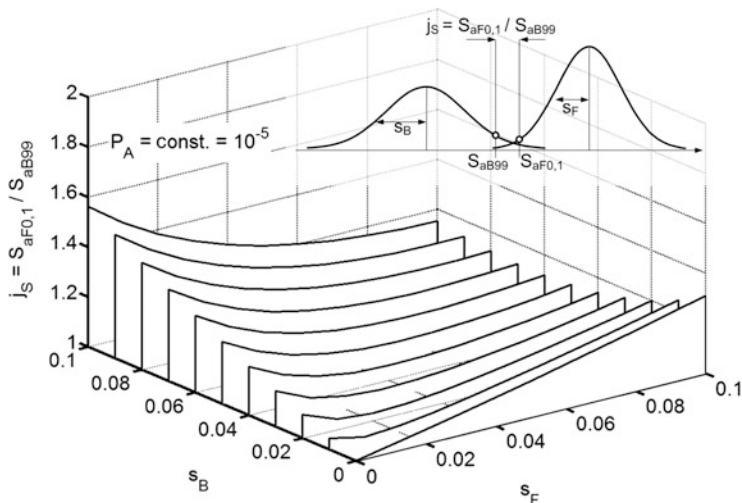


Fig. 11.19 Effect of scatter on the safety index for quantiles at constant probability of failure

Table 11.6 Causes of uncertainty in the specification of statistical characteristic parameters

Stresses	Strength
• Size of random samples for stress measurements	• Size of random samples for fatigue-strength testing
• Type of evaluation (counting method)	• Type of evaluation (regression)
• Assumption for determining the distribution	• Assumption for determining the distribution
• Expected value for the mean	• Expected value for the mean
• Expected value for the standard deviation	• Expected value for the standard deviation

Thus, the reliability concept can be recommended for application if sufficient information on the scatter of the fatigue strength and of the stress is available. If no concrete deviations have been detected or demonstrated to be present, and if there is no evidence of such deviations, log-normal distributions can be assumed. The calculated probability of failure  $P_A$  represents an expectation value whose permissibility must be verified on the basis of experience. For many applications, experience of this kind has been gained over a period of decades. For electronic components and programmable electronic systems, DINEN61508, [DINEN61508], provides a measure of the system reliability as a function of the risk which is involved. Mean probabilities of failure are indicated in four steps. Even though a direct application to mechanical components is not permissible, useful guideline values can still be derived for allowable probabilities of failure.

**Table 11.7** Causes of uncertainty in the determination of characteristic parameters for the stress and strength, as well as the performance of the fatigue-strength analysis

Stresses	Strength
<ul style="list-style-type: none"> <li>• Accuracy of measurement as well as errors in measurement</li> <li>• Neglect of relevant operational states</li> <li>• Insufficient model quality and suitability of calculation and simulation methods</li> </ul>	<ul style="list-style-type: none"> <li>• Insufficient applicability of material characteristics to the structural component</li> <li>• Changes in the manufacturing process</li> <li>• Insufficient model quality for the fatigue-strength calculation</li> <li>• Accuracy (reliability) of the damage-accumulation hypotheses</li> </ul>

### 11.6.4 Differences in the Scope of Application

A characteristic feature of the safety consideration is the fact that completely different procedures are employed for individual technical applications. On the one hand, there are historical reasons; on the other hand, the technical conditions for application can differ enormously. The prevention of personal injury receives the first priority. Financial damage can result from the loss of factories and machines as well as transport media (in the air, on water, on land), from production stoppage, from expenditures for the repair of damage, and from detriment to a company's image. Furthermore the prevention of environmental damage caused by the mechanical breakdown of technical equipment has become a requirement of great importance for the operator as well as for society in general.

Safety considerations for designing and dimensioning also differ for other reasons, for instance, whether or not maintenance and inspection are possible during operation, whether an individual construction, a limited series, or high-volume manufacture is involved, whether innovative or traditional methods and technology are applied. Finally, the potential impact on the environment and the duration of adverse environmental effects are of paramount importance for the safety consideration (oil wells, tankers for transporting chemicals and oil, nuclear power plants).

Load measurements on unique facilities, such as machine plants, are performed only over a limited period of time. Conclusions about the distribution of the load events during the design life must be derived from the scatter of the measured events. The fatigue strength of structural components in such machine plants can be determined nondestructively only on the basis of indirect characteristics such as hardness or characteristic values from material samples.

Load measurements on series products, such as motor vehicles, can be performed only over a limited period and for specific user behaviour. Consequently, the statistical distribution of the loads must be estimated for the purpose. The strength properties of series components can be determined by performing random tests, for instance, by quality-assurance testing. From the results of the random tests, conclusions concerning the basic population can be reached.

That is, specific procedures have gradually developed and become established in the various individual fields of application, and the experience thus gained is usually quite comprehensive. However, a method which has been successfully applied in one field usually cannot be directly applied in another field.

During the past decades, the accuracy of load assumptions has been improved in many fields of application. Measuring technology and data processing usually do not present any major problems. Improvements are still necessary in some fields, partially for technical reasons associated with measurement, for instance, in the case of recreation and leisure-time products, sporting equipment, as well as prosthetic and implantation devices.

### ***11.6.5 Safety-Relevant Measures***

The fatigue-strength analysis constitutes the basis for reliable designing and dimensioning. The importance of the load assumption, the strength characteristics, and the methods applied for the purpose has been explained in detail. Furthermore, a number of measures are highly relevant to safety. These measures are considered in the following section. In this context, measures which are implemented during the development of products are distinguished from those which are applicable during the use of the products.

**Measures Implemented During Product Development** At the beginning of the development process, decisions on design and construction as well as the selection of materials must first be reached. These decisions are dependent on the specific application. Is an individual construction or a high-volume series involved? Must particular requirements be satisfied and thus taken into account for performing a given function, for instance, light-weight construction, exposure to elevated temperatures, corrosive media, or fretting corrosion? Will the expected fatigue stresses occur seldom (low-cycle fatigue) or very often (ultra-high-cycle fatigue)? Is overload expected, for instance, during special events? How high will the overload be, and how frequently will it occur? Will possibilities be provided for inspection and repair? Is the occurrence of crack initiation permissible during the design life? Does macroscopic crack initiation impose a limit on the design life, or can a crack be tolerated up to a critical crack length? Of course, the production possibilities, costs, and further management aspects are also decisive factors which must be taken into account with all of these questions.

A decision in favour of a given design can imply, for instance, monolithic construction or several load paths (statically overdeterminate), cast or forged components, welded or unwelded, riveted or removable screw connections.

The selection of materials is of special importance for safety. Since the inception of fracture mechanics as a scientific field (at the beginning of the 1960s), it has become a well-established fact that the application of a high-strength material alone is not sufficient for achieving a high level of safety and reliability. This objective

can be achieved only by a compromise between strength and ductility of the material. If it is assumed that a material contains defects (inclusions, pores, shrink holes, microcracks), these defects can severely impair the capacity of high-strength materials to withstand loads under static and especially oscillating stress. On the other hand, this impairment is less severe in the case of low-strength materials; in fact, such defects may not even cause any decrease in strength at all in low-strength materials. Especially in aircraft and automotive construction, so-called damage-tolerant materials are employed; these materials are characterised by higher ductility. A further effect must be considered in this context: Under fatigue stress the notch sensitivity and the mean-stress sensitivity increase with increasing static strength. Strength (tensile strength) and ductility (strain upon fracture) were formerly regarded as diametrically opposed properties. With the continuing development of materials during recent years, however, it has been shown that higher ductility can also be achieved with high-strength materials; see for instance [Stein05].

Considerations of this kind have resulted in a distinction between two design philosophies for aircraft construction; see for instance [Engl84]:

**safe-life philosophy**, which implies designing and dimensioning for a safe lifetime without crack initiation,  
and

**fail-safe philosophy**, which implies that a crack up to a critical length is permissible.

Prerequisites for fail-safe designing and dimensioning are the use of damage-tolerant materials and the possibility of performing inspections at specified, regular intervals. Relatively slow, stable crack propagation must be ensured by the type of construction (sheet-metal structure, frame construction, stringer). Several load paths must be provided for preventing the failure of the overall structure in the event that an individual load path fails. An example of this is a wire cable, where numerous individual wires are twisted together to form the cable. If an individual wire breaks, the function of the cable is not significantly compromised. Thus, the broken wire is detectable during inspection, and appropriate measures can be taken.

The separate consideration of one individual structural component is often not sufficient, since the failure may be caused by the overall system. By means of analytical and experimental simulation, conclusive parameter analyses can be performed for optimising a system in many cases [Lasc88, Dres01].

In some cases, the avoidance of overload is feasible by constructive measures such as the use of predetermined breaking points (separation of the primary and secondary units), slipping clutches (drive trains), pressure-relief valves (pressure vessels), and mechanical limit stops (for avoiding winding contact with helical springs, for instance).

In many cases, a decrease in the amplitudes of oscillations is desirable or necessary. Various possibilities exist for achieving this objective:

**Closed-Circuit Control Measures** For electrical drive trains in industrial processing machines, such as shredders, it may be necessary to determine possible means of decreasing the stresses in the drive train. The overall system consists of an asynchronous motor, a hydrodynamic clutch, a transmission gear, a brake, an articulated shaft, and the components of the shredder itself. For the present purpose, measurements were first performed under operational conditions. The stator voltage and current, the motor and rotor moments, as well as the rotational speed of the motor and rotor were measured under various operating conditions. A model was constructed for the numerical simulation. This model is adapted and validated with the use of the results from the measurements. With the simulation of various closed-circuit control concepts, it has been shown that the set objective can be achieved with the use of a drive train which reacts elastically to the rotational speed. Thus, the load peaks and torque oscillations can be reduced, and the electrical losses can be decreased at the same time [Sour96].

**Passive Vibration Absorber** The simplest method of reducing the amplitudes of oscillations is the use of a passive absorber. By means of an additional mass, an interfering oscillation (eigenfrequency) is decomposed into two individual oscillations which are situated above and below the eigenfrequency, and the amplitudes decrease as a result of this process. The effect can be further enhanced with the use of a damper. The increase in the total mass may prove to be a disadvantage, however.

**Active Vibration Absorber** With the use of an active vibration absorber, the oscillations are damped by means of an actor, a sensor, and a controller. An example is the application of the piezoelectric effect as operating principle for the actor [Enns10].

An essential prerequisite for the successful development of products is the knowledge of the areas where possible failures may occur in a product. Internal as well as external experience is necessary for this purpose. In cases of damage, experts have repeatedly discovered that the causes of the damage have been known for a very long time. Evidently, the avoidance of damage is also a problem of documentation and information. Special sensitivity is required when new technology is being introduced and previous experience is not applicable. On the other hand, some cases of damage occur unexpectedly and are not included in the series of technical rules. In 1988, for instance, multiple-site damage occurred on a Boeing 737 of Aloha Airlines and nearly resulted in a crash. The damage was caused by corrosion fatigue as well as defective adhesive fastening of a crack stopper. This incident occurred after more than 90,000 flights with this aircraft [Schü08]. This leads to the conclusion that long-term experience with a product can lead to decreasing alertness and an erroneous perception of safety.

**Measures Implemented During Operation** In this case, too, very different measures have been developed in various fields of technology for ensuring safety in applications. The particular measures adopted depend on the consequences which the damage can have in each case, on the type of process involved, and on the

possibilities of performing inspections and repairs. These measures extend over a very wide range: For instance, machine plants require maintenance, but are usually not inspected for ensuring fatigue strength during their design life. Critical components are inspected periodically. Some machines and other facilities are subject to continuous condition monitoring. Individual measures are indicated in the following. Refer to Chap. 13 for publications concerning the individual specialised fields.

The difficulties associated with the load assumption, as in the case of unique designs and constructions, is illustrated for the example of a machine employed for grinding. If new grinding processes or new sizes of construction are implemented, only a rough estimate of the operational loads is usually possible. A description of the grinding processes may be possible for idealised conditions, but the mutual interactions involved in a complex process sequence are open to question (mills, shredders). In this case, measurements can be performed during the start of operation with these machine plants. The measured values may be forces on a bearing or moments in the drive train, as well as local strains at points where high stresses are expected. The local strain measurements can result in one of the following conclusions:

- The local stress at this point is not critical.
- The local stress at this point is critical. Appropriate measures must be implemented immediately for reducing the local stress.
- The local stress at this point is a borderline case. Inspections must be performed at regular intervals, or a longer-term modification is necessary, or both.

In the event of changes in use or application, the manner in which these changes affect the operational stresses and strains must be examined. For example, damage is known to have occurred on long-distance aircraft in cases where they were employed for short distances and thus much more frequently subjected to severe ground-air-ground load alternations.

In machine plants as well as in aircraft it is often desirable to know the instantaneous operating condition as exactly as possible. The effect on the fatigue life, that is, the “consumption” of fatigue life, as well as the remaining fatigue life must be determined. As a matter of principle, it is possible to perform a cycle counting on-line by continuous measurement of the operational stress and to estimate the “consumption” of fatigue life by means of an analytical fatigue-life estimate for specific critical points. Instead of the operational stresses, indirect parameters such as the rotational speed and temperature (jet engines, turbines) or centre-of-gravity acceleration (aircraft) can also be employed. For general applications, however, for instance in drive-systems engineering, a procedure of this kind is still an exception. An advantage of this approach is that inspections, maintenance, repairs, or the replacement of components can be performed as required by the condition of the system or component concerned. That is, the intervals for these measures are not based on the elapsed time of operation or some other parameter (throughput, distance travelled). Furthermore, the operational stress, and thus possibly also the cause, would be known in the event of damage. It is not in general possible for the operator of a machine to derive a relationship between the

parameters with which the machine is being operated and the type of pay load as well as the loads which result. By means of load measurements, the situation would become transparent and capable of being influenced. The operator could thus participate in the process of ensuring safety. It may even be possible to recognise damage which is developing in a machine. These concepts are applied especially where stringent demands are imposed on the availability of machines and machine plants.

In the past, several attempts were made to employ so-called fatigue gauges for measuring the consumption of fatigue life. This was accomplished by measuring the variation of the electrical resistance (zero-point shift) or the variation of the surface roughness (variation of the optical reflection) caused by fatigue stress. In the laboratory, relationships were demonstrated between the number of load cycles, the amplitudes, and these specific variations, and this in turn resulted in exaggerated expectations: *“The first practical method which promises accurate monitoring of the fatigue damage experienced under actual service conditions”* [Hart65]. No long-term applications have ever developed for these sensors.

Of course, knowledge of the fatigue-life consumption is of strategic interest if a very large number of similar systems exist and are subject to different conditions of application. Thus, inspections which require high expenditures can be performed on the specimens that are operated under the most severe conditions of application. Procedures of this kind are known from the operation of jet engines and aircraft [Keen59, Akte99].

As a rule, machine plants are systems capable of oscillation. As a result of fatigue damage, especially in the case of crack initiation and growth, the oscillatory behaviour can change because of variations in stiffness. By continuous measurement of specific variables, the condition of the system can thus be monitored, see for instance [Lang10, Cegl10]. Condition monitoring usually offers advantages for manufacturers, operators, as well as insurers.

As shown by damage statistics, for instance in [Lang87], damage is frequently due to faulty or inadequate maintenance and repair work (faulty welding, alignment, or assembly). In many cases of damage, including disastrous accidents, the damage often begins in areas which tend to be regarded as marginal as far as the load on the overall construction is concerned. An example of this kind is the damage which occurred on the drilling platform, Alexander L. Kielland, which sank in 1980. In this case, the crack propagated from a defective weld in a connecting piece on a cross beam [Hobb83]. Of course, technical safety must receive the highest priority. Nevertheless, cases of damage have always been associated with technology, as already demonstrated, and no one will ever succeed in completely preventing this in the future either. In fact, some experts even predict an increase in the frequency and severity of damages as well as more serious consequences of such damage [Perr87, Jisc05].

This topic is much too comprehensive and involves far too many aspects for receiving an adequate treatment in a technical textbook. Learning from experience, assessing new developments carefully and sensibly, improving scientific methods and the series of technical rules, increasing the safety and reliability of

manufacturing processes, continuing development of the analytical and experimental strength assessment: all of these processes must continue in the future. Desirable objectives include a more rational presentation to, and perception by the public. As a matter of principle, a certain residual risk must always be accepted for enjoying the advantages of technical progress. Recognition of this fact is an absolute prerequisite for this purpose. The extent of the measures which must be implemented for minimising this risk always depends on the possible consequences of the damage which results from failure of the technical system concerned.

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# Chapter 12

## Load Assumptions in Various Special Fields

As already pointed out in the preceding chapters, the procedures applied for making load assumptions differ from one specialised field to another in correspondence with the specific boundary conditions involved. Examples include such fields as automotive construction, machine and plant designing, as well as high-volume manufacture and individual constructions. In the following chapter, the load assumption in the series of technical rules is first considered as an example. The procedure applied for deriving a load assumption in the absence of such technical rules is illustrated for the example of automobile construction in Sect. 12.2. Further important publications on load assumptions in various specialised fields are indicated in Chap. 13.

### 12.1 Load Assumption in Technical Rules and Standards

Load assumptions in the form of load spectra are rarely indicated in the series of technical rules. This is hardly surprising, since the conditions under which equipment, machines, machine plants, and means of transport are operated differ greatly [Zenn89]. In the case of machine elements such as clutches and transmission gears, agreements concerning the loads which occur during the design life must be reached between the operator and manufacturer. Cases of damage are often the result of a failure to reach an adequate agreement. Moreover, the use or operation of a machine plant or means of transport can change with time, and the stress on the components can thus vary. For some technical applications, furthermore, the loads, which occur during operation, are still not sufficiently well known, for instance, in sporting equipment and recreation products.

On the other hand, a comprehensive series of technical rules is available with a detailed description of the fatigue-strength analysis for specific fields of technology. This analysis allows reliable designing and dimensioning for ensuring the necessary

fatigue strength under variable stress amplitude. Chapter 13 includes a list of references to publications on load assumptions in various specialised fields.

## 12.2 Examples in Various Fields

In the following a number of examples for load assumptions in the engineering fields of cranes, rail vehicles, bicycles and automobiles are described.

### 12.2.1 Example: Cranes

The standard procedure for designing and dimensioning of cranes is described in DIN 15018, which has been applied for decades [DIN15018a, DIN15018b]. The load assumption for the certification of fatigue strength under variable stress amplitude is briefly explained in the following:

- Four load spectra, which are designated as very light, light, moderately heavy, and heavy, are taken as basis. The spectrum designated as “very light” is an idealised normal distribution. The other three spectra are so-called p-value spectra, see Sect. 8.2. The p-value describes the degree of fullness for the spectrum and varies from zero for very light crane operation, through 1/3 for light, 2/3 for moderately heavy, and 1 for heavy crane operation. For a p-value  $> 0$ , the amplitude does not decrease to zero upon attainment of the maximal cumulative frequency. The spectrum designated as “heavy” is a rectangular spectrum, that is, the hardest possible (constant-amplitude stress). In the plot of the spectrum, normalised values are given for the load amplitude (ordinate) as well as for the cumulative frequency (abscissa).
- The other decisive parameter is the assumed load-cycle range. For this purpose, four cases are distinguished:
  - occasional use at irregular intervals with long idle periods
  - regular use with interrupted operation
  - regular use with sustained operation
  - regular use with sustained heavy-duty operation

The assumption of four load spectra and four load-cycle ranges results in 16 stress groups. However, this number is reduced to six, for instance, by assigning the cases of “heavy load spectrum with use at regular intervals and interrupted operation”, “moderately heavy load spectrum with use at regular intervals for sustained operation”, and “light load spectrum with use at regular intervals for heavy-duty operation” to the same stress group.

### 12.2.2 Example: Rail Vehicles

#### General

In Central Europe, dimensioning of rail vehicles is performed in conformance with European standards (EN). From a historical standpoint, these standards are the result of continuing development and harmonising of national standards and documents of the International Railway Union UIC (Union Internationale des Chemins de Fer). By way of the UIC, the experience and requirements of the Railway Undertakers (RU) have been incorporated into the series of technical rules which are applicable at present.

The European standards are reviewed and revised at regular intervals by European standardisation committees with the participation of the rail vehicle industry and the RU. Thus the standards are continually adapted and updated in correspondence with new requirements and knowledge, as well as improved verification methods.

For the approval of rail vehicles, numerous proofs must be furnished for the individual components as well as for the complete vehicle and submitted to the approval authorities (such as the German Eisenbahn Bundesamt). For this purpose, the fatigue-strength analyses of the load-bearing components of the vehicle constitute an essential part of these proofs.

By way of departure from the automobile sector, for instance, the construction of test specimens or prototypes is not usual in the case of rail vehicles. Consequently, the load assumption for dimensioning must be derived without the stepwise validation of the loads by measurements on prototypes during the development process. For the load assumption, therefore, the objective is to indicate simple analytical procedures which are valid for all rail vehicles, but which still take all decisive parameters into account.

In the past, numerous measurements have been performed for checking the load assumptions in the series of technical rules. Especially changes in the conventional operating conditions, such as an increase in the speed of rail vehicles to more than 300 km/h, have resulted in the necessity of performing validation tests. Essentially, the load assumptions in the series of technical rules have been confirmed by these measurements. If necessary, however, the series of technical rules have also been altered appropriately. An example of such alterations is the modification of the load assumptions for vehicles with inclination technology, for which the transverse accelerations to be considered have been increased.

In the following, the load assumptions specified in the European standards for individual components of rail vehicles are described:

#### DIN EN

1. [EN13979]: 2011 Monobloc Wheels
2. [EN13104]: 2013 / 13103:2012 Powered and Non Powered Axles
3. [EN13749]: 2011 Bogie frames
4. [EN12663a]: 2014 Vehicle Bodies
5. [EN12663b]: Bahnanwendungen—Festigkeitsanforderungen an Wagenkästen von Schienenfahrzeugen—Teil 2: Güterwagen (2010)

## Determination of the Vehicle Weight

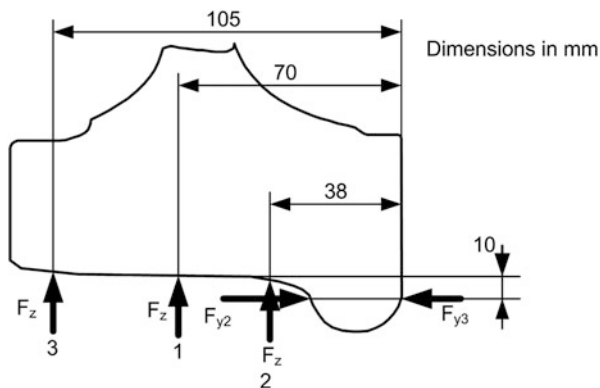
The essential input parameter for load assumptions in all of the aforementioned series of technical rules is the vehicle weight or vehicle mass, in addition to the geometrical boundary conditions. If the vehicle mass is known, the specifications given in the standard can be applied for determining the static forces which act on the components under investigation.

For locomotives, the vehicle mass is unambiguously defined and simple to determine. In the case of freight cars, the sum of the dead weight and payload is clearly given in advance. For passenger cars, however, the procedures and assumptions specified in the respective series of technical rules differ. In the various European standards, for instance, different payloads (passengers, baggage, accompanying personnel) are assumed to be transported in the vehicles. If the calculations performed for designing and dimensioning the various components of a vehicle are compared, a conspicuous feature is the fact that the payload (number of passengers) may differ for the respective component under consideration.

Endeavours by the standardisation committees to achieve standardisation in this respect resulted in the publication of EN 15663, “Railway Applications—Definition of Vehicle Masses” in 2009. The purpose of this standard is to ensure that the vehicle mass is determined identically for designing and dimensioning of each component concerned. However, the specification of different safety factors in the payload dimensioning for the individual standards presents problems in this respect. For some components, standardisation of the payload dimensioning would decisively affect the load assumption for designing and dimensioning of these components. Alteration of the dimensioning rules for the individual components would require considerable effort and expense for testing because of the associated shift in the safety requirements. Consequently, harmonisation cannot be expected for the short term. In any event, dimensioning in conformance with European standards must always be considered as a closed overall system, for which the load assumptions sometimes include safety factors which are not explicitly indicated.

The distinction with respect to seating and standing accommodations as well as luggage areas in determining the payload is consistent for all European standards. As a rule, one passenger per seat and two passengers per  $\text{m}^2$  are assumed for the standing area in main-line rail vehicles. For local commuter service, up to five passengers per  $\text{m}^2$  are assumed for the standing area in the vehicles. For streetcars, even higher numbers of occupants are sometimes assumed for dimensioning. A few operators impose more stringent requirements on the possible payload, if pertinent information is available to them. However, a uniform European specification is difficult to achieve in view of the considerable differences in the permissible numbers of seated and standing passengers in various individual European countries. Accommodation of the highest possible numbers of passengers in all rail vehicles in operation in the European Union would decidedly result in overdimensioning in many cases.

**Fig. 12.1** Points for the introduction of the forces for dimensioning of solid wheel discs



### Determination of the Loads

After determination and dimensioning of the mass from the dead weight of the vehicle and the payload, the loads on the structural components are determined for performing the fatigue-strength analysis. For this purpose, the loads are calculated by applying the procedures specified in the European standards. Thus, the dynamic forces acting on a structural component are determined from the quasistatic forces with the use of dynamic factors (such as the vertical impact factor, transverse acceleration factor, rocking factor).

For non-rotating vehicle components such as bogies and superstructure, a static strength analysis as well as a fatigue-strength analysis must be performed. For the static strength analysis, exceptional load cases which seldom occur are defined with the proviso that plastic deformation is not permissible. Exceeding of the fatigue strength is permissible in this case. In addition, a fatigue-strength analysis is necessary. For this purpose, the loads and stresses which occur during operation (operational loads or fatigue loads) must be taken as basis.

In the case of rotating components such as wheels and wheel-set axles, the fatigue-strength analysis is the decisive method for dimensioning because of the large number of load cycles during the design life of the components. Additionally static loads have to be considered in exceptional load cases.

### Examples of Load Assumptions

As an example, the determination of the stresses for dimensioning of solid wheel discs in conformance with EN 13979 is described in the following.

The forces at the wheel-contact point which must be considered for dimensioning of solid wheels are plotted in Fig. 12.1 [EN13979]. In this representation, the force  $P$  is the static wheel load with a full payload as specified in [EN13103] (dimensioning of wheel-set axles).

Three cases of loading are distinguished, Table 12.1.



**Table 12.1** Forces and cases of loading for dimensioning of solid wheels

Load case	Designation	$F_z$ (vertical force)	$F_y$ (horizontal force)
1	Straight track	$F_z = 1.25 P$	$F_{y1} = 0$
2	Curved track	$F_z = 1.25 P$	$F_{y2} = 0.6 \sim 0.7 P$
3	Switch point	$F_z = 1.25 P$	$F_{y2} = 0.36 \sim 0.42 P$ $F_{y3} = 0.6$

With the forces from the three load cases, FEM calculations are performed for the mechanical fatigue-strength analysis for the entire wheel.

Besides the three standard load cases, additional forces are also considered as functions of the operating conditions for the vehicle, for instance, centrifugal forces on the wheels of high-speed vehicles or thermal effects on wheels with block brakes.

Additional load cases are specified for axles and bogies. For this purpose, braking and driving forces are also considered in addition to the forces which occur during travel through curves and switch points. The order of occurrence and the frequency of occurrence are not taken into account by the assumptions in this case, since the consideration of these factors would unduly increase the complexity of the analyses because of excessive emphasis on the frequencies of the respective load combinations. As a result, it would not be possible to ensure restriction-free operation throughout Europe. For this reason, the assumptions are conservative. For example, permanent braking with full braking force and simultaneous travel through a curve with full transverse acceleration are thus assumed for the case of the braking load on a wheel-set axle.

For attachments installed on a bogie or on the vehicle superstructure, for instance, air-conditioning equipment or transformers, the load assumption can be derived with the use of acceleration values for the respective plane of installation in the vertical, horizontal, and transverse directions. If the acceleration values under consideration are multiplied by the mass of the attachment, the assumed forces can be calculated and taken into account for designing and dimensioning.

## Conclusions

The load assumptions specified in the European standards for dimensioning of rail vehicles have proved to be very useful in the past. These assumptions constitute a simple instrument for determining the expected stresses in a structural component on the basis of load cases, dynamic factors, as well as computational information, and thus ensuring adequate dimensioning. These values have been determined by a tedious, time-consuming process, for which adverse experience from the past has also been taken into account. The load assumptions specified in the series of technical rules should not be considered as isolated entities. That is, they yield a closed analytical system of proof for the respective component only in combination with the calculations, fatigue-strength tests, and safety concepts. Under some circumstances, variations at individual points of the analytical concept can

considerably affect the dimensioning and thus the safety of the component. Under varying conditions of operation, such as an increase in speed, in mileage, or in the number of passengers, the input parameters for designing and dimensioning must be validated.

### 12.2.3 Example: Bicycles

For designing and dimensioning of bicycles, European standards on safety requirements and testing methods have existed for some time:

- [EN14764]: 2005 City and trekking bikes
- [EN14765]: 2005 Children's bicycles
- [EN14766]: 2005 All-terrain bicycles (mountain bikes)
- [EN14781]: 2005 Racing bikes

In view of the fact that there are more bicycles than automobiles and that the bicyclist is almost completely unprotected in the event of a technical failure, the significance and usefulness of such standards is evident. Since bicycles and bicycle components are manufactured and sold all over the world, these European standards are of paramount importance.

These standards include comprehensive requirements on design, construction, assembly, and safety aspects. Static testing, impact testing, and fatigue testing are necessary for determining the fatigue strength under variable stress amplitude. Fatigue testing is specified for the handlebar and front subassembly, frame, front-wheel fork, pedal and pedal shaft, drive train, as well as the seat and seat support. For example, the pedal is separately tested under the action of pedalling, horizontal, and vertical forces. The oscillation range and mean value of the force as well as the required allowable number of load cycles are specified in the standard. That is, only minimal fatigue lives are required on the basis of component-fatigue tests at constant amplitude. The load specifications are derived from measurements under operational conditions and experience gained from operation.

In comparison with operational fatigue-strength testing in other fields, for instance, in the automotive industry, this test is a very simple matter. However, it is quite obvious that an industrial testing procedure which has been in practice for decades cannot be simply applied to a different sector such as sporting gear or recreational and leisure-time products. The organisational structure of the bicycle manufacturers (many small and medium-sized enterprises), the scale of production (often in small quantities), the manufacturing processes (predominantly the assembly of components from suppliers) and the market situation (low-price segments, specific customer wishes) differ considerably from the corresponding factors for other products.

The existing standards can be regarded as a basic safeguard. For more stringent requirements, operational fatigue-strength tests with variable stress amplitude are already being performed at various points on components and on the complete bicycle, see for instance [Groß07, Issl07, Kien07, Blüm08].

A characteristic feature for bicycles is the extremely wide range of scatter for the stress from customer to customer, for various riding habits, terrain, and distance travelled. Thus, the following types of terrain can be assumed for mountain bikes, for example, and completely different spectra will be obtained in correspondence with their frequency of occurrence: Uphill, downhill, paving setts and cobblestones, sidewalk edges, crushed stone roads, rocky ground, single trail, jumps, impact, and full brake application [Iss107].

A series of technical rules is never finished, because continuing developments are always in progress. Thus, experience must first be gained with the existing series of rules. For the continuing development of these rules, the following aspects should be considered [Hans10]:

- How can the load assumption be improved by further measurements under operational conditions, and how can the weighting of the individual operating conditions be improved by further statistical investigations?
- To what extent can the constant-amplitude fatigue test be replaced by the fatigue-strength test under variable stress amplitude in the future?
- How can the behaviour of the complete bicycle be assessed more exactly?
- How should special events and multiaxial loads be assessed?
- To what extent should corrosive ambient conditions be considered?
- What is the effect of the progressively more frequent material mix with components of carbon-fibre-reinforced plastic on the stresses and on the loading capacity?
- How can customer expectations concerning design life, terrain, and cycling habits be more concretely described than in the past?

### ***12.2.4 Example: Automobile***

The specification of designing and dimensioning loads for the fatigue-strength analysis in the automotive industry is not defined in a generally applicable manner in series of technical rules. As specified for the condition of a motor vehicle in the Ordinance for Approval in Road Traffic (for example the German Road Traffic Licensing Regulations StVZO) the normal operation of a motor vehicle in road traffic shall not result in injury to anyone or in any safety hazard which exceeds the unavoidable residual risk [Brae05]. Thus, the manufacturer must ensure that the legal requirements are satisfied, for instance, with respect to product liability, with the application of appropriate processes for designing and dimensioning [Bund70]. Furthermore, he must ensure the achievement of company objectives, such as fulfilment of the customer's wish and expectation for quality in the long term. Although no universal, standard methods are available for deriving load assumptions throughout the automotive industry, comparable standards have developed in correspondence with "best practice". Hence, the applied methods and principles are indicated in a general manner in the following sections, but without describing the designing and dimensioning process of any particular automobile manufacturer in detail.

As a rule, the design of a structural component results from a multitude of requirements on the components and systems of a motor vehicle. Some of these requirements may be quite difficult to satisfy simultaneously. Besides the demands on the strength for withstanding loads caused by operation, special events and misuse, requirements are also imposed on the passive safety performance and stiffness characteristics, among other items. The satisfaction of these demands, in turn, is vital for achieving objectives associated with vehicle-dynamic properties, acoustics, or driving comfort.

A further feature of the product-development process in the automotive industry is the repeated passage through development loops, in which virtual or physical prototypes are designed and tested. The objective of this procedure is to verify properties and to confirm or optimise assumptions, including load assumptions. During the early phase of development, the load assumptions are often based on the results of measurements on previously produced motor vehicles and physical models. These results are modified to take into account the deviations of the motor vehicle under development from its predecessor, for instance, with respect to maximal axle weight, wheel base, track, or centre of gravity. These load assumptions are usually referred to the maximal forces which occur at the wheel. At this initial stage, the load assumptions are not intended primarily for dimensioning of structural components. Instead, the main purpose of these assumptions is to provide support for conceptual decisions on matters such as the selection of the axle design, the specification of load paths, or the use of modular components. At a later stage, numerical multibody simulation models are applied. These models take all structural components which transmit forces into account and simulate the elastokinematic behaviour of the axles. With the use of these models, the loads on the structural components can be determined for constant driving manoeuvres, such as travel through curves, braking, and acceleration on a level road, as well as for dynamic driving manoeuvres, such as driving on bumpy roads. The loads thus determined constitute the basis for designing and dimensioning of the structural components, provided that sufficient experience is available.

During the development process, operational load measurements and operational tests are performed with motor-vehicle prototypes for confirming the load assumptions and for determining the fatigue strength of the structural components.

### **Operational Loads**

Operational loads include the use of the motor vehicle for the intended purposes. All loads which occur during normal driving operation are included in this category. All differences in customer behaviour must be considered in this context, from the inexperienced or hesitant driver all the way to the sports-car enthusiast. Furthermore, many different operational scenarios or situations must be taken into account with the intended use of a motor vehicle, from predominantly urban traffic (for instance, taxi operation) all the way to long-distance driving with a large share of motorway driving. For the assumption or determination of loads, especially the

payload in the motor vehicle (maximal axle weight, centre of gravity) the wheel dimensions (unsprung masses) and special operating conditions, such as trailer operation or uphill driving, must be considered.

As a rule, the forces which occur can be attributed to defined driving manoeuvres, such as braking, acceleration, or travel through curves. For constant driving manoeuvres on a level road, the resultant forces which occur on a wheel in the longitudinal, transverse, and vertical directions, as well as the couples of forces which act on the wheel, can be determined by means of the so-called basic dynamic equations for driving operation [Mits04, Brae05]. For positive frictional transmission of forces at the wheel-contact point between wheel and road, the relationship between the forces in the longitudinal ( $F_x$ ) and transverse directions ( $F_y$ ) is given by the following equation [Kamm36]:

$$F_z \cdot \mu \geq \sqrt{F_x^2 + F_y^2} \text{ with } \mu : \text{friction coefficient} \tag{12.1}$$

The forces are calculated from individual components of force, for instance, in the vertical direction ( $F_z$ ), from the static wheel load, the dynamic displacement of the maximal axle weight during acceleration or travel through curves, as well as further effective forces, such as aerodynamic resistance and buoyant forces, if appropriate, Fig. 12.2 [Brae05].

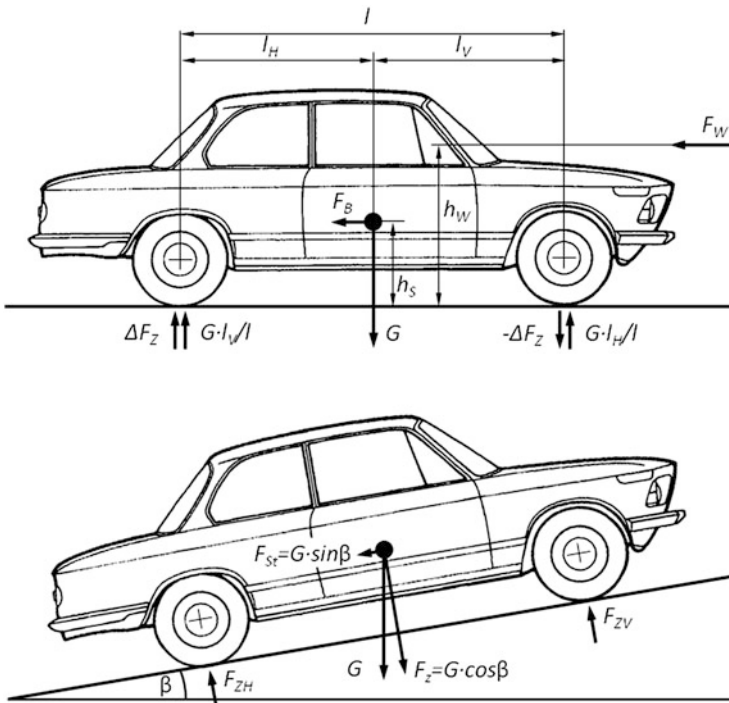


Fig. 12.2 Static and dynamic axle loads during travel on a level course and on an upward slope

The following is obtained for the front axle:

$$F_z = F_{z,stat} + \Delta F_{z,brake} + \Delta F_{z,curve} \pm \Delta F_{z,resistance} \quad (12.2)$$

Static wheel-contact force:

$$F_{z,stat} = G \cdot \left( \frac{l_H}{l} \cdot \cos \beta - \frac{h_S}{l} \cdot \sin \beta \right) \text{ with } G = m \cdot g \quad (12.3)$$

Dynamic axle load during braking:

$$\Delta F_{z,brake} = m \cdot a_x \cdot \frac{h_S}{l} \quad (12.4)$$

Dynamic axle load on a curve:

$$\Delta F_{z,curve} = m \cdot a_y \cdot \frac{h_S}{s} \quad (12.5)$$

Resistance:

$$\Delta F_{z,resistance} = F_w \cdot \frac{h_w}{s} \quad (12.6)$$

with

$a_x$ : longitudinal acceleration

$a_y$ : transverse acceleration

$F_w$ : air resistance

$G$ : weight of the motor vehicle

$h_S$ : height of the centre of gravity

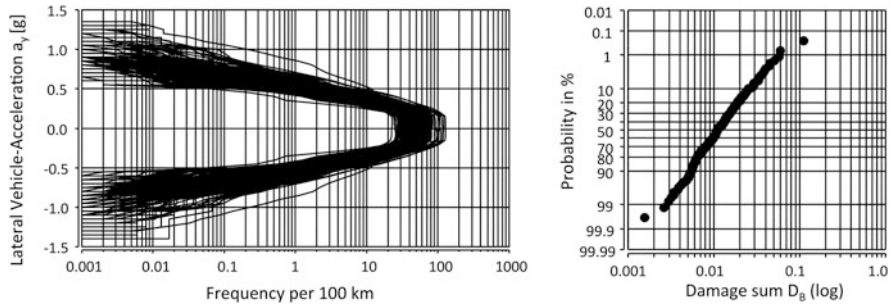
$l$ : wheel base

$m$ : mass of the motor vehicle

$\beta$ : angle of road inclination

Dynamic effects, such as those which result from road-induced excitation, can be taken into account by dynamic factors. In correspondence with the driving manoeuvre, the vehicle parameters, and the characteristics of the excitation on the road, the dynamic factor can exceed the load during a stationary driving manoeuvre by a multiple of 3.

Besides the load amplitude, the frequency of the loads is also decisive for the specification of load spectra for designing and dimensioning. In this context,



**Fig. 12.3** Spectrum for transverse acceleration of a motor vehicle from 250 vehicle measurements and a statistical analysis of the characteristic stress parameters

automobile manufacturers have gained experience over a period of many years. Furthermore, systematic investigations are performed on a random-sampling basis for determining the stresses on motor vehicles during operation by customers. For this purpose, the investigations are conducted by mutual agreement with selected customers [Hors02]. The load spectra are determined for the various load components on each motor vehicle or for each customer. By means of a damage calculation in conformance with the Palmgren-Miner rule, these load spectra are then reduced to the characteristic scalar damage parameter (calculated damage  $D_B$ ). For this purpose, the spectra are plotted against a reference S-N curve which is independent of the structural component, without consideration of a fatigue limit (Palmgren-Miner elementary). The characteristic damage parameters exhibit pronounced scatter, which depends on the specific driving habits of the customer, and must therefore be regarded as statistically distributed values. By means of the statistical characteristic parameters thus determined, probabilities of occurrence can be associated with the individual spectra. This result indicates that the characteristic scalar damage parameters  $D_B$  approximate a log-normal distribution, Fig. 12.3 [Pött11].

As a rule, a distinction is made in correspondence with the characteristic operating conditions for a motor vehicle: driving on urban streets, country roads and highways, or motorways. Moreover, bad road conditions and, if appropriate, uphill driving and trailer operation are considered. The characteristic load parameters are determined for the respective operating condition.

The mix of road conditions, that is, the share of each characteristic operating condition in the total distance travelled during the lifetime of each motor vehicle, also exhibits pronounced scatter. This share can be determined by statistical evaluation of street- and road-utilisation data or by comprehensive field observations.

A decisive factor for ensuring the required fatigue strength under variable stress amplitude is the specification of the design life or total distance of travel to be taken

as basis for the motor vehicle. Statistics on the age distribution of motor vehicles, such as those compiled by the approval authorities, can provide the necessary information for this purpose.

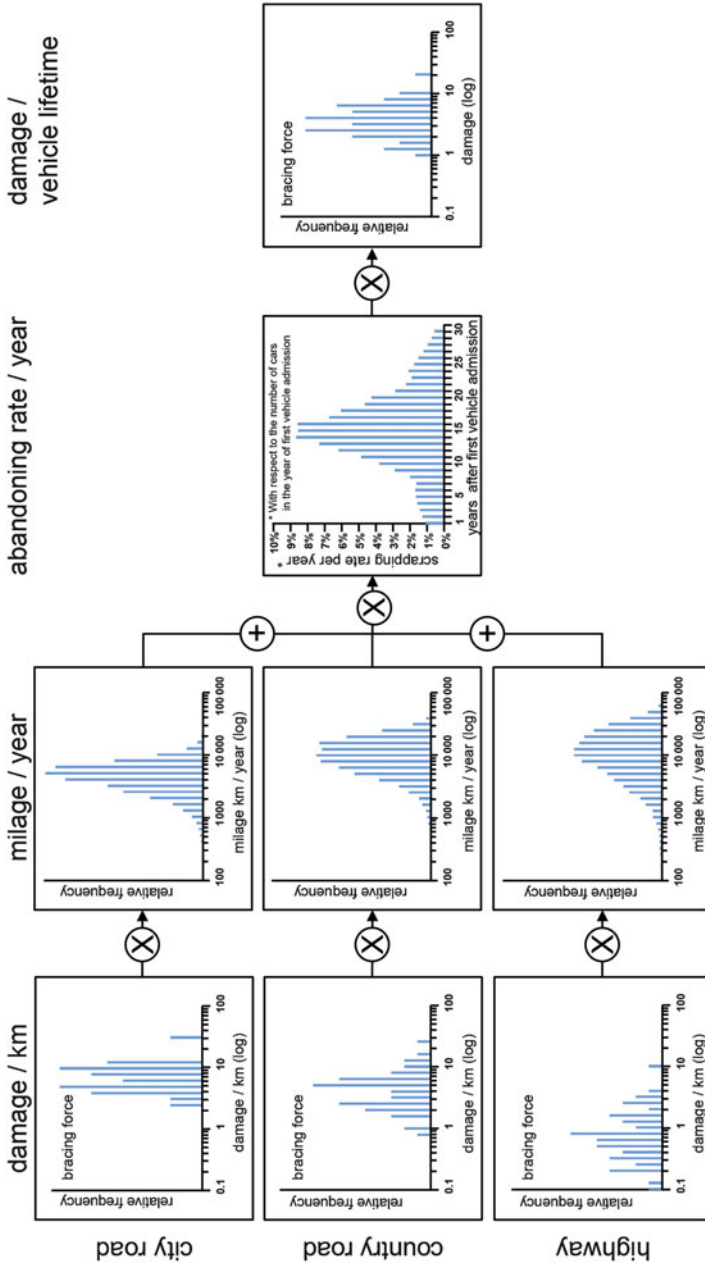
In Fig. 12.4, the determination of the frequency distribution of characteristic load parameters for the design life of a motor vehicle is illustrated for the example of one load component [Pött11]. For instance, this distribution can be obtained by convolution (multiplication of the frequency distribution) for the damage per kilometer of customer operation for each operating condition (type of road) by the annual distance travelled per operating condition and by the expected design life (from the abandoning rate). The scatter of the load on motor vehicles during operation by customers is caused by differences in driving habits, the mix of road conditions, and distance travelled, and is represented by the frequency distribution of the characteristic load parameter.

In this context, it should be noted that the statistical distributions plotted in Fig. 12.4 are not mutually independent. Thus, a sports-car enthusiast will subject a motor vehicle to high transverse acceleration in urban traffic as well as on a rural highway or on the motorway. Furthermore, a correlation exists between the annual distances travelled and the mix of road types and conditions. The expected age of a motor vehicle or the total distance travelled with the vehicle until scrapping depends on the utilisation profile. A further point which must be considered is the fact that the owner of a motor vehicle can change several times during the design life, and that the utilisation profile can change correspondingly. For the purpose of safe and reliable designing and dimensioning, therefore, a certain value is usually specified for the total distance travelled with a motor vehicle (for instance 300,000 km), and a certain variation of the route mix is also defined.

As referred to an individual load component or structural component, the characteristic load parameter depends on the type of route (urban street, country road, highway, or motorway) and can therefore differ between various structural components. Thus, for the example of a wheel or wheel bearing, the value of the characteristic stress parameter is high during travel on a country road because of lateral forces in combination with the higher speed (roll-over frequency). For the rotating components of the drive train (bearings, gear teeth), motorway driving is an essential criterion for designing and dimensioning.

Load spectra from measurements on customer vehicles are not usually employed for dimensioning and verification. On the one hand, the load spectrum for an individual customer vehicle never represents the reference stress for all motor-vehicle components, as already described. On the other hand, testing of motor vehicles during operation by the customer would necessitate measurements over a distance of several hundred thousand kilometres. The duration of such an investigation would be much too long in comparison with the short development periods, and an approach of this kind is therefore not realistic. Consequently, the effective duration of operational testing must be appropriately shortened by omission of all driving conditions which are not relevant to the actual damage. Hence, test programs are accomplished on special test courses consisting of bad road sections and on racing courses with forced driving manoeuvres and individual events. This





Calculation performed per:

- load direction ( $F_x, F_y, F_z, \dots$ )
- type of car (e.g. sedan, sports car, SUV, convertible, minivan)
- market area (regional, worldwide)
- special-purpose vehicle (e.g. taxi, courier service)

**Fig. 12.4** Exemplary determination of the frequency distribution of characteristic load parameters from measurements performed during operation by customers, annual distance travelled, and scrapping rate for motor vehicles

procedure ensures safe, reliable designing and dimensioning of all critical structural components for achieving the required fatigue strength under variable stress amplitude.

As shown in Sect. 11.3, the reliability concept is based on the statistical distribution of the characteristic stress parameters during operation by customers and the statistical distribution of the tolerable characteristic stress parameters for a structural component, that is, for the scatter of the fatigue strength for the structural component [Heul03]. In Sect. 11.4, guideline values are given for the standard deviation or scatter range of the fatigue strength for various structural-component categories (materials, manufacturing processes, joining methods), see also [Aden01, Haib02]. In correspondence with the results presented in Tables 11.2–11.5, an average standard deviation of  $s_{50,N} = 0.2$  or a scatter range of  $T \approx 1:3$  can be assumed, as a rule of thumb.

A failure of a component of a customer's motor vehicle could only result from fatigue if the fatigue strength of a structural component in this vehicle is lower than the stress on this component, that is, if a "bad" structural component happened to have been installed in a vehicle subject to high stresses. For probability-density functions with a log-normal distribution, the probability of occurrence can be determined with the use of Eq. (11.2). Hence, designing and dimensioning can be regarded as technically safe and reliable if the required mean fatigue strength of a structural component is sufficiently higher than the stress during operation by the customer, with the known scatter taken into account. In order to achieve this objective, the structural component is often required to tolerate the stress corresponding to the 1% quantile during customer operation with a total probability of survival of 99.9%. In this context, the expression "1% customer" is often employed as a simplification, see [Grub73].

As already mentioned in the preceding section, the concept of the 1% driver and the 1%-customer is repeatedly encountered in connection with the design and dimensioning of structural components. The term was coined more than four decades ago for designating a customer whose driving habits result in severe stresses [Grub73]. This approach has frequently been a subject of discussion because of the numerous new questions which were always associated with the tremendous increase in the volume of test data and with the processing of these data. In the following section, these questions will be considered in more detail.

If the stresses on a motor vehicle are measured in the x-, y- and z directions on a specified test course for a sufficiently large number of drivers, different load-time functions will result for each individual driver. If the respective load spectra are considered, the maximal amplitude  $\hat{S}_a$ , the cumulative frequency  $H$ , and the shape of the spectrum will differ. If the damage sums  $D_i$  are calculated under various assumptions and specifications (damage accumulation hypothesis, S-N curve, test course, driving habits, ...) considerable scatter will be observed. If the result is plotted on a probability grid,  $P = f(\log D_i)$ , the variation can usually be approximated by a straight line, see for example Figs. 11.13 and 12.3. That is, not only distribution-independent quantities, such as the mean value and standard deviation

can be calculated. Extrapolation or interpolation is also possible, and the damage sum can be determined analytically for the 1% driver. The value thus obtained is restricted to the load alone. Reservations have repeatedly been expressed with respect to the concept of the 1% driver. These objections include the following:

The 1% customer for longitudinal forces is not necessarily the 1% customer for lateral forces. If a 1% customer is defined for designing and dimensioning, however, very high stresses are required in all three directions. Certain boundary conditions must be taken into consideration in a representative manner. These conditions include factors such as driving habits, payload, road conditions, weather conditions, etc., and may depend on the location. In the case of special events, in particular, the occurrence of an extremely severe stress which is not taken into account by the selected distribution function cannot be excluded. Thus, it may be concluded that a representative load assumption requires a synthetic procedure which takes numerous effects into consideration. These effects cannot be represented by a single driver and by a single specific test course. Furthermore, it is not always possible to demonstrate the statistical relevance for specific cases of loading. Consequently, the concept of the 1% customer is now often regarded as a simplification, as a slogan, or as an artificial driver, for instance. As a result of the global use of their motor vehicles, of the corresponding load measurements on site, and last but not least, the damages which have occurred, motor-vehicle manufacturers have at their disposal comprehensive information for reliable designing and dimensioning. The individual procedures may, of course, vary.

For this purpose, the test strategy must also be considered in designing an appropriate test program for dimensioning and verification. In correspondence with the methods of performing the tests and evaluating the results, fatigue-strength tests on a few components from a batch allow only conclusions on the expected mean value of the fatigue strength. Consequently, the characteristic load parameter for the test program must correspond at least to the required mean fatigue strength  $D_{F,50\%}$ . The reliability of the conclusion depends on the extent of random sampling. The mean value for the random sample is scattered around the true mean value for the basic population. Thus, there is a risk of overestimating the true mean value. If only a few tests are performed, this risk must be taken into account by an additional safety factor, which depends on the number of individual tests  $n$  and on the standard deviation of the fatigue strength  $s_F$ , which is assumed to be known. For a logarithmic-normal distribution of characteristic stress parameters, the safety factor can be calculated for a confidence probability of 90% [Haib06]:

$$j_{C,n} = 10^{\frac{1,28 \cdot s_F}{\sqrt{n}}} \quad (12.7)$$

As a rule of thumb for determining the characteristic load parameter, it may be assumed that the requirement imposed on the load-bearing capability of a structural component in a test program must exceed that imposed by the average customer by

a factor of about twenty. In view of this requirement, as well as the necessity of time compression for testing under fatigue stress, the extreme demands often imposed during test programs in the automotive industry are quite justified.

### Exceptional or Special Events

Exceptional or special events constitute the limits on the intended or correct use of a motor vehicle. Events of this kind seldom occur, but when they do, they must be withstood or tolerated without damage and without impairment of the planned design life. As a rule, the manufacturer defines a catalogue of pertinent driving manoeuvres. Such a list includes the essential special events in the longitudinal, transverse, and vertical directions. Examples include the following:

Vertical: Travel over obstacles (pot-holes, humps)

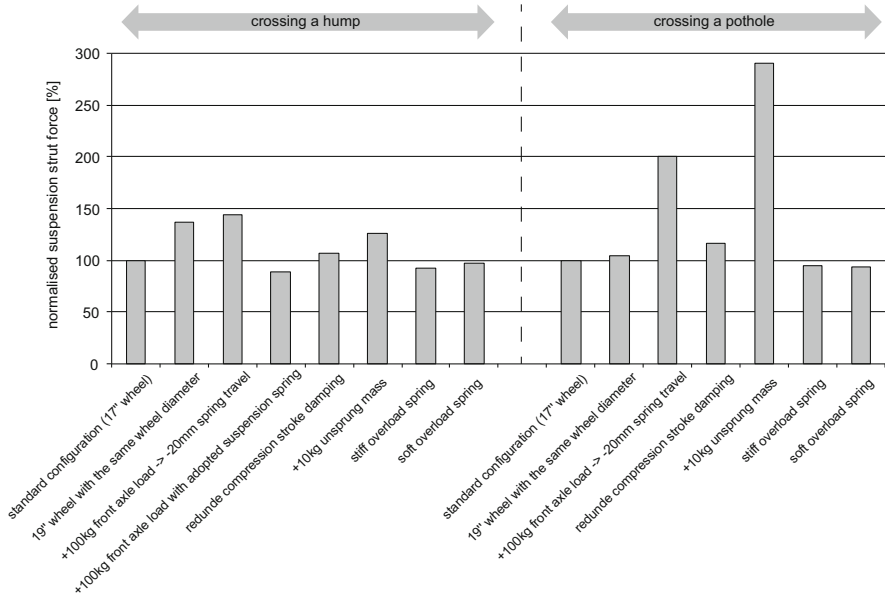
Longitudinal: Braking and acceleration over bumpy roads

Transverse: Controlled full-skidding turn

Further special events are of a component-specific nature, for instance, for the drive train (sudden start, jerky driving-school motion), the steering system (steering of a stationary vehicle, turning of a front wheel against a curbstone in order to push the vehicle away from the curbstone) or bumping against an obstacle with the towing hook, etc. Load assumptions for these events present problems because of the highly dynamic processes which are often involved in such cases. During the early phase of the development process, the load assumptions are usually based on older values measured on previous automobile production. Decisive parameter models are applied for deriving the load assumption. These models must describe the effect of the essential vehicle parameters for the respective load case qualitatively and, if possible, quantitatively. The results of experimental decisive parameter analyses are presented in various publications. For this purpose, decisive parameters such as the type of wheels and tyres, vehicle weight and payload, as well as tuning of springs or shock absorbers are usually varied, and the effect of the variation on the loads which result from special events is measured [Grab07, Zeic07, Häge09]. A corresponding evaluation is illustrated in Fig. 12.5, [Grab07]. In this example, the shock-absorber support force on the front axle during travel over humps and transverse channels is plotted as a function of the variation in the vehicle configuration (for instance, a change in the wheel diameter or in the load on the front axle). In this figure, the force is normalised with respect to the standard configuration of the motor vehicle.

As a rule, empirical investigations do not result in any conclusions which can be generalised or applied to more general cases, since the mutual interactions among the various decisive parameters are not taken into account. Conclusions of this kind apply only to comparable motor vehicles.

For this reason, numerical methods are applied in an attempt to derive load assumptions for special events. Highly dynamic events with high (elastic or plastic) deformation energy require calculations on the basis of the explicit finite-element



**Fig. 12.5** Experimental decisive-parameter analysis: effect of changes in configuration on the shock-absorber support force, as referred to the standard configuration

method. Such calculations are also applied for complete-vehicle crash models [Hauk03].

### Conclusions

The methods applied for dimensioning and verification of fatigue strength under variable stress amplitude in the automotive industry are intended to ensure optimised and reliable designing to meet requirements. The achievement of this objective is ensured primarily by the competition among manufacturers in providing highly reliable products for their customers, rather than by the use of standards or series of technical rules.

During the various phases of development, loads are determined, and frequencies are specified on the basis of experience, analytical estimates, as well as numerical simulation and measurement on test courses and during operation by customers. Besides the load assumptions and spectra, the scatter in the strength of structural components and the reliability of the verification concept are also considered in the test programs.

For ensuring the safety and reliability of their products, automobile manufacturers have gained experience over a period of more than 100 years. However, new markets and products will demand continuous examination and updating of requirements in the future, too. Cars propelled by an electric drive train and the

replacement of the driver by assisted and finally even automated driving will have an impact on load assumptions as well as on safety aspects in the near future.

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# Chapter 13

## Supplementary References to Publications on Load Assumptions in Various Specialised Fields

In two publications from 1858 and 1861, A. Wöhler already described measurements performed on railway axles under service conditions. By means of a self-developed compound lever system, the deflection of the axle was scratched on a zinc plate by a scribe. This was accomplished for a number of four-wheeled and six-wheeled freight and passenger cars on trips between Breslau (Wrocław) and Berlin and between Frankfurt an der Oder and Berlin. Only the maximum moments for bending and torsion per trip could be measured [[Wöhl1858](#), [Wöhl1860](#), [Engi1867](#), [Schü96](#), [Zenn15](#), [Zenn17](#)].

As indicated by W. Schütz [[Schü08](#)], the first measurements on load spectra were performed on springs by Batson and Bradley (1929) and on agricultural machines by Kloth and Stroppe (1932, 1936). Since that time, the techniques for measuring and evaluation have undergone a revolutionary development. The technical literature on the methods of performing measurements as well as on the results is very comprehensive, but the associated scatter is considerable.

In the following, publications in a variety of specialised fields are cited. Some of these articles are of historical interest, whereas others are the result of more recent research. New developments are described in many articles. Some of the publications themselves include comprehensive lists of references. The publications are listed in chronological order, but a separate list has been compiled for each particular field.

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## 13.12 Symposia and Conferences

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- 7th International Conference on Biaxial/Multiaxial Fatigue & Fracture, 7 ICBMFF, Berlin, 2004
- 1st Symposium on Structural Durability, Darmstadt, 2005
- International Conference Residual Fatigue Life and Life Extension of In-Service Structures, JIP, Paris, 2006
- 8th International Conference on Multiaxial Fatigue & Fracture, ICMFF8, Sheffield, 2007
- 2nd Symposium on Structural Durability, Darmstadt, 2008
- 6th International Conference on Low Cycle Fatigue LCF 6, Berlin, 2008
- 2nd International Conference on Material and Component Performance under Variable Amplitude Loading, Darmstadt, 2009
- 9th International Conference on Multiaxial Fatigue & Fracture, ICMFF9, Parma, 2009
- 18th European Conference on Fracture, Fracture of Materials and Structures from Micro to Macro Scale, Dresden, 2010
- 3rd Symposium on Structural Durability, Darmstadt, 2011
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